Dynamic simulation of gas-liquid two-phase flow: effect of column aspect ratio on the flow structure

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Abstract—In this paper an Eulerian/Lagrangian model, describing the hydrodynamics of a gas-liquid bubble column, is presented. The model resolves the time dependent, two-dimensional motion of small, spherical gas bubbles in a liquid using the equation of motion. The model incorporates all relevant forces acting on a bubble as it rises through the liquid, and additionally accounts for direct bubble-bubble interactions. The liquid-phase hydrodynamics are described using the volume-averaged Navier-Stokes equations. This model is used to study the hydrodynamic behaviour of bubble columns with aspect ratios ranging from 1.0 to 11.4. In addition to these theoretical results, experimental observations are presented of the flow structure in a pseudo-two-dimensional bubble column with different aspect ratios. A clear transition in the gas-liquid flow pattern could be observed, both experimentally and theoretically, from the well-known 'cooling tower' mode of circulation ($L/D = 1.0$) to the staggered vortices mode of circulation ($L/D \geq 2.0$). The computational results clearly showed the presence of vortical structures in the liquid phase at aspect ratios exceeding 2.0. These vortical structures in the liquid phase were studied experimentally using neutrally buoyant tracer particles and streak photography. The experimentally observed vortical structures are shown to resemble the computed structures. © 1997 Elsevier Science Ltd

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1. INTRODUCTION

Bubble column reactors are used throughout the biological, chemical and petrochemical industry. Typical applications are encountered in processes involving absorption, catalytic slurry reactions, bioreactions and coal liquefaction. Bubble column reactors are often used because of their relatively simple construction, low operating costs, excellent heat transfer characteristics to immersed surfaces and the ease with which the liquid residence time can be varied (Shah *et al.*, 1982). Despite their frequent application, knowledge on the fluid mechanical behaviour of bubble column reactors is still limited. The structure of the two-phase flow, and its dependence on design parameters such as height and diameter of the column, is especially important but fundamental knowledge regarding this topic is lacking. Designers therefore rely on simplified models that neglect many of the important physical aspects of the gas-liquid two-phase flow.

The structure of the two-phase flow has been the subject of both experimental and theoretical investigations for many years. For example, Freedman and Davidson (1969) developed a liquid circulation model known as the 'Gulfstream or cooling tower model'. This type of liquid circulation consists of two vortex cells with liquid flowing upward in the centre of the bubble column and downward along the walls of the column. In later years Joshi and Sharma (1979) proposed a model postulating the flow structure in bubble columns to consist of multiple circulation cells which were thought to be located in a regular pattern above each other. Finally, Chen *et al.* (1989) conducted experimental research in two pseudo-two-dimensional bubble columns. They investigated the effect of the liquid depth, and thus, of the aspect ratio of the bubble column, on the structure of the two-phase flow. Chen *et al.* observed the 'cooling tower' mode of circulation cells only when the aspect ratio was less than or equal to unity. At greater liquid depths the observed flow pattern consists of two rows of circulation cells, where the vortices in these rows possess a staggered orientation with respect to each other. In this respect the vortical structures resemble those shed behind a circular cylinder in cross-flow (Batchelor, 1967).

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Fluid dynamic modeling of the gas–liquid two-phase flow prevailing in bubble columns can significantly contribute to our understanding of the (fluid) dynamics of these industrially important systems. In recent years a number of investigators (Lapin and Lübbert, 1994; Sokolichin and Eigenberger, 1994; Sokolichin *et al.*, 1997; Delnoij *et al.*, 1997) have published interesting results obtained from different types of CFD models. In these studies it was clearly demonstrated that the complex, time-dependent, flow structures observed in gas–liquid bubble columns could be obtained from these models.

Delnoij *et al.* (1997) developed a detailed Eulerian/Lagrangian model for a gas–liquid bubble column operating in the homogeneous regime. The model describes the time-dependent motion of small, spherical gas bubbles in a Newtonian liquid. It is based on the volume-averaged Navier–Stokes equations to describe the liquid-phase hydrodynamics and separate equations of motion to track each individual bubble contained in the column. The model incorporates all relevant forces acting on a bubble as it rises through the liquid. Full exchange of momentum between the phases is accounted for, and direct bubble–bubble interactions are described in detail using a collision model. In the present paper this model will be used to study the structure of the two-phase flow prevailing in a bubble column at various aspect ratios. In addition to the theoretical results, experimental observations of the flow structure in a pseudo-two-dimensional bubble column will be presented. Streak photography and video imaging techniques were used to visualize the structure of the two-phase flow in the bubble column.

