CONTACT EFFECTS IN BENDING AFFECTING STRESS AND FORMABILITY

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ABSTRACT: If a strip is pulled over a curved tool there is a contact stress acting on the strip. This contact stress changes the stress state in the material, which is analysed with a simple model. One effect is that the yield stress in tension is reduced. Predictions by the model agree with observation from a 90-degree bending test found in literature, and indirectly with observation from a stretch-bend test also found in literature. Another effect is that a change in stress state also affects the formability. This is analyzed by applying the maximum force condition on this situation. The predictions agree with a more thorough analysis of the effect of thickness stress in general, but the predictions of both methods are lower than actually observed in tests. There may be other mechanisms at work, and one candidate is presented.

KEYWORDS: Bending, Formability, Contact stress

1 INTRODUCTION

The common FLC is valid only under certain conditions, one of them is the absence of bending. The occurrence of bending in a forming operation can cause several effects, the most relevant of which is the raising of the formability. The latter is for example encountered in the measurement of FLCs using Nakazima strips on tools of various radii. That effect is being investigated by several institutes at the moment.

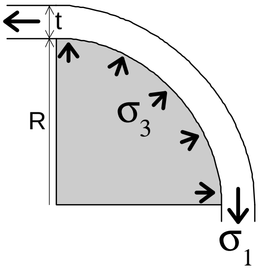

***Figure 1**. Bending over a radius.*

If a strip is pulled over a curved tool there is a contact stress acting on the strip as pictured in figure 1. This contact stress changes the stress state in the material, so that it cannot be considered as plane-stress anymore. However that effect seems to be ignored in the literature. A well known effect is that a change in stress state affects the yield stress in tension. A lesser known effect is that it also affects the formability. Both will be discussed here, and the effects will be compared to results found in the literature if possible.

2 EFFECTS ON TENSILE FORCE

For the situation in Figure 1 it can easily be shown that $s_{3,i} = -s_1.t/R$ where $s_{3,i}$ is the normal contact stress at the inner side of the bend strip (s_3 is compressive, so negative). s_3 will vary over the thickness of the strip, but we will simplify matters and only refer to the mean value. In general we can write $s_{3,\mathrm{mean}} = s_{3,i}/a$ where the constant a may depend on the material and the geometry, so:

$$s_3 = -s_1 \frac{t}{a.R} \qquad (1)$$

If s_3 would vary linearly we would get $a = 2$ but this is not known beforehand. In this paper several values for a will be used, but note that these are arbitrary. The effect of change of stress state is that it affects the yield stress in tension s_1. Tresca's criterion simply states s_1-s_3 = s_f where s_f is the material's flow stress. This yields:

$$s_1 = s_f /(1 + t/aR) \qquad (2)$$

This shows that the yield stress in tension will reduce if there is bending over some radius.

2.1 90-DEGREE BEND TEST

The literature reports several types of bending-under-tension tests. One is a bending over a 90^O radius under back-tension, just as pictured in Figure 1. Several researchers have reported results for this kind of test, but generally report pulling forces and not stresses. If forces are measured at the same elongation (read: reduction of thickness) they may me interpreted as stresses but in general this is not known. So interpretation of the results has to be done with care.

Wagoner and co-workers have used this type of test recently on DP steel [1,2] and have indeed observed a reduction in pulling force, and have published the normalized maximum pulling stress (= measured stress / UTS), as a function of R/t; their results are compiled in Figure 2. The general relation predicted by (2) agrees

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with the observed data, notably with the simulation for series 2. Note also that formula (2) with a = 3.5 seems to over-estimate the effect for some series.

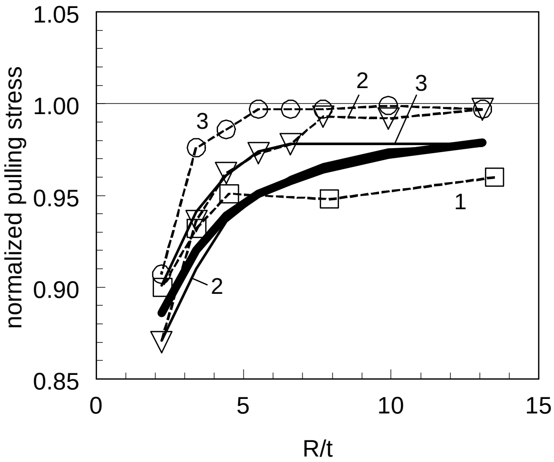

Figure 2. *Comparison to literature data. 1 = measured data from [1]. 2,3: data from [2], both measured (points and dashed lines), and simulated (solid lined). Thick black line: relation according to formula (1), a = 3.5.*

