Nanopositioner actuator energy cost and performance

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This article was accepted for publication in *Sensors & Actuators: A. Physical*, doi:10.1016/j.sna.2013.05.020

Abstract

We investigate the energy consumption and seek-time performance of different actuator types for nanopositioners, with emphasis on their use in a parallel-probe-based data-storage system. Analytical models are derived to calculate the energy consumption and performance of electrodynamic (coil and permanent magnet) and comb-drive actuators. The equations are used to simulate the operation of probe-storage devices with these actuator types under a realistic file system load. The electrostatic comb-drive actuators are more energy efficient than the electrodynamic actuators, by an order of magnitude for slow movements and a factor of 2.5 for high-acceleration movements. Overall in a probe-storage device, comb-drive actuation is a factor of 3.3 more energy efficient than electrodynamic actuation, at the same level of performance. The analytical model presented in this work can be used to direct the optimization of nanopositioners and their use, for example, in terms of the data layout on the medium and the ‘shutdown’ policy of probe-storage devices.

Keywords: MEMS, nanopositioner, energy cost, electrostatic comb drive, electrodynamic actuator

1. Introduction

Accurate nanopositioning is an essential component in many applications on the micro and nano scales, most notably in scanning probe microscopy. A two-dimensional nanopositioner, or ‘scanner’, features a scan table suspended by springs that is moved in x and y directions by a set of actuators. This scan table is actuated with nanometer precision. The ability to position objects on the nanometer scale enables exciting new applications. Examples of applications that require a nanopositioner are probe-based data storage [1], dip-pen lithography [2], 3D nanopatterning [3], assembly of nanoscale parts [2], and precision manipulation, probing, or injection of biological samples such as cells or DNA [4, 2, 5]. For portable applications, the scanner has to be shock-resistant, small, and also energy efficient.

In this work, we present an analytical model to calculate the lower bound on the energy cost and seek performance of nanopositioners. The analytical model is developed for generic nanopositioning tasks: staying at a fixed location, moving at constant speed, and moving to a location as fast as possible (seeking).

We will discuss the model in the framework of a parallel-probe-based data storage device because of the challenging requirements it puts on the nanopositioner and because of the relative maturity of the technology. Parallel-probe-based data storage, or ‘probe data storage’ in short, is a storage technology that combines advances in microelectromechanical systems (MEMS) with scanning-probe-based microscopy [6]. A probe data-storage system contains an array of atomic-force-microscope-like probes that read and write data to a medium on a movable platform. The nanopositioner’s scan table forms the medium platform and is moved relative to the array of probes. See Ref. [1] for a recent review on probe storage.

Several actuator types have been proposed for probe data-storage devices, including electrostatic comb-drives [7, 8, 9, 10, 11], electrodynamic actuation [12, 13, 14], and piezoelectric actuation with mechanical amplification [15]. Electrostatic comb-drive actuation and electrodynamic actuation have been studied extensively for probe storage and will be compared in this article. To the best of our knowledge, a complete probe data-storage prototype has only

(a) IBM's probe storage nanopositioner uses electrodynamic actuators (image reprinted from [16]).

(b) Nanopositioner using electrostatic comb-drives [17].

Figure 1: Two probe data-storage scanners (approx. $2\text{ cm} \times 2\text{ cm}$). Both use similar mass-balancing suspensions for enhanced shock-resistance, but different actuator types.

been reported by IBM, whose prototype uses electrodynamic actuation [16, 6] and is shown in Fig. 1a. Fig. 1b shows a scanner with the same mechanical structure, but with electrostatic comb-drive actuators instead [17]. Electrostatic comb-drive actuation may offer advantages over electrodynamic actuation in terms of both fabrication and energy costs.

The seek performance of electrostatic comb-drive and electrodynamic actuators has been extensively studied in Refs. [18, 19, 20]. In this work, we extend those results to comb drives that are optimized for shock-resistant nanopositioning [11]. The energy cost of using an electrodynamic actuator in a probe data-storage system has been studied by Khatib [21]. However, the differences in energy cost of the different actuator types have not yet been studied in detail. Unfortunately, the scanners shown in Figure 1 differ too much in for example their suspension stiffness to make a fair comparison. In this work, the characteristics of the scanner in IBM's prototype [14] are used to define designs for the two actuator types, which can then be compared with each other. The performance and energy cost for basic operations (idle, reading/writing, and seeking) are presented. Also, results are presented of a simulated workload using measured file-system traces [21] for a more complete comparison.