2. THE TWO-PHASE FLOW MODEL

The model presented in this paper applies to the homogeneous or dispersed bubble regime. Relatively

low gas velocities and small, spherical bubbles characterize this regime. The model consists of two intimately coupled parts; a part describing the liquid-phase hydrodynamics and a part that models the behaviour of each individual bubble using the equation of motion. Both parts, the coupling between these parts and thus the coupling between the gas and the liquid phase will, subsequently, be discussed in more detail.

2.1. Bubble dynamics

As stated previously, our model uses an individual equation of motion to calculate each bubble's new position from its previous time level position and its present time level velocity, accounting for possible bubble–bubble collisions. The required new bubble velocity is calculated with a simple, explicit integration formula

$$\mathbf{v}^{n+1} = \mathbf{v}^n + \left(\frac{d\mathbf{v}}{dt} \right)^{n+1} \Delta T. \quad (1)$$

Each bubble's acceleration is calculated from a force balance over the bubble under consideration

$$m_b \frac{d\mathbf{v}}{dt} = \mathbf{F}_{\text{total}}. \quad (2)$$

It is assumed that the force $\mathbf{F}_{\text{total}}$ acting on a non-deformable spherical bubble rising in an unsteady, and non-uniform liquid flow field is composed of separate and uncoupled contributions from pressure gradient, drag, virtual or added mass, vorticity and gravity (Auton, 1983)

$$\mathbf{F}_{\text{total}} = \mathbf{F}_P + \mathbf{F}_D + \mathbf{F}_{VM} + \mathbf{F}_L + \mathbf{F}_G. \quad (3)$$

Table 1 summarizes the equations and constants used to calculate the various forces acting on a bubble. For details concerning these expressions the interested reader is referred to Delnoij *et al.* (1997).

Table 1. Forces acting on a bubble rising in an unsteady and non-uniform liquid flow field

Force	Expression
Gravity and far-field pressure	$\mathbf{F}_G + \mathbf{F}_P = \rho_g V_b \mathbf{g} - V_b \nabla P$
Drag force	$\mathbf{F}_D = -\frac{1}{2} C_D \rho_l \pi R_b^2 \mathbf{v} - \mathbf{u} (\mathbf{v} - \mathbf{u})$ $C_D = \begin{cases} \text{Re} < 1000 \Rightarrow \frac{24}{\text{Re}} (1 + 0.15 \text{Re}^{0.687}) \\ \text{Re} > 1000 \Rightarrow 0.44 \end{cases}$
Lift force	$\mathbf{F}_L = -C_L \rho_l V_b (\mathbf{v} - \mathbf{u}) \times \boldsymbol{\Omega}$ $\boldsymbol{\Omega} = \nabla \times \mathbf{u}$ $C_L = 0.53$
Virtual mass force	$\mathbf{F}_{VM} = -\left(\frac{D\mathbf{I}}{Dt} + \mathbf{I} \cdot \nabla \mathbf{u} \right)$ $\mathbf{I} = C'_{VM} \rho_l V_b (\mathbf{v} - \mathbf{u})$ $C'_{VM} = C_{VM} (1 + 2.78(1 - \epsilon_l))$ $C_{VM} = 0.5$

2.2. Direct bubble-bubble interaction

In gas-liquid dispersions bubbles have been observed to coalesce, to bounce and, subsequently, coalesce or to bounce and separate (Duineveld, 1994; Tsao and Koch, 1997). In our model it has been assumed that the bubbles always bounce and separate, which is probably a fair assumption for relatively small bubbles rising in water at high Reynolds numbers. In experiments bouncing and separation of bubbles can be achieved by adding salt to the suspending water (Tsao and Koch, 1994). There are various approaches to model collisions between bubbles in a gas-liquid dispersion. Sangani *et al.* (1995) employed a soft-core repulsive potential to model the collision process. In the present paper however, a hard sphere collision model resembling the model developed by Hoomans *et al.* (1996) is used to process the sequence of collisions between bubbles in a bubble column.

