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1aEA3. Calculation of thermoacoustic functions with computational fluid dynamics**Simon Bühler*, Douglas Wilcox, Joris P. Oosterhuis and Theo H. Van der Meer**

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Thermoacoustic functions are important parameters of one-dimensional codes used for the design of thermoacoustic engines. The thermal and viscous thermoacoustic functions allow the inclusion of three dimensional effects in one-dimensional codes. These functions are especially important in the regenerator of a thermoacoustic engine, where the thermoacoustic heat pumping occurs. Even though analytical solutions were derived for uniform pores, the thermoacoustic functions for complex geometries such as stacked screen or random fiber regenerators cannot be calculated analytically. In order to gain more insight into the geometry induced complex flow fields, the procedure of Udea, et al. (2009) to estimate the thermoacoustic functions was applied in computational fluid-dynamic simulations. By using two measurement locations outside of the regenerator and modeling the regenerator as an array of uniform pores it is possible to estimate the thermoacoustic functions for complex geometries. Furthermore, a correction method is proposed to quantify the entrance effects at the beginning and end of a regular pore. The simulations are first validated for a uniform cylindrical pore with the help of the analytical solution. Then the correction method is successfully applied to a cylindrical pore with the results closely matching the analytical solution.

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INTRODUCTION

Thermoacoustic engines are of raising interest in recent years as a cost effective and reliable alternative to traditional Stirling engine. Thermoacoustic engines can achieve these good characteristics as the number of moving parts is reduced. The displacement and compression is not executed by displacer and power pistons, but by an acoustic wave thermally interacting with a solid in a regenerator. The thermoacoustic effect converts heat into acoustic power.

The thermoacoustic effect mainly occurs within one thermal penetration depth of the solid. Gas parcels further away from the solid undergo adiabatic wave propagation as the parcel is too far away from the solid to interact. As the heat pumping occurs within the thermal penetration depth, narrow pores are necessary in the regenerator to reach high power densities. However, the viscous effects in these narrow pores can lead to large losses. In order to reach a high efficiency the thermal and the viscous effects have to be balanced. For this optimization the thermal and viscous thermoacoustic functions, f_κ and f_ν , respectively [1], are used as they allow the inclusion of three-dimensional effects in one-dimensional thermoacoustic codes such as DELTAEC [2]. This code allows users to achieve a quick performance prediction of an entire engine. For some simple geometries the thermoacoustic functions can be calculated analytically [3].

The thermoacoustic functions cannot however be estimated analytically for geometries such as porous media or staggered screen regenerators. This is due to the complex flow field. Ueda *et al.* [4] therefore developed an experimental method to estimate the thermoacoustic functions of staggered screens. In this paper the method is validated for application in Computational Fluid Dynamics (CFD) simulations and extended to characterize the end-effects at the pore entrance and exit. This allows a reduction of the computational domain and thus the computation time.

In the first section of this paper the method of Ueda *et al.* [4] for experimental evaluation of the acoustic properties of staggered screen is adapted for CFD and extended to account for the end effects. Furthermore, the CFD model is presented. In the subsequent section the numerical results are presented and discussed.

METHOD

Four cylindrical pores are simulated in CFD to estimate their thermoacoustic functions. From the simulations the time dependent pressure and velocity at the inlet and the outlet are exported for post processing. The time dependent data is rewritten in complex notation to calculate the thermoacoustic functions with the method of Ueda *et al.* As the CFD simulation also accounts for end effects at the inlet and outlet of the pore, the end correction method, which uses two simulations with different pore lengths, is applied. The value without the end effects represents the value for an infinite pore and can then be compared with the analytical solution to validate the end correction method and the CFD model.

Evaluation of Thermoacoustic Functions

The propagation of a sound wave through the regenerator of a thermoacoustic engine can be modeled as wave propagation through a narrow tube, reducing the three dimensional Navier Stokes Equations to a set of two one dimensional equations [5]:

$$\frac{dp_1}{dx} = -\frac{i\omega\rho_m}{1-f_\nu} \frac{U_1}{A} \quad (1)$$

$$\frac{dU_1}{dx} = \frac{i\omega A}{\gamma p_m} [1 + (\gamma - 1)f_\kappa] p_1 + \frac{f_\kappa - f_\nu}{(1-f_\nu)(1-\sigma)} \frac{1}{T_m} \frac{dT_m}{dx} U_1 \quad (2)$$