The damping by air is relatively small and will be neglected in the calculations to simplify the analysis and to obtain more compact equations without losing much accuracy of the estimates. Moreover, the energy lost because of air damping is similar for all actuator types and would not contribute to differences between them. The displacement sensors, control loops and driving circuitry are omitted in the discussion. Here we focus on comparing the actuation mechanisms themselves to obtain a lower bound on the energy cost using 100% efficient driving electronics. Because each actuator type will require a specific type of driving circuitry, there may be fundamental differences in obtainable driving-circuit efficiencies that future research could address.

2. General aspects

A general actuation subsystem of a nanopositioner is shown in Fig. 2. The scan table, or medium sled in a probe-storage system, is suspended by springs such that it is constrained to move along the x and y axes. Two actuators

Figure 2: General nanopositioner actuation system. R_{air} will be neglected in the derivations.

Table 1: Typical characteristics of a probe data-storage scanner, compiled from Refs. [14, 22].

Quantity	Symbol	Value
Actuated mass	m	100 mg
Suspension stiffness	k	90 N m^{-1}
Avail. force at $x = 0$	$F_{\text{avail},0}$	5 mN
Scan range		$\pm 50 \mu\text{m}$
Read velocity	v_{read}	1 mm s^{-1}
Write velocity	v_{write}	15 mm s^{-1}

enable independent movement in positive and negative x and y directions. The spring suspension is made such that the x and y axes are almost completely decoupled and can be treated as being independent of each other. Each axis can be modelled accurately by a second-order mass-spring-damper system [14, 10]. The total spring stiffness k is the sum of the medium sled suspension stiffness k_{table} and the actuator suspension stiffness k_{actuator} ,

$$k = k_{\text{table}} + k_{\text{actuator}}. \quad (1)$$

The total actuated mass m is the sum of the masses of the medium sled and the actuator. In case of a mass-balanced system [14, 10], m equals the effective total masses on the actuator side of the pivot. Characteristic values for the actuation subsystem are listed in Table 1. Note that the read and write velocities differ.

The displacement x ranges from $-50 \mu\text{m}$ to $50 \mu\text{m}$. The scanner is assumed to be a symmetric device, behaving the same for negative and positive displacements. The actuation mechanisms are symmetric under transformations $x \mapsto -x$ and $y \mapsto -y$. Using this symmetry, the text assumes that all motion is in the positive direction, $x_0 < x_1$. If $x_0 > x_1$, a mirror transformation $\mathcal{M}$ should be applied to obtain correct results,

$$\mathcal{M} = \{x \mapsto -x, v \mapsto -v\}. \quad (2)$$

In general, the x and y axes have different characteristic values, and the scanner is therefore not symmetric under transformation $\{x \mapsto y, y \mapsto x\}$. There is a ‘fast’ axis with a higher resonance frequency than the other, ‘slow’, axis. We will assign the x axis to the ‘fast’ axis along which data is read, and y to the ‘slow’ axis. We will assume the

Figure 3: Mechanical model and electrical circuit for one axis of an electrostatic comb-drive scanner.

conventional x/y -rasterized read-out path, where the y axis is used to select a row of bits that is read by scanning along the x axis (conforming to Ref. [23], but opposite to Ref. [24]).

At any given moment, the scanner is in one of four modes: idle, stay, read/write, seek. The behaviour of the scanner can be described as a consecutive string of modes. For example: idle, seek(0 to 25), read(25 to 26), seek(26 to 12), write(12 to 11), stay(11), write(11 to 10), seek(10,0)¹, idle. In idle mode, no energy is consumed by the actuator. The energy cost of the other modes will be denoted as follows: (1) staying at a certain displacement x for some time t , $E_{\text{stay}}(x, t)$; (2) moving from x_0 to x_1 at speed v , $E_{\text{read}}(x_0, x_1, v)$; and (3) moving as quickly as possible from x_0 with initial velocity v_0 to a new displacement x_1 with final velocity v_1 , $E_{\text{seek}}(x_0, x_1, v_0, v_1)$. The seek time will be denoted as $t_{\text{seek}}(x_0, x_1, v_0, v_1)$. ‘Bang-bang’ control, where the driving signal is always at its maximum, is assumed for seek operations. ‘Bang-bang’ control is time-optimal for linear systems [25] and for the scanners discussed here, because the force is, even if non-linear, a strictly increasing function of the input voltage or current. The actual control output in IBM’s prototype is quite similar to ‘bang-bang’ control [26].