2.3. Liquid-phase hydrodynamics

The liquid-phase hydrodynamics are described with the volume-averaged mass and momentum conservation equations given, respectively, by

$$\frac{\partial(\varepsilon_l \rho_l)}{\partial t} + \nabla \cdot \varepsilon_l \rho_l \mathbf{u} = 0 \quad (4)$$

and:

$$\frac{\partial(\varepsilon_l \rho_l \mathbf{u})}{\partial t} + \nabla \cdot \varepsilon_l \rho_l \mathbf{u} \mathbf{u} = -\varepsilon_l \nabla P - \nabla \cdot \varepsilon_l \tau_l + \varepsilon_l \rho_l \mathbf{g} + \Phi. \quad (5)$$

The spatial resolution with which the liquid velocity field is resolved is small compared to the size of the bubbles; the liquid-bubble interaction is therefore superimposed on the liquid velocity field. This liquid-bubble interaction is modeled with the source term Φ that accounts for the momentum exchange between the bubbles and the liquid.

In the present study two-dimensional, isothermal motion of both the gas- and the liquid-phase is assumed. The basic variables that are calculated by the model are pressure, liquid velocity and the velocity and position of each individual bubble. All other variables appearing in the balance equations must be specified in terms of these variables.

The liquid-phase viscous stress tensor τ_l is assumed to obey the general form for a Newtonian fluid

$$\tau_l = [(\lambda_l - \frac{2}{3}\mu_l)(\nabla \cdot \mathbf{u})\mathbf{E} + \mu_l((\nabla \mathbf{u}) + (\nabla \mathbf{u})^T)]. \quad (6)$$

The bulk viscosity λ_l is set to zero in all simulations presented in this paper. In most simulations water is used as the liquid phase which corresponds to a shear viscosity of $1.0 \times 10^{-3} \text{ kg m}^{-1} \text{ s}^{-1}$.

2.4. Coupling between the phases

The two parts that constitute our hydrodynamic model are coupled through the liquid volume fraction ε_l and through the source term Φ that accounts for the

momentum transfer from the bubbles to the liquid. Both this liquid volume fraction ε_l and the source term Φ have to be calculated in accordance with the number of bubbles present in a computational cell. In order to evaluate the reverse momentum transfer rate, i.e. from the liquid to a specific bubble, all quantities pertaining to the liquid phase (i.e. pressure and velocity components) have to be available at the centre of mass position of this bubble. These so-called local liquid properties are calculated from the values of the volume-averaged liquid properties at the grid nodes surrounding the bubble under consideration.

This two-way momentum coupling that is included in our model is of crucial importance, as has been shown by Delnoij *et al.* (1997). Delnoij *et al.*, showed that a model not using two-way momentum coupling could not account for the time-dependent behaviour generally observed in pseudo two-dimensional bubble columns.

2.4.1. Liquid volume fraction. The liquid volume fraction in a computational cell is calculated from the volume occupied by the bubbles present in the cell under consideration

$$\varepsilon_l = 1 - \frac{\sum_i V_{bi}}{V_{\text{cell}}}. \quad (7)$$

When calculating this liquid-phase volume fraction, it is very important to account for bubbles overlapping with more than one computational cell. In the present code, the volume occupied by these bubbles is divided over the respective computational cells.

2.4.2. Momentum transfer term Φ . The momentum transfer rate from the bubbles to the liquid per unit volume of gas-liquid dispersion, Φ , is just the opposite of the forces due to drag, lift and virtual mass exerted by the liquid on the bubbles per unit volume of gas-liquid dispersion. From these forces, a local momentum transfer rate ϕ_i can be calculated using

$$\phi_i = -(\mathbf{F}_{Di} + \mathbf{F}_{VMi} + \mathbf{F}_{Li}). \quad (8)$$

Subsequently, an area-weighted averaging technique is used to calculate volume-averaged momentum transfer rates Φ at the four nodes surrounding the bubble, from the local momentum transfer rate ϕ_i . Figure 1 illustrates this principle. The volume-averaged momentum transfer rate is calculated from:

$$\begin{aligned} \Phi_1 V_{\text{cell}} &= \frac{A_1}{dx dy} \phi_i(\mathbf{r}_i) \\ \Phi_2 V_{\text{cell}} &= \frac{A_2}{dx dy} \phi_i(\mathbf{r}_i) \\ \Phi_3 V_{\text{cell}} &= \frac{A_3}{dx dy} \phi_i(\mathbf{r}_i) \\ \Phi_4 V_{\text{cell}} &= \frac{A_4}{dx dy} \phi_i(\mathbf{r}_i). \end{aligned} \quad (9)$$

Fig. 1. Area-weighting of the local momentum transfer rate ϕ_i to obtain the volume-averaged momentum transfer rates Φ at the four grid nodes surrounding the bubble.