where f_ν and f_κ are the viscous and the thermal thermoacoustic functions. These functions allow the modeling of three dimensional effects in the one dimensional equations. Both thermoacoustic functions are defined by the same differential equation. For simple geometries the differential equation can be solved analytically assuming no slip or isothermal boundaries at the walls for the viscous and the thermal thermoacoustic function. As the boundary conditions and the differential equations are similar, both thermoacoustic functions have the same dependency on R/δ , the pore radius over the corresponding penetration depth. For a circular tube, as investigated in this paper, the thermoacoustic function can be calculated using [3]:

$$f_j = \frac{2 J_1 \left[(i-1) \frac{R}{\delta_j} \right]}{(i-1) \frac{R_0}{\delta_j} J_0 \left[(i-1) \frac{R}{\delta_j} \right]}, \text{ for } j = \kappa \text{ or } \nu \quad (3)$$

Both thermoacoustic functions are dependent on the geometry, the regenerator material parameters and the frequency of oscillation. Assuming no temperature gradient and a uniform pore cross section A , the two differential Equations (1) and (2) can be solved as to relate the pressure and the velocity at the point $i+1$ to the pressure and velocity at point i :

$$p_{1,i+1} = p_{1,i} \cos(k\Delta x_{i,i+1}) - i Z \frac{U_{1,i}}{A} \sin(k\Delta x_{i,i+1}) \quad (4)$$

$$\frac{U_{1,i+1}}{A} = \frac{U_{1,i}}{A} \cos(k\Delta x_{i,i+1}) - \frac{i}{Z} p_{1,i} \sin(k\Delta x_{i,i+1}) \quad (5)$$

With the wave number k

$$k = \frac{\omega}{c_0} \sqrt{\frac{1 + (\gamma - 1)f_\kappa}{1 - f_\nu}} \quad (6)$$

and the complex impedance Z

$$Z = \frac{\rho_m \omega}{k (1 - f_\nu)}. \quad (7)$$

To calculate the thermoacoustic functions, a method similar to the one from Udea *et al.* [4] is used. They use four pressure measurements points, two in front of the regenerator and two after the regenerator. With these four pressure values, they can compute the pressure and the velocity at both ends of the regenerator. In the case of CFD the pressure and the velocity can be measured easily at the same location, the number of measurement points can thus be reduced from four to two. Figure 1 shows the investigated geometry used in this study for the estimation of the thermoacoustic functions. The incoming pressure wave is monitored at point 1 and the exiting wave is monitored at point 4. The regenerator, in this case a circular pore, is located between the points 2 and the 3.

FIGURE 1. Definition of the subdomains: Δx_{12} inlet length; Δx_{23} investigated pore length; Δx_{34} outlet length

The inlet (1-2) and outlet (3-4) parts are used to model the free wave propagation upstream and downstream of the pore. A direct measurement in point 2 and 3 is not meaningful due to the pore end effects which occur there. Assuming that the thermoacoustic functions are known for the inlet and the outlet, the pressure and the velocity at point 2 and 3 can be calculated from Equation (4) and (5). With this the wave number k and the characteristic impedance Z can be calculated as follows [4]:

$$k = \frac{1}{\Delta x_{23}} \arccos \left(\frac{p_{1,2} U_{1,2} + p_{1,3} U_{1,3}}{p_{1,2} U_{1,3} + p_{1,3} U_{1,2}} \right) \quad (8)$$

$$Z = A \sqrt{\frac{p_{1,3}^2 - p_{1,2}^2}{U_{1,3}^2 - U_{1,2}^2}} \quad (9)$$

With k and Z known, the thermoacoustic functions can be calculated according to Equation (6) and (7).

Evaluation of Thermoacoustic Functions with pore end correction

The method of the previous section is now extended to estimate the effects occurring at the ends of the pore. These effects get increasingly important as the length of the pore becomes smaller. In order to reduce the computational power necessary to compute the thermoacoustic functions, the pore and thus the computational domain should be as short as possible. The end correction method used, allows the simulation of two short pores instead of one very long one, where the pore end effects become negligible compared to the length. Furthermore, any pore length can be modeled as the pore end effects are known. The method is validated by comparing the thermoacoustic function of the pores without end effects to the analytical solution given in Equation (3).