Hudgins has carried out similar experiments on DP steel subjected to different heat-treatments [3]. He has published values of maximum pulling stress, that have been converted here to normalized stress by dividing the value by the material's UTS (also published), the results are shown in Figure 3. There is a good agreement between the measured data and the relation according to (2), with the exception of series 6. Series 6 refers to a material that after heat treatment showed very little ductility, meaning that the UTS is measured at a very low elongation. So it is quite possible that the maximum force in the bending tests occurs at a different elongation than the measured UTS, but this is not known. Noteworthy is that Hudgins models the relation by fitting two straight lines, and concludes that the values for series 1 and 2 at $R/t \approx 4$ are 'outliers'. However these data agree excellently with relation (2).

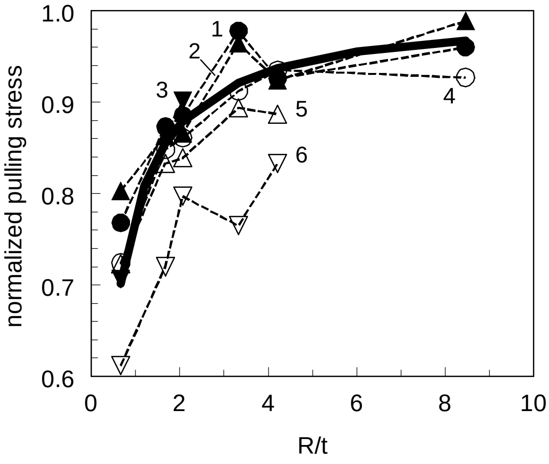

Figure 3. *Comparison to literature data. All series (1-6) are measured data from [3], some series are incomplete. Thick black line: relation according to formula (1), a = 3.5.*

In the above we have ignored thinning of the strip as a function of its elongation and have taken t in (2) as constant. However this is a second order effect that does not affect the conclusions in this section. Correctly calculated forces will be presented in Figure 8 in section 3.2

2.2 STRETCH-BEND TEST

A second type of test is a stretch-bend test where a clamped strip is bent by pushing a cylindrical tool at its centre as shown schematically in Figure 4. This is the most simple test as any movement of the strip over the radius can be ignored.

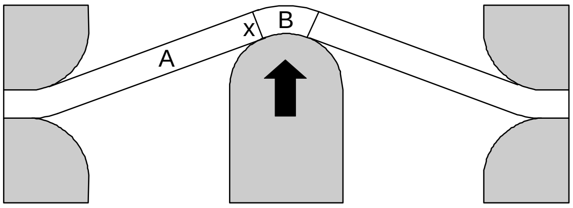

Figure 4*. Schematic presentation of the stretch-bend test.*

Experiments have shown that there is a difference between the strain in the straight part and in the bend part and this will be analyzed now. Part A is not supported; part B however is pulled against the punch, and therefore its strain state is affected. At the interface x there must be an equilibrium of forces, so: $F_A = F_B$. As part B requires a lower tensile force for stretch, it must be stretched to a higher level than part A to obtain equilibrium. Common to what is used in the stability criterion for local necking, we will apply this equilibrium on a piece of unit width, so $F = s.t$ and the condition becomes: $s_A.t_A = s_B.t_B$.

Assuming a power-law hardening relation and substituting (2) we can write:

$$s_A = C.e_A{}^n, \quad s_B = C.e_B{}^n.\frac{1}{1+t_B/a.R}, \quad t = t_0.e^{e3} \qquad (3)$$

For any given strain in part A (s_A) the strain in part B can now be calculated, depending on the tool radius R, the original sheet thickness t_0, the hardening coefficient n, and the strain state determining the relation between thickness strain e_3 and length strain e_1.

The strain distribution in some actual tests has recently been determined and published by Kitting et. al. [4]. The results obtained with a low bending radius showed a high amount of strain localization which makes them unsuitable for our analysis. So we will analyze only the results obtained with a high bending radius (R=20), and only in cases that showed a fairly uniform strain in part B. The strain state is approximately uniaxial, therefore we will use $e_3 = -e_1/2$. Note that Kitting has measured the strain at the outer surface, so the strains predicted by our model in part B have to be corrected for the curvature of the strip. The results are presented in Figure 5 using t_0 = 1.5 mm and n = 0.14, in which only the strain levels predicted by the model are to be considered, not the transitions. There is a good agreement between model and experiments, for low bending depths the model with a = 5 seems to underestimate the effect, but for larger bending depths the model seems to overestimate the effect..