The area's $A_1, \dots, A_4$ depicted in Fig. 1 can be calculated form:

$$\begin{aligned} A_1 &= (dx - \delta x)(dy - \delta y) \\ A_2 &= \delta x(dy - \delta y) \\ A_3 &= (dx - \delta x)\delta y \\ A_4 &= \delta x\delta y \end{aligned} \quad (10)$$

In both eqs (7) and (9) the volume V_{cell} represents the volume of a computational cell based on a virtual third dimension, which is due to the two-dimensional nature of our model. This third dimension can be derived from the relation between the two-dimensional liquid hold up and the three-dimensional liquid hold up assuming similar geometrical arrangement in a two-dimensional and a three-dimensional space. The resulting expression for the volume of a computational cell, V_{cell} , is given by

$$V_{\text{cell}} = dx dy 2.3^{-0.75} D_b. \quad (11)$$

2.4.3. Local liquid-phase properties. In order to calculate the force acting on a bubble with eq. (3), local values of the pressure, of the liquid velocity, of the partial and substantial derivatives of the pressure and of the partial and substantial derivatives of the liquid velocity have to be available at the centre of mass position ($\mathbf{r}_i$) of the bubble. However, these volume-averaged variables and their derivatives are only available at discrete nodes in the computational domain. Therefore an area-weighted averaging technique, similar to that employed to calculate the momentum transfer rate, is used to obtain local values from the volume-averaged variables at the four nodes surrounding the bubble. This local value of a quantity

f can be calculated using

$$f(\mathbf{r}_i) = \frac{1}{dx dy} \sum_{n=1}^4 A_n f_n \quad (12)$$

where f_n represents some volume-averaged quantity at node n , and the areas A_n are given by Eq. (10).

The method described here to incorporate momentum transfer between the gas and the liquid phase is slightly different from our previous technique (Delnoij *et al.*, 1997). This new method ensures rigorous momentum conservation, as Newton's third law is strictly obeyed.

2.5. Boundary conditions and numerical solution

The boundary conditions for the Navier-Stokes equations are specified using the flag matrix concept. This concept, also used by Kuipers *et al.* (1993), allows boundary conditions to be set for each individual computational cell. Using this flag matrix concept a variety of boundary conditions can be set by specifying the value of the cell flag $fl(i, j)$

Bubble-wall interaction is accounted for by the collision submodel; only instantaneous, hard sphere collisions are assumed to occur. The model treats these collisions similar to bubble-bubble collisions.

The model discussed in this paper has been implemented in a computer code written in C. This code takes consecutive steps in time, during each of which the code calculates the forces acting on the bubbles present in the system. It then calculates the new bubble velocities using eqs (1) and (2), and the new bubble positions taking into account possible bubble bubble collisions. Finally, the code solves for the new time level liquid velocity field. For details concerning the solution procedure the interested reader is referred to Kuipers *et al.* (1993) and Delnoij *et al.* (1997). CPU time requirements depend on the size of the bubble column under consideration, on the number of grid cells used in the computation and on the number of bubbles contained in the bubble column. On average, calculating 1 min of time-dependent behaviour of a typical bubble column (1.0 m in height, 0.25 m in width and containing ≈ 4000 bubbles) requires approximately 4 h dedicated CPU time on a Silicon Graphics Indigo² workstation.

3. EXPERIMENTAL SET-UP

A key issue in computational fluid dynamics, especially in connection with multiphase flows, is assessing the validity of the computational results. Therefore, an experimental set-up was constructed to visualize the structure of the two-phase flow. This pseudo-two-dimensional bubble column is depicted in Fig. 2. It has a cross section of 250 mm $\times$ 20 mm and is 2000 mm in height; the column provides data up to an aspect ratio of 7.0. The bubble column is constructed from clear transparent plexi-glass sheets and is fitted with a gas distributor made from a 6.0 mm diameter stainless-steel tube. The upward side of this tube has 14 evenly spaced 0.2 mm diameter holes in the central

Fig. 2. Pseudo-two-dimensional bubble column used in flow visualisation experiments.

part of the tube. The gas flow rate is controlled through a calibrated mass flow controller.

The structure of the two-phase flow was studied using video imaging techniques and streak photography. Neutrally buoyant tracer particles ($d_p = 250 \mu\text{m}$) were employed to visualize the vortical structures in the bubble column.