3. Comb-drive actuation

The force generated by an electrostatic comb drive equals

$$F_{\text{comb}} = \frac{1}{2} \frac{\partial C}{\partial x} V^2, \quad (3)$$

where x is the displacement, C the capacitance between the electrodes, and V the applied voltage [27]. The generated force is always attractive, such that two comb drives are required to actuate in both positive and negative x directions. Comb drives suffer from energy losses due to electrical resistances. A mechanical model and the electrical circuit of one axis of a comb-drive scanner are shown in Fig. 3.

Analogous to using electrodes and an electric field to generate a force, it is possible to generate a force using interdigitated magnetic pole shoes and a magnetic field [28]. We will derive equations for an electrostatic comb drive, without losing their applicability to an electromagnetic comb drive. The necessary substitutions for magnetic comb drives have been detailed in Ref. [29]. In the following, we will assume the use of electrostatic comb-drives unless explicitly stated otherwise.

An optimized comb-drive finger shape was developed for use in a shock-resistant nanopositioner [11]. We will assume that this shark-fin ‘optim’ finger shape is used. Note that the ‘optim’ finger shape, with slight modifications, is also applicable to electromagnetic comb-drive actuators. The available force of ‘optim’-shaped comb drives including the suspension springs is constant,

$$F_{\text{avail,comb}}(x) = F_{\text{avail,0}}. \quad (4)$$

¹To save energy, one can decide to let the scanner passively settle to zero instead of actively moving to the rest position.

Using ‘optim’-shaped fingers, the ‘positive’ comb drive’s total capacitance depends non-linearly on the position,

$$C_+(x) = \frac{kx^2}{V_{\max}^2} + \frac{2F_{\text{avail},0}x}{V_{\max}^2} + C(0), \quad (5)$$

where $C(0)$ represents the capacitance at zero displacement (mostly due to the initial overlap between comb fingers). The ‘negative’ comb drive’s capacitance C_- is equal to C_+ with mirrored x ,

$$C_-(x) = C_+(-x) = \frac{kx^2}{V_{\max}^2} - \frac{2F_{\text{avail},0}x}{V_{\max}^2} + C(0). \quad (6)$$

The voltage V_{eq} required to position the comb drive in equilibrium at displacement x is

$$V_{\text{eq}}(x) = \pm \sqrt{\frac{k|x|V_{\max}^2}{k|x| + F_{\text{avail},0}}}, \quad (7)$$

The absolute value of x is used to allow for negative displacements when C_- is charged. Because of the quadratic nature of the force, the sign of the applied voltage is irrelevant. Hereafter, we will assume that the voltages applied to comb drives are positive.

Operating an electrostatic comb drive involves charging and discharging a capacitance. The stored energy $E_{\text{c,stored}}$ in equilibrium equals

$$E_{\text{c,stored}}(x) = \frac{1}{2}C(x)V_{\text{eq}}^2(x) + \frac{1}{2}kx^2.$$

The dissipated energy depends on the charge process. When charging fast, a high current i is required, and much energy is dissipated in R_s because the dissipated energy scales with i^2 . Charging slowly, much energy will be dissipated though R_p , because there will be a large voltage across R_p for a relatively long time. The optimal charge process for a capacitor, minimizing the energy cost with series and parallel resistors, is calculated in Appendix A. Instead of charging with a constant voltage, it is more energy efficient to charge with a voltage that increases as a function of time. For our parameter values and charging in 10 ns, the result shows that the dissipated energy is two orders of magnitude smaller than the stored energy. We can therefore approximate the required energy to charge the comb-drive capacitance, with resistors, simply by $\frac{1}{2}kx^2 + \frac{1}{2}C(x)V^2$.