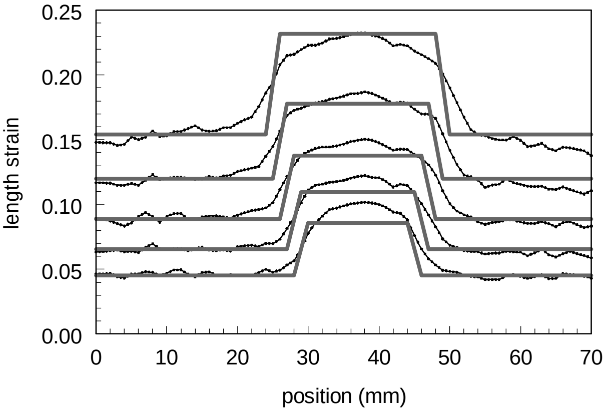

Figure 5. *Strain distribution in stretch-bend tests as measured by Kitting (lines with dots), and as predicted by the model with a = 5 (thick grey lines).*

3 EFFECTS ON FORMABILITY

A thickness stress also has an effect on the formability. Popular speaking: a thickness stress eases the elongation of the material, meaning that an instability occurs later than without thickness stress.

3.1 GENERAL CONSIDERATIONS

A detailed analysis in general is far from easy. The most recent is a very thorough M-K analysis carried out by Allwood and Shouler [5], and some of their findings are presented in Figure 6. This figure shows clearly that the formability increases when a thickness stress is applied. For the plane-strain situation the effect is roughly linear and can be approximated by:

$$\frac{e_0(s_3)}{e_0(0)} \approx 1 + \frac{-s_3}{s_1} \tag{4}$$

where e_0 is the necking limit at plane strain conditions. This relation is confirmed by other models not mentioned here.

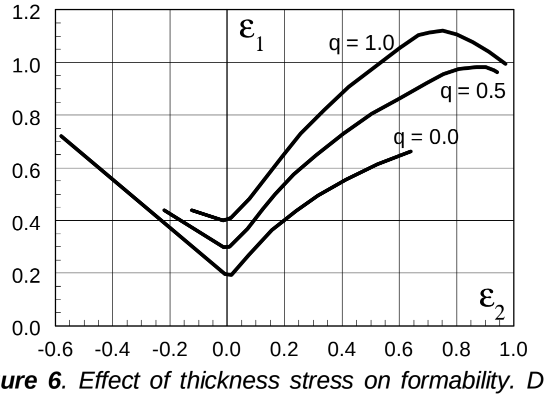

Figure 6. *Effect of thickness stress on formability. Data from [5],* $q = -s_3/s_1$

It is difficult to check these findings with results from bending tests. Kim has published values for 'displacement to failure' for his tests [2] but these cannot be converted to actual strains simply. Kitting has presented actual measured strains at fracture [4], and the strain state in tests with small bending radii are approximately plane-strain. However the measured strains are considerably higher than predicted by formula (4), notably in cases of low bending radius. Please keep in mind that models like the one developed by Allwood or presented in the next section predict the strain at onset of necking, while Kitting has presented actual measured values. This may be an effect of measurement or definition of limit strain. Also, for materials that show considerable strain-rate hardening the actually measured strains are expected to be higher than the strains at the onset of necking predicted by simple models.

Another possibility however is that there are other mechanisms at work, and one will be presented here. If there is indeed a severe neck the material in the neck may loose contact with the tool as shown in Figure 7 at location A. This means that there is no thickness stress any more, and the 'softening' of the material at B does not take place at A. Consequently at A the material becomes stronger than at B, and this will slow down further development of the neck. However this mechanism is speculative, and it is not known if it does actually happen in a practical operation.

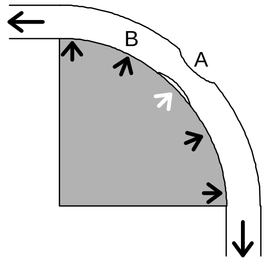

Figure 7*. In case of a severe neck (at A) the material looses contact with the tool.*

3.2 ANALYSIS WITH THE MAXIMUM FORCE CONDITION

The effects of contact stress on formability can also be analysed by applying the so-called maximum force condition. This has been done indeed using equation (2), albeit slightly simplified. The analysis will not be presented here in detail, but it finally yields the following equation for the necking limit expressed in the thickness strain e_3:

$$\frac{-e_3}{n} = 1 + a.t_0.\mathrm{e}^{e_3}, \quad a = \frac{1}{a.R} \tag{5}$$

This is a transcendental equation that has to be solved numerically. The outcome is e_3 at the onset of local necking as a function of n and $a.t_0 = t_0/aR$.