4. RESULTS AND DISCUSSION

4.1. Effect of column aspect ratio—experiments by Chen *et al.* (1989)

Chen *et al.* studied the liquid circulation in a bubble column with aspect ratios ranging from 0.5 to 11.4. At low liquid depths the 'cooling tower' pattern was observed whereas at liquid depths exceeding unity two rows of staggered vortices, resembling the Von Karman Vortex Street were observed. As a validation step, our model was used to simulate the behaviour of bubble columns with various aspect ratios. The column considered in our computations was aerated through the central part of its cross-section, as opposed to the experimental set-up used by Chen *et al.* (1989) that was aerated over the entire cross-section. Table 2 lists the numerical value of every important variable used in the simulations.

Figure 3(a) shows a sequence of snapshots that depict the position of the bubbles and the associated

Table 2. Parameters used in computations to assess the effect of the aspect ratio on the flow structure, geometry and gas flow rate as used by Chen *et al.* (1989)

Column dimensions	
Width	0.1750 (m)
Height	
$L/D = 1.0$	0.1750 (m)
$L/D = 2.0$	0.3500 (m)
$L/D = 4.8$	0.8400 (m)
$L/D = 7.7$	1.3475 (m)
$L/D = 11.4$	1.9950 (m)
Superficial gas velocity	35.0 (mm s^{-1})
Physical properties (air-water)	
Density liquid	1000.0 (kg m^{-3})
Viscosity liquid	1.0×10^{-3} (Ns m^{-2})
Density gas	1.2 (kg m^{-3})
Bubble diameter	2.0×10^{-3} (m)
Computational parameters	
Number of computational cells in lateral direction	20
Number of computational cells in axial direction	
$L/D = 1.0$	20
$L/D = 2.0$	40
$L/D = 4.8$	96
$L/D = 7.7$	154
$L/D = 11.4$	228
Time step	5.0×10^{-3} [s]

instantaneous liquid-phase velocity field in a bubble column with an aspect ratio of 1.0 (grid used: 20×20 computational cells). It can be seen that the bubble plume rises along the column's centreline. In this case the computed bubble trajectories change remarkably little over 300 s of simulation time. The liquid velocity fields clearly exhibit the same characteristics, with liquid up flow in the column's centre (in the wake of the rising bubbles), and liquid down flow along the containing walls. This is the classical 'cooling tower' mode of circulation also observed by Chen *et al.*, at low column aspect ratios. The two circulation cells that constitute the typical 'cooling tower' flow pattern can be seen most clearly from the liquid velocity fields presented in Fig. 3(a).

Figure 3(b) depicts the position of the bubbles and the associated instantaneous liquid-phase velocity field in a bubble column with an aspect ratio of 1.0 after 400 s of simulation time. The grid used in this computation differs from the grid used to calculate the flow patterns in Fig. 3(a): in the case of Fig. 3(b) 25×25 computational cells were used. The flow structure depicted in this figure clearly resembles the classical 'cooling tower' mode of circulation, with liquid up flow in the wake of the rising bubbles and liquid down flow along the walls of the column. The use of computational cells smaller than those associated with the 25×25 grid is not recommended. This is because our model employs volume-averaged conservation equations that require computational cells that are relatively large with respect to the bubbles. It can,

therefore, be concluded that the numerical solution obtained by our model is not sensitive to changes in the grid size.

Figure 4 depicts snapshots of the flow pattern in a bubble column with an aspect ratio of 2.0. Again the instantaneous bubble positions and the correspond-

ing liquid-phase velocity fields are shown. Initially, the bubble plume rises in a rectilinear fashion along the centreline of the column. A 'cooling tower' mode of circulation seems to develop, as is evident from the liquid velocity field at 1.0 s after startup. With increasing operation time, however, the flow structure

Fig. 3(a). Computed structure of the two-phase gas-liquid flow in a bubble column with an aspect ratio of 1.0. Superficial gas velocity equals 35 mm s^{-1} . Both instantaneous bubble positions and liquid velocity fields are shown 10 s. after start up (I), 100 s. after start up (II) and 300 s. after start up (III). A 10 cm s^{-1} reference vector is provided with the liquid-phase velocity fields.

Fig. 3(b). Computed structure of the two-phase gas-liquid flow in a bubble column with an aspect ratio of 1.0. Superficial gas velocity equals 35 mm s^{-1} . Both instantaneous bubble positions and liquid velocity fields are shown 400 s. after start up. A 10 cm s^{-1} reference vector is provided with the liquid phase velocity field. Grid 25×25 computational cells.

changes to a more complex one with two staggered circulation cells, one cell in the lower left corner and one cell in the upper right corner of the column. These circulation cells influence the bubble plume causing it to rise through the bubble column along an S-shaped path. From Fig. 4, and from video representation of the computed results, it could clearly be seen that the flow pattern in the bubble column changed remarkably little during the 120 s of simulation time. None of the circulation cells moved through the bubble column and the bubble plume did not change its position considerably.