4. Electrodynamic actuation

The probe data-storage prototype by IBM uses an electrodynamic actuator with permanent magnets and coils [30]. The actuator uses the Lorentz force generated by flowing a current through a wire in the magnetic field of permanent magnets. Fig. 4 shows the mechanical model of such an actuator. The permanent magnets and coils can be arranged in a ‘Lorentz-force’ configuration [31, 12, 13] or in a voice-coil configuration [14]. In the former, the coil slides in or out of the magnetic field, perpendicular to that magnetic field. In the latter, the coil moves between two permanent magnets, parallel to the magnetic field. The Lorentz-force and voice-coil configurations have the same characteristics and are treated as one.

The electrodynamic scanners in the literature are (to a good approximation) linear devices [31, 14, 12], where the current required for equilibrium at a certain displacement x is proportional to that displacement,

$$i_{\text{eq}}(x) = \frac{kx}{n}, \quad (8)$$

where n is the actuator force constant. This linearity implies that the actuator force is proportional to the applied current i and is independent of the displacement,

$$F_{\text{Ltz}} = ni.$$

This is consistent with a flux linkage λ that increases linearly with x ,

$$\lambda(x, i) = Li + n(x + x_0), \quad (9)$$

Figure 4: Mechanical model of an electrodynamic Lorentz force actuator.

where the inductance L of the coil is constant, and x_0 is the initial overlap of the coil with the magnetic field. For the Lorentz-force configuration, (9) is intuitively correct; it is also valid for voice-coil actuators linearised by the push-pull configuration of the permanent magnets [14]. The stored co-energy $E^*(x, i)$ in the actuator (without springs) then equals [32, chap. 3],

$$E^*(x, i) = \int_0^i \lambda(x, i') \, di' = \frac{1}{2} Li^2 + n(x + x_0)i,$$

leading to the force when the actuator is current controlled,

$$F_{\text{Ltz}} = \frac{\partial E^*(x, i)}{\partial x} = ni.$$

The energy E stored in the actuator equals [32, chap. 3],

$$E = \lambda i - E^* = \frac{1}{2} Li^2.$$

Note that here the stored energy is independent of x , in contrast to the stored energy for a comb-drive actuator. Whereas for a comb-drive actuator the force stems from the change in energy of both the source and the comb drive, the force of the electrodynamic actuator stems from the change in energy of only the source. The total stored energy (including springs) for the electrodynamic actuator equals

$$E_{\text{stored, Ltz}} = \frac{1}{2} Li^2 + \frac{1}{2} kx^2.$$

Given a maximum current i_{max} , the maximum actuator force is constant and equals

$$F_{\text{max, Ltz}} = i_{\text{max}} n. \quad (10)$$

Therefore, the available force has its maximum at $x = 0$ and decreases for increasing displacements, similar to the available force of an electrostatic comb-drive with straight fingers [11],

$$F_{\text{avail, Ltz}}(x) = F_{\text{avail, 0}} - kx. \quad (11)$$

The maximum obtainable equilibrium displacement x^* is the displacement at which $F_{\text{avail}} = 0$,

$$x^* = \frac{F_{\text{avail, 0}}}{k}. \quad (12)$$

As in the electromagnetic comb-drive, the dissipative element of the electrodynamic actuator is the resistance R_{coil} of the coil.

5. Seek-time calculations

To move a linear system from x_0 to x_1 as fast as possible, i.e. to perform a time-optimal seek operation, one must use the maximum available force to accelerate and then use the maximum available force to decelerate [25]. This is called ‘bang-bang’ control. This applies to the Lorentz actuator, where the force generated is independent of position. A comb drive with shaped fingers is a nonlinear system, because the force generated depends on the position. However, the comb-drive system can be rewritten such that the spring force and comb-drive force are combined as one effective actuator force (effectively becoming a system with $k = 0$). Using the ‘optim’ shape, the maximum of this effective actuator force (available force) is constant, and therefore ‘bang-bang’ control is also time-optimal for ‘optim’ comb drives.