The pore end correction method is similar to the method presented above, the only difference is that the wave number k_E and the characteristic impedance Z_E are not known for the end part Δx_E , see Figure 2. Therefore, two numerical experiments with a different pore length Δx_t are used.

FIGURE 2. Geometric definition of the subdomains for the end-correction method: Δx_{12} inlet length; Δx_E pore end effect length; Δx_I investigated inner pore length; Δx_E pore end effect length; Δx_{34} outlet length; Δx_t total pore length

To apply the pore end correction method it is assumed for all numerical simulations that the pore end effects are the same on both ends of the pore. Using the fact that the parts where the pore end effects occur and the inner part of the pore are in series, the respective effects sum up for both simulations, indexed A and B respectively, as follows:

$$2 k_E \Delta x_E + k_I \Delta x_{I,A} = k_{t,A} \Delta x_{t,A} \quad (10)$$

$$2 k_E \Delta x_E + k_I \Delta x_{I,B} = k_{t,B} \Delta x_{t,B} . \quad (11)$$

The exact pore end length Δx_E is not critical as long as the sum of both end lengths is not larger than the total pore length, thus it should hold:

$$2\Delta x_E < \Delta x_t \quad (12)$$

For the calculations discussed in the result section of this paper the pore end length is set to the viscous penetration depth. Equations (10) and (11) can be rewritten such that the wave number of the two subdomains is given as follows:

$$k_I = \frac{k_{t,A} \Delta x_{t,A} - k_{t,B} \Delta x_{t,2}}{\Delta x_{t,A} - \Delta x_{t,B}} \quad (13)$$

$$k_E = \frac{1}{2} \frac{\Delta x_{I,B} k_{t,A} \Delta x_{t,A} - \Delta x_{I,A} k_I \Delta x_{I,A}}{2 \Delta x_E (\Delta x_{t,B} - \Delta x_{t,A})} \quad (14)$$

Using again the two simulations and the fact that the subdomains are in series, one can solve for the characteristic pore end impedance as follows:

$$Z_E = -\frac{1}{2} \frac{-\sin(k_I \Delta x_{I,A}) \sin(k_{t,B} \Delta x_{t,B}) Z_{t,B} + \sin(k_I \Delta x_{I,B}) \sin(k_{t,A} \Delta x_{t,A}) Z_{t,A}}{\cos(k_E \Delta x_E) \sin(k_E \Delta x_E) [\cos(k_I \Delta x_{I,B}) \sin(k_I \Delta x_{I,A}) - \cos(k_I \Delta x_{I,A}) \sin(k_I \Delta x_{I,B})]} \quad (15)$$

Knowing Z_E and k_E , the same method as described at the beginning of this section can be applied to one of the simulations to get the characteristic impedance of the center part of the pore.

CFD Model

Numerical domain

The numerical experiments are carried out with the finite volume code Fluent [6]. Four circular pores of radius $R = 2 \cdot \delta_v$ and different lengths, as defined in Table 1, are numerically investigated. All simulations have an inlet and outlet zone length of $L_a = 10 \cdot \delta_v$. This allows the modeling of the adiabatic wave propagation outside of the pore and the pore end effects. The characteristic impedance Z_a and the wave number k_a of the adiabatic zone are computed beforehand with a simulation without any pore:

$$Z_a = 166.7 + 0.0674 i \quad (16)$$

$$k_a = 0.6145 + 5.64 \cdot 10^{-5} i \quad (17)$$

The pore itself is modeled as a wall with isothermal and no-slip boundary conditions. The pore temperature is set to $T_{wall} = 300\text{K}$. As the problem is rotational symmetric it simplifies to a 2-D axisymmetric case. The wave enters the domain on the left through a dedicated extrapolation nonreflecting pressure boundary condition and exits at the right through another extrapolation nonreflecting pressure boundary condition (Same idea as: [7]). A summary of all boundary conditions can be found in Figure 3.

TABLE 1. Pore length for the different numerical experiments.