If the aspect ratio of the bubble column is increased to 4.8 a highly dynamic, time-dependent flow pattern develops. This can be seen most clearly from Fig. 5, which depicts the instantaneous positions of the bubbles and the liquid-phase velocity fields at various times. From this Figure and video representation of the computed results, it could be inferred that the bubble plume continuously shifts from left to right and vice versa in the bubble column. The meandering shape and continuous movement of the bubble plume is caused by vortices that develop at the free surface on opposite sides of the column's centre and descend along one of the walls of the column. From Fig. 5 it can clearly be seen that the flow structure is dominated by rows of staggered vortices, which agrees well with the observations of Chen *et al.* Finally, it can be seen from Fig. 5 that the bubbles are dispersed over the cross section of the column. Delnoij *et al.* (1997) showed that this is most likely due to the lift force acting on the bubbles in the bubble plume. As a result of this dispersion, bubbles are caught by liquid flowing downwards along the column walls.

Reconsidering the computed flow structure at an aspect ratio of 2.0 and taking into account the predicted flow structure at an aspect ratio of 4.8, a gradual transition from the 'cooling tower' mode of circulation ($L/D = 1.0$) to the fully developed staggered vortices configuration ($L/D = 4.8$) is predicted by our model.

Chen *et al.*, also investigated the flow structure in a bubble column with an aspect ratio of 7.7 and in a column with an aspect ratio of 11.4. The computed results for these cases are summarized in Figs 6 and 7. Despite the difference in aspect ratio, the results depicted in Figs 6 and 7 appear quite similar and will therefore be discussed together. Figs 6 and 7 indicate that both bubble columns consist of two distinctly different sections. The flow structure in the first section, or the lower section, of the bubble column resembles that of the multiple staggered vortices mode of operation as was observed in the bubble column with an aspect ratio of 4.8. In the lower section of the bubble column a distinct bubble plume can be observed; its behaviour is highly dynamic and it is influenced by staggered vortices that develop at the 'interface' between the lower and upper section of the bubble column. These vortices move downward along the walls of the column. In the second section, or the upper section of the column, the bubbles are dispersed over the entire cross section of the column; the bubble plume has apparently spread out due to lift forces acting on the bubbles and velocity differences in the liquid phase. The liquid velocity field in the upper section of the column is very different from the velocity field in the lower section. Vortices cannot be observed, and the flow field resembles the classical flow field thought to prevail in the homogeneous regime, with liquid up flow in the wake of a bubble and liquid down flow in between the bubbles. Finally, it has to be noted that Chen *et al.*, did not observe these two different sections; instead they observed vortices over the entire height of the column.

4.2. Effect of column aspect ratio—comparison with experimental observations

In addition to model simulations, experiments have been conducted in the pseudo two-dimensional bubble column described in Section 3. An important theoretical result is the predicted transition of the flow structure from the 'cooling tower' flow pattern to the

Fig. 4. Computed structure of the two-phase gas-liquid flow in a bubble column with an aspect ratio of 2.0. Superficial gas velocity equals 35 mm s^{-1} . Both instantaneous bubble positions and liquid velocity fields are shown 1 s after start up, 60 s after start up and 120 s after start up. A 10 cm s^{-1} reference vector is provided with the liquid-phase velocity fields.

flow pattern with staggered vortices. At an aspect ratio of 1.0 the model predicted a pseudo steady state flow pattern, with the bubble plume rising along the center of the bubble column. This bubble plume

did not oscillate. At an aspect ratio of 4.8 however, the model predicted the occurrence of a strongly time-dependent flow structure with an oscillating bubble plume. In order to assess the validity of these

Fig. 5. Computed structure of the two-phase gas-liquid flow in a bubble column with an aspect ratio of 4.8. Superficial gas velocity equals 35 mm s^{-1} . Both instantaneous bubble positions and liquid velocity fields are shown 20 s after start up, 40 s after start up and 60 s after start up. A 10 cm s^{-1} reference vector is provided with the liquid phase velocity fields.

computational results the frequency of oscillation of the bubble plume at various aspect ratios was determined experimentally. This frequency of oscillation was obtained from longtime video observation of the bubble plume in columns with aspect ratios ranging from 1.0 to 7.0.