We define the acceleration a_0 to be the total maximum acceleration (including springs) at $x = 0$ in positive direction,

$$a_0 = \frac{F_{\text{avail},0}}{m}, \quad (13)$$

defined by the specifications in Table 1. For ‘optim’ comb drives, the available force is constant (see (4)) and the maximum acceleration is independent of position and equal to a_0 ; in contrast, for the electrodynamic actuator the available force decreases with position (see (11)) and the maximum acceleration equals $a_0 - \frac{kx}{m}$. Because of this difference, the time t_{hw} and location x_{hw} of the transition from acceleration to deceleration will be different for each scanner type (the subscript ‘hw’ stands for ‘half-way’).

There is an important difference between the fast and the slow axis with respect to seeking: for the fast axis, along which data is read/written, there is an initial velocity v_0 and also a desired final velocity v_1 after the seek; for the slow axis, both the initial and the final velocity are zero.

The calculations below assume that the seek operation can be performed by applying first positive and then negative acceleration. The correct result for seek operations that first require negative acceleration and then positive acceleration, for example, a seek operation for which the start and the end point are identical, $x_0 = x_1$, with initial velocity zero, $v_0 = 0$, but with a positive final velocity, $v_1 > 0$, is obtained after applying mirror transformation $\mathcal{M}$.

5.1. Comb drives with ‘optim’ fingers

For the comb drives, the acceleration during a seek operation is independent of the displacement x and equal to a_0 . The difference between the final and the initial velocity is therefore equal to

$$v_1 - v_0 = a_0(t_{\text{accel}} - t_{\text{decel}}). \quad (14)$$

The distances travelled during the acceleration and deceleration phases, x_{accel} and x_{decel} respectively, are

$$x_{\text{accel}} = \int_0^{t_{\text{accel}}} v_0 + a_0 t \, dt, \quad (15)$$

$$x_{\text{decel}} = \int_0^{t_{\text{decel}}} (v_0 + a_0 t_{\text{accel}}) - a_0 t \, dt. \quad (16)$$

Combining (14), (15), and (16) with the knowledge that $x_{\text{accel}} + x_{\text{decel}} = x_1 - x_0$, we obtain a solution for the seek time,

$$t_{\text{seek,comb}} = 2 \sqrt{\frac{v_0^2 + v_1^2}{2a_0^2} + \frac{x_1 - x_0}{a_0}} - \frac{v_0 + v_1}{a_0}. \quad (17)$$

To calculate the seek energy cost, we need to know the half-way point x_{hw} (equal to x_{accel}),

$$x_{\text{hw,comb}} = \frac{x_0 + x_1}{2} + \frac{v_1^2 - v_0^2}{4a_0}, \quad (18)$$

and the half-way time t_{hw} , at which the deceleration phase begins (equal to t_{accel}),

$$t_{\text{hw,comb}} = \frac{-v_0 + \sqrt{v_0^2 + 2a_0(x_{\text{hw}} - x_0)}}{a_0}. \quad (19)$$

5.2. Electrodynamic actuator

For the electrodynamic actuator, the available force (11) decreases for increasing displacements. This means that seek operations in the positive direction will be faster for more negative displacements. Because data is read along the x axis, the seek time along x is more complex to calculate because the start and end velocities are not zero as is the case for the y axis. When the start and/or end velocities are negative, a turnaround happens, but the seek operation is still split into an acceleration phase and a subsequent deceleration phase.

The seek time for an electrodynamic scanner has been calculated by Hong and Brandt [24]. Their calculation of the half-way point assumes that the magnitudes of the start and end velocities are equal, $|v_0| = |v_1|$. The more general solution for the half-way point, valid for all velocities and $x_0 < x_1$, is

$$x_{\text{hw,Ltz}} = \frac{k}{4F_{\text{avail},0}} (x_1^2 - x_0^2) + \frac{m}{4F_{\text{avail},0}} (v_1^2 - v_0^2) + \frac{x_0 + x_1}{2}. \quad (20)$$