	Pore length
Experiment 1	$10 \cdot \delta_v$
Experiment 2	$20 \cdot \delta_v$
Experiment 3	$30 \cdot \delta_v$
Experiment 4	$100 \cdot \delta_v$

FIGURE 3. Definition of the boundary conditions of the CFD simulation

Physical properties

The simulations are carried out at atmospheric pressure and a mean temperature of $T_m = 300\text{K}$. Helium is chosen as the working gas, as it is frequently used in thermoacoustic engines for its favorable thermoacoustic properties [5]. Helium is modeled according to the ideal gas law with the following properties at atmospheric pressure:

TABLE 2. Properties of Helium at $p_m = 1\text{atm}$ and $T_m = 300\text{K}$.

Description	Symbol	Value	Units
Mean density	ρ_m	0.1626	kg/m^3
Dynamic viscosity	μ	$1.99 \cdot 10^{-5}$	kg/(m s)
Thermal conductivity	λ	0.152	W/(m K)
Specific heat ratio	γ	1.667	1
Specific heat at constant pressure	c_p	5193	J/(kg K)

The pressure wave entering at the left has a frequency of $f = 100\text{Hz}$ and an amplitude of $p_1 = 100\text{Pa}$. The low pressure amplitude allows further reduction of the computational domain. As the particle displacement is on the order of magnitude of the penetration depth, the non-reflecting boundary condition is far enough from the pore such that wave propagation can be assumed adiabatic at the boundary. No turbulence model is switched on as according to Olson and Swift [8] the flow field is laminar for a dimensionless radius of $R/\delta_v = 2$ and a Reynolds number of $Re \approx 12$.

In total, two periods are simulated. This is equal to a total time of $t = 0.02\text{s}$. At $t = 0$ the domain is initialized with zero velocity, zero relative pressure and temperature $T_m = 300\text{K}$. Thus, the first period is necessary for the transient effects to build up, while only the second is used for post-processing.

Numerical discretization

The full time dependent compressible Navier Stokes equations are solved with a second order upwind scheme in space and a second order implicit scheme in time. The coupling between the pressure and the velocity is done in a coupled manner [6].

As gradients are much higher in the radial direction than in the axial direction, a rectangular mesh is chosen. This allows a much finer grid in the radial direction compared to the axial direction. The mesh in the axial direction is uniform with three elements per viscous penetration depth, while the mesh in radial direction consists of 25 elements growing geometrically by a factor of 1.2 from the wall to the symmetry boundary. The time step size is limited to $\Delta t = 5 \cdot 10^{-7}\text{s}$ by the extrapolation non-reflecting boundary condition. Pressure and velocity are exported at the monitor points 1 and 4 (See Figure 1) every 40 time steps to get 500 measurement points per wave period.

RESULTS AND DISCUSSION

By applying the method of Udea *et al.* on the CFD simulations, the thermoacoustic functions are calculated and summarized in Table 3. These results are compared to the analytical values calculated from Equation (3) and the physical parameters presented earlier:

$$\delta_v = 0.5367 - 0.3693i \quad (18)$$

$$\delta_\kappa = 0.6781 - 0.3728i \quad (19)$$

The results for the viscous thermoacoustic function match the analytical solution to 5%, while the results for the thermal thermoacoustic function match to 30%. In both cases the results get continuously better with increasing pore length, L . The difference between the simulation and the analytical solution can be explained by the pore end effects. The method from Udea *et al.* gives averaged values of the viscous and thermal effects over the whole length of the pore. Assuming that the same pore end effects occur in all experiments, they will occupy a larger proportion of the pore the shorter the pore length, Δx_t , becomes. The influence will thus be bigger for short pores. On the other side a very long pore should converge to the analytical solution, as the analytical solution holds for an infinite pore.

One assumption to apply the pore end correction method is that the pore end effects are the same and independent of the pore length. This is now verified. Figure 4a shows the axial velocity normalized with the velocity amplitude at $R = 0$, over the x axis, in the vicinity of the left pore end. The axial velocities of all four experiments are plotted at four moments, separated by a quarter period each. The gray and white area represent the pore and the adiabatic domain respectively. The profiles lay close over each other, showing that the same pore end effects occur independently of the total length of the pore. The end correction method can thus be applied to get the viscous thermoacoustic function of the inner part.