Figure 8 shows the frequency of oscillation of the bubble plumes as a function of the aspect ratio and the superficial gas velocity. It can clearly be seen that the frequency of oscillation of the bubble plume decreases rapidly as the aspect ratio of the bubble column approaches unity. Additionally, the frequency of oscillation is observed to be fairly constant at aspect ratios exceeding 3.0. Based upon these observations there appears to exist a transition region, corresponding to aspect ratios ranging from 1.0 to 3.0, where the frequency of oscillation increases with increasing aspect ratio. This behaviour is most likely due to a transition from the 'cooling tower' mode of circulation ($L/D = 1.0$) to the staggered vortices flow pattern ($L/D > 3.0$). It is therefore concluded that these observations support the results obtained with our model.

In a separate experiment, neutrally buoyant tracer particles were added to the two-phase flow in order to visualize the vortical structures present in the liquid. Figure 9 shows these structures in a bubble column with an aspect ratio of 2.2 (additional parameter

values are listed in Table 3). The bubble plume exhibits an oscillatory behaviour; Fig. 9 shows the two extreme positions of the bubble plume. The vortices determining the structure of the two-phase flow can clearly be seen from these streak photographs. The bubble plume is influenced by a large vortex, pushing it towards the left or towards the right wall of the column. These vortices were observed to develop at the free surface and to descend along the walls of the bubble column, thereby causing the time-dependent, oscillatory behaviour of the bubble plume.

The time-dependent behaviour of this pseudo two-dimensional bubble column has been investigated with the model presented in this paper. Figure 10 depicts the instantaneous bubble positions and associated liquid velocity field 50 s after gas was first supplied to the column. The parameter values used in the computation are listed in Table 3. Comparison of Fig. 9 with Fig. 10 indicates points of difference and points of resemblance between experiment and theory. At first sight, the bubble plumes in Figs 10 and 9 show similar characteristics; the model predicts the S-shaped path followed by the bubbles and it also predicts the large vortex near the left wall of the column. This vortex however, does not move downward along the wall of the column as was observed in the experimental set-up. The model also overestimates the amount of bubbles captured by

Fig. 6. Computed structure of the two-phase gas-liquid flow in a bubble column with an aspect ratio of 7.7. Superficial gas velocity equals 35 mm s^{-1} . Both instantaneous bubble positions and liquid velocity fields are shown 30 s after start up, 50 s after start up and 70 s after start up. A 10 cm s^{-1} reference vector is provided with the liquid-phase velocity fields.

the downward liquid motion along the walls of the column.

5. CONCLUSIONS

In the present paper an Eulerian/Lagrangian model for a gas-liquid bubble column has been presented. The model resolves the time-dependent, two-dimensional motion of small, spherical gas bubbles in a liquid. The model incorporates all relevant forces acting on a bubble in a liquid, and additionally accounts for direct bubble-bubble interactions. The liquid-phase hydrodynamics is described using the volume-averaged Navier-Stokes equations.

This model has been used to investigate the effect of the aspect ratio of the bubble column on the overall

liquid circulation pattern. This effect has been studied experimentally by Chen *et al.* (1989). Computer simulations were performed for bubble columns with aspect ratio of 1.0, 2.0, 4.8, 7.7 and 11.4. A clear transition in the gas-liquid flow pattern could be observed. At an aspect ratio of 1.0 the 'cooling tower' flow structure prevailed. This flow structure is characterized by two circulation cells with liquid up flow in the centre of the column and liquid down flow along the walls of the column. At an aspect ratio of 2.0, the staggered vortices mode of circulation prevailed, although at this aspect ratio the vortices were not observed to move downward along the walls of the column. Increasing the aspect ratio to 4.8 revealed a highly dynamic flow pattern with multiple staggered vortices. These vortices were observed to develop at the free surface, and to subsequently, descend along

Fig. 7. Computed structure of the two-phase gas-liquid flow in a bubble column with an aspect ratio of 11.4. Superficial gas velocity equals 35 mm s^{-1} . Both instantaneous bubble positions and liquid velocity fields are shown 30 s after start up, 60 s after start up and 90 s after start up. A 10 cm s^{-1} reference vector is provided with the liquid-phase velocity fields.

the walls of the column, thereby causing the bubble plume to oscillate. At aspect ratios of 7.7 and higher, the flow structure in the bubble column was found to consist of two different regions. A clear bubble plume and a time-dependent, multiple staggered vortex mode of circulation, characterize the lower region. In the upper region of the bubble column, bubbles are dispersed over the entire cross section of the bubble column and vortices do not appear in this section of the column.