For negative velocities, Hong and Brandt [24] add an approximate turnaround time to the seek time. However, this turnaround time is only roughly accurate when $|v_0| = |v_1|$. In Appendix B, a more general solution for the seek time is derived that is valid for all velocities and $x_0 < x_1$,

$$t_{\text{seek,Ltz}} = \sqrt{\frac{m}{k}} \left[\arcsin \left(\frac{x_{\text{hw}} - x^*}{\sqrt{(x_0 - x^*)^2 + (v_0 \sqrt{\frac{m}{k}})^2}} \right) - \arctan \left(\frac{x_0 - x^*}{v_0 \sqrt{\frac{m}{k}}} \right) + \pi_0 \right. \\ \left. + \arcsin \left(\frac{-x_{\text{hw}} - x^*}{\sqrt{(x_1 + x^*)^2 + (v_1 \sqrt{\frac{m}{k}})^2}} \right) + \arctan \left(\frac{x_1 + x^*}{v_1 \sqrt{\frac{m}{k}}} \right) + \pi_1 \right], \quad (21)$$

where

$$\pi_i = \begin{cases} \pi & \text{if } v_i < 0, \\ 0 & \text{if } v_i \geq 0. \end{cases} \quad (22)$$

The maximum equilibrium displacement x^* is given by (12). The half-way time t_{hw} is equal to t_{accel} given in Appendix B.

6. Energy cost calculations

In idle mode, no voltage or current is applied on the actuator and no energy is consumed. The energy cost of the other three modes is discussed below.

6.1. Stay energy

When the scanner is held in equilibrium away from the origin, the voltage (or current) is held constant and no charging is required. However, energy must still be supplied because of the dissipative elements. For the electrostatic comb drive, there is leakage through R_p ; for the electrodynamic actuator, R_{coil} is dissipating energy. The dissipated power equals

$$p_{\text{stay}}(x) = \frac{R_p + R_s}{R_p^2} V_{\text{eq}}^2(x) = R_{\text{coil}} i_{\text{eq}}^2(x). \quad (23)$$

The energy cost of standing still at position x for time t equals

$$E_{\text{stay}}(x, t) = \frac{R_p + R_s}{R_p^2} V_{\text{eq}}^2(x) \cdot t = R_{\text{coil}} i_{\text{eq}}^2(x) \cdot t. \quad (24)$$

6.2. Read/write energy

A read/write operation uses a constant velocity v to move from x_0 to x_1 ,

$$x(t) = x_0 + vt. \quad (25)$$

As mentioned in section 2, we assume $x_0 < x_1$; note that this implies that $v > 0$.

During read and write operations, the applied voltage is increased or decreased relatively slowly. Therefore, the charge current during read/write for a comb-drive is much lower than the current used when rapidly charging the comb drive. The dissipation of the charge current can therefore be neglected. The total energy cost is the sum of the energy cost of an ideal scanner without resistors and the dissipated energy by R_p ,

$$E_{\text{read}}(x_0, x_1, v) = E_{\text{read,ideal}}(x_0, x_1) + E_{\text{read,diss}}(x_0, x_1, v). \quad (26)$$

The energy cost of a read/write operation for an ideal scanner is equal to the change in stored energy. Because the velocity is constant, there is no change in kinetic energy.

When both x_0 and x_1 are negative, $x_0 < x_1 < 0$, the (negative) actuator and spring should be slowly discharged, and no extra energy has to be supplied.

When x_0 and x_1 have opposite signs, $x_0 < 0 < x_1$, the stored energy at x_0 cannot easily be used to load the spring in the positive direction or to charge the actuator up to x_1 , because the directions are opposite. The stored energy at x_0 is completely discarded before charging the system to x_1 .

When x_0 and x_1 are both positive, the extra required energy is the difference between the stored energy at x_0 and x_1 .

These three cases can be summarized as follows,

$$E_{\text{read,ideal}}(x_0, x_1) = \begin{cases} 0 & \text{if } x_0 < x_1 \leq 0, \\ E_{\text{stored}}(x_1) & \text{if } x_0 \leq 0 < x_1, \\ E_{\text{stored}}(x_1) - E_{\text{stored}}(x_0) & \text{if } 0 < x_0 < x_1. \end{cases} \quad (27)$$

The dissipated energy is the integral of the dissipated power p_{diss} ,

$$E_{\text{read,diss}}(x_0, x_1, v) = \int_0^{\frac{x_1 - x_0}{v}} p_{\text{diss}} \, dt. \quad (28)$$