If ε_3 is not too large we can make a first approximation by applying $\mathrm{e}^{\varepsilon 3} \approx 1+e_3$:

$$\frac{-e_3}{n} = 1 + a.t_0.(1+e_3), \quad e_3 = -n.\frac{1+a.t_0}{1+n.a.t_0} \tag{6}$$

For a situation of plane strain we have $\varepsilon_3 = -\varepsilon_1$, and setting as a further approximation in (6) $t \approx t_0$, valid only for situations of $\varepsilon_3 \ll 1$, we get:

$$e_1 \approx n.(1+a.t_0) = n.(1+\frac{t_0}{a.R}) \qquad (7)$$

This in fact the same relation as deduced from the Allwood results expressed in equation (4), note that $t_0/aR = -s_3/s_1$ (see equation 1).

Equation (5) has been solved for some cases and the result is shown in Figure 8. The parameter '$a.R/t_0$' can be interpreted as the ratio of mean thickness stress and mean pulling stress at zero elongation (the onset of the operation). In an actual stretch-bending operation the value of that parameter will not likely be lower than 10.

The upper part shows the increase of formability, in fact the effect on the thickness strain at the onset of necking; this does not depend on the strain state. The simple model is just formula (7) that can be interpreted as a limiting situation for $n \rightarrow 0$, the other two curves have been derived from equation (5). There is some influence of n: the effect reduces for increasing n, this can also be seen in the simplified equation (6). The effect is limited in size, for $a.R/t_0 = 10$ it is less than 10%.

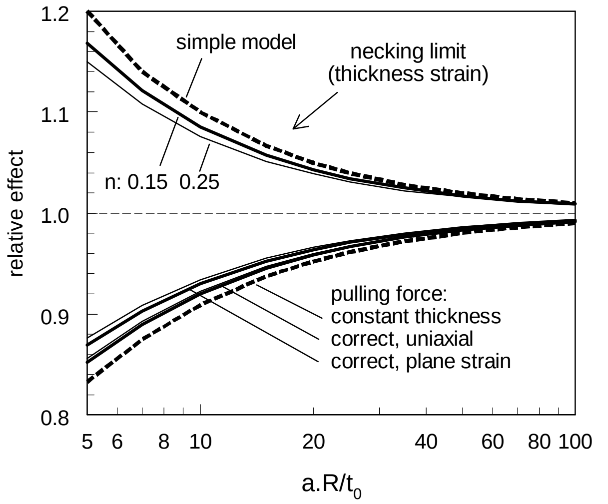

***Figure 8**. Effect of bending on necking limit (upper part) and pulling force UTS (lower part) as analysed with the maximum force condition. Thick lines are for n=0.15, thin lines for n = 0.25.*

In addition, the lower part of the figure shows the effect on the UTS, or better: the pulling force at $e_1 = n$, for a situation of constant thickness, and correctly for two strain states (uniaxial: for isotropic material). The fact that the uniform strain actually increases slightly by bending is neglected here. The difference between the three situations is caused by the amount of thinning of the strip which is the highest for plane strain. The effect of n is even lower than was found for the necking limit.

Note that the effects are roughly each others inverse, so that as a rule of thumb: if the force decreases by P %, then the necking limit increases by P %.

4 DISCUSSION AND CONCLUSION

In the previous sections a simple model has been developed that predicts certain phenomena occurring in a bent strip by looking at the change in stress state. This model showed good agreement with observation found in the literature. However the model was developed with some severe simplifications:

- the phenomena assumingly depend only on the mean thickness stress s_3 (equation 1);
- Tresca's yield criterion is used ignoring any effects of the transverse stress s_2 (equation 2);
- values of 3.5 and 5 for the parameter a are used without any motivation, other than that the results look OK.

Furthermore, some second order effects have been ignored. Consequently, one cannot simply conclude that the model is correct. However, that was not the intention of this paper. The intention of this paper is only to study possible effects that can be caused by the change in stress state created by the tool contact. The conclusion is that even modest values of the thickness stress, in this case as indicated by values for a of 3.5 and 5, can cause effects very much comparable to effects observed in actual experiments, at least concerning the stresses and forces.

Therefore the overall conclusion is:

> *the change in stress state in a situation of bending over a tool radius caused by a normal stress at the contact must not be ignored in an analysis.*

Nevertheless, more research is still required to fully understand the increased formability in a situation of combined bending and stretching.

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