Finally, experimental results obtained in a pseudo-two-dimensional bubble column, support our model findings. The frequency of oscillation of the bubble plume was determined experimentally for several aspect ratios and the data obtained indicate that a clear

and distinct transition in flow pattern occurs in case the aspect is changed from 1.0 to 3.0. At $L/D = 1.0$ the frequency of oscillation approached zero, indicating the 'cooling tower' mode of liquid circulation whereas at $L/D > 3.0$ the frequency of oscillation approached a constant value resembling the staggered vortices mode of circulation.

Neutrally buoyant tracer particles were used to visualize the vortices in a pseudo two-dimensional bubble column with an aspect ratio of 2.2. The computed flow structure showed strong resemblance to the experimentally observed flow pattern although the time-dependent behaviour predicted by our model differed from that observed in the bubble column.

Fig. 8. Frequency of oscillation of the bubble plumes as a function of the aspect ratio at various superficial gas velocities. Based on measurements in a pseudo-two-dimensional air-water bubble column.

Fig. 9. Streak photographs depicting flow structure in a pseudo-two-dimensional bubble column with an aspect ratio of 2.2. Neutrally buoyant tracer particles were used to visualize the liquid flow field. Additional parameter values are listed in Table 3.

Table 3. Conditions used in experimental study of flow structure in pseudo-two-dimensional bubble column

Column dimensions	
Width	0.2500 (m)
Height	
$L/D = 2.2$	0.5500 (m)
Superficial gas velocity	2.0 (mm s ⁻¹)
Physical properties (air-water)	
Density liquid	1000.0 (kg m ⁻³)
Viscosity liquid	1.0×10^{-3} (Ns m ⁻²)
Density gas	1.2 (kg m ⁻³)
Bubble diameter	2.0×10^{-3} (m)
Computational parameters	
Number of computational cells in lateral direction	25
Number of computational cells in axial direction	
$L/D = 2.2$	55
Time step	5.0×10^{-3} (s)
Tracer particles	
Type	XAD - 7
Particle size	250 (μ m)
Density	1070 (kg m ⁻³)
Terminal velocity in water	2.7×10^{-3} (m s ⁻¹)
Photography	
Exposure time	0.1250 (s)

NOTATION

μ_l	shear viscosity liquid, kg m ⁻¹ s ⁻¹
A	area, m ²
C_D	drag coefficient, dimensionless
C_L	lift force coefficient, dimensionless
C_{VM}	virtual mass coefficient, dimensionless
D	diameter of the bubble column, m
D_b	bubble diameter, m
Δt	time step, s
Δx	grid size in x direction, m
Δy	grid size in y direction, m
E	unit tensor, dimensionless
ϵ_l	volume fraction liquid in computational cell, dimensionless
ϕ	local momentum transfer gas to liquid, N
Φ	volume averaged momentum transfer, N m ⁻³
F_D	drag force on a bubble, N
F_G	force on bubble due to gravity, N
F_L	lift force on bubble, N
F_p	force on bubble due to pressure gradient, N
F_{total}	total force on a bubble, N
F_{VM}	virtual mass force on bubble, N
g	acceleration due to gravity, m s ⁻²
i	bubble index number, dimensionless
I	Kelvin impulse deformable body, Ns

Fig. 10. Computed structure of the two-phase gas-liquid flow in a bubble column with an aspect ratio of 2.2. Superficial gas velocity equals 2 mm s⁻¹. Both instantaneous bubble positions and liquid velocity fields are shown. A 10 cm s⁻¹ reference vector is provided with the liquid-phase velocity field.

L	height of the bubble column, m
λ_l	bulk viscosity liquid, $\text{kg m}^{-1} \text{s}^{-1}$
m_b	mass of a bubble, kg
P	pressure, N m^{-2}
$\mathbf{r}$	position of bubble, m
R_b	radius of a bubble, m
Re_b	Reynolds number for flow around a bubble, dimensionless
ρ_g	density gas phase, kg m^{-3}
ρ_l	density liquid phase, kg m^{-3}
τ_l	stress tensor liquid, N m^{-2}
$\mathbf{u}$	liquid velocity, m s^{-1}
$\mathbf{v}$	velocity of bubble, m s^{-1}
V_b	volume of a bubble, m^3
V_{cell}	volume of a computational cell, m^3
Ω	vorticity in liquid phase, s^{-1}

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