TABLE 3. Computed values of viscous and thermal thermoacoustic functions, separated in real and imaginary part, compared with the analytical solution (Equation (18),(19)) for the four different lengths.

	f_v		Difference with analytical solution		f_κ		Difference with analytical solution	
	Re	Im	Re	Im	Re	Im	Re	Im
Experiment 1	0,5629	-0,3748	4,9%	1,5%	0,6179	-0,4806	-8,9%	28,9%
Experiment 2	0,5493	-0,3708	2,3%	0,4%	0,6356	-0,4279	-6,3%	14,8%
Experiment 3	0,5447	-0,3697	1,5%	0,1%	0,6442	-0,4097	-5,0%	9,9%
Experiment 4	0,5381	-0,3678	0,3%	-0,4%	0,6532	-0,3855	-3,7%	3,4%

The same reasoning can be applied to the thermal thermoacoustic function. Figure 4b shows the temperature changes at $R = 0$, over the x axis, in the vicinity of the left pore end. Again the temperature oscillation of all four experiments is plotted at four moments, separated by a quarter period each. Just as for the axial velocity, the profiles lay nearly over each other. The only deviation occurs in the temperature profile of Experiment 1 ($H = 10 \cdot \delta_v$) at $\frac{x}{\delta_v} = 14 \dots 15$, the center of this pore. In this case the pore is too short and the end effects fill the entire pore. For the other lengths the profiles are the same, it can thus be concluded that the end correction method can be applied to get the thermoacoustic functions of the inner part of the pore.

FIGURE 4. a: Axial velocity normalized with the velocity amplitude at $R = 0$, over the x axis, in the vicinity of the left pore end at four different moments, separated by a quarter period each - gray and white area represent the pore and the adiabatic domain respectively. b: Temperature changes at $R = 0$, over the x axis, in the vicinity of the left pore end at four different moments, separated by a quarter period each - gray and white area represent the pore and the adiabatic domain respectively

Furthermore, Figure 4a and 4b show that the domain of the pore in which the pore end effects occur, is larger for the thermal effects ($\sim 5\delta_v$) than for the viscous effects ($\sim 2\delta_v$). This can explain why in Table 3 the differences with the analytical solution are bigger for the thermal thermoacoustic function compared to the viscous thermoacoustic function.

In the next step the end correction method is applied to Experiment 1 and 2, the two shortest simulations. The thermoacoustic functions of the “inner” part of the pore, which are not affected by end effects, are calculated. These two values are then compared with the analytical solutions, the results are summarized in Table 4. With this method the difference with the analytical solution can be reduced to less than one percent except for the real part of the thermal thermoacoustic function. The difference for the real part of the thermal thermoacoustic function can be

reduced compared to the two experiments, but the difference in this case stays at a higher value, approximately 4%. Even if the end correction method is applied to different combinations of the four experiments, the differences stay the same. It can thus be concluded, that this deviation is not due to a too short pore. Further simulation with pores having a slip boundary instead of a no slip wall boundary condition exclude viscosity driven streaming as a cause, because the deviation occurs there too. A reduction of the pressure amplitude to $p_{amp} = 1Pa$ reduces the deviation to roughly 1.5%. A possible reason could be the reduced convection.

TABLE 4. Computed values of viscous and thermal thermoacoustic functions, separated in real and imaginary part, compared with the analytical solution (Equation (18),(19)) for the first two experiments and the estimated value of a pore without end effects computed from the first two experiments.

	f_v		Difference with analytical solution		f_κ		Difference with analytical solution	
	Re	Im	Re	Im	Re	Im	Re	Im
Experiment 1	0,5629	-0,3748	4,9%	1,5%	0,6179	-0,4806	-8,9%	28,9%
Experiment 2	0,5493	-0,3708	2,3%	0,4%	0,6356	-0,4279	-6,3%	14,8%
End corrected	0,53545	-0,366	-0,2%	-0,9%	0,65295	-0,3754	-3,7%	0,7%

CONCLUSION

In a first step, the experimental method of Ueda *et al.* is validated for the CFD model of the cylindrical pore. CFD is thus a valid tool to predict the thermoacoustic functions. In a second step the end correction is successfully applied, leading to a very good agreement with the analytical solution. A deviation between the numerical results and the analytical solution concerning the real part of the thermal thermoacoustic functions was noted and could not yet be explained. As this slight difference has a big influence on the efficiency of a regenerator, further research will be done.

The working model will be extended in a next step to account for the finite thickness of the pore wall. In addition, the successful validation of the CFD model will open the door for more complex geometries for which no analytical solution exist. For example the model could be extended to staggered screen regenerators which are commonly used in today's thermoacoustic devices.

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