The dissipated power is equal to p_{stay} , see (23). Using (25),

$$E_{\text{read,diss}}(x_0, x_1, v) = \frac{1}{v} \int_{x_0}^{x_1} p_{\text{stay}}(x) \, dx, \quad (29)$$

6.2.1. Comb drives with ‘optim’ fingers

Working out the integral in (29) for ‘optim’ comb drives, using (7) and (23), we find

$$E_{\text{read,diss,comb}}(x_0, x_1, v) = \frac{R_p + R_s}{R_p^2} \frac{V_{\text{max}}^2}{v} \int_{x_0}^{x_1} \frac{|x|}{|x| + \frac{F_{\text{avail},0}}{k}} \, dx. \quad (30)$$

Because p_{diss} depends only on the instantaneous position and not on the direction of motion, the energy dissipated moving from x_0 to x_1 is equal to the dissipated energy when moving from x_1 to x_0 . Moreover, because the scanner is symmetric, the dissipated energy is equal for moving from 0 to x and moving from $-x$ to 0. Defining an intermediate solution f equal to the total dissipated energy for moving from 0 to x , or from x to 0 when $x < 0$,

$$f(x) = \frac{R_p + R_s}{R_p^2} \frac{V_{\text{max}}^2}{v} \int_0^{|x|} \frac{x'}{x' + \frac{F_{\text{avail},0}}{k}} \, dx', \quad (31)$$

we obtain

$$E_{\text{read,diss,comb}}(x_0, x_1, v) = \text{sgn}(x_1)f(x_1) - \text{sgn}(x_0)f(x_0), \quad (32)$$

$$f(x) = \frac{R_p + R_s}{R_p^2} \frac{V_{\text{max}}^2}{v} \left[|x| - \frac{F_{\text{avail},0}}{k} \ln \left(\frac{k|x| + F_{\text{avail},0}}{F_{\text{avail},0}} \right) \right], \quad (33)$$

where $\text{sgn}(x)$ is the sign function.

Note that decreasing V_{max} decreases the dissipated energy. However, the comb-drive size is inversely proportional to V_{max}^2 (to obtain an equal $F_{\text{avail},0}$), and R_p may decrease proportionally with the comb-drive size because the area through which leakage occurs increases, depending on the design. Therefore, the effect of decreasing V_{max} may be cancelled by a decrease in R_p .

6.2.2. Electrodynamic actuator

Working out the integral in (29) for the electrodynamic actuator, we find

$$E_{\text{read,diss,Ltz}}(x_0, x_1, v) = R_{\text{coil}} \frac{k^2}{3vn^2} \cdot (x_1^3 - x_0^3). \quad (34)$$

6.3. Seek energy

During a seek operation, the actuator moves from x_0 to x_1 as fast as possible, with initial velocity v_0 and final velocity v_1 . The calculations below assume that the seek operation can be performed by first accelerating the sled and subsequently decelerating it, see section 5.

6.3.1. Comb drives with ‘optim’ fingers

Seeking from x_0 to x_1 using comb drives can be subdivided in five steps: (1) C_- is discharged and C_+ is charged to V_{max} , (2) the sled is accelerated while the voltages are kept constant, (3) at x_{hw} , C_+ is discharged and C_- is charged to V_{max} , (4) the sled is decelerated while the voltages are kept constant, and finally (5) at x_1 , C_+ or C_- is charged up to $V_{\text{eq}}(x_1)$, for positive or negative x_1 , respectively. Because the mechanical resonance is much slower than the electrical resonance ($\sqrt{mk^{-1}} \gg RC$), the charging and discharging of C_- and C_+ are much faster than the movement of the sled. Therefore, the capacitance is considered constant during steps 1 and 3.

The energy cost of a seek operation is the sum of five terms corresponding to the five steps:

E_1 Charge C_+ .

$$E_1 = \frac{1}{2} V_{\text{max}}^2 C_+(x_0).$$

E_2 Keep voltage constant on C_+ . The required supply current i is equal to the current required to maintain V_{max} across the comb drive plus the leakage current through R_p ,

$$i = V_{\text{max}} \frac{dC_+}{dt} + \frac{V_{\text{max}}}{R_p}. \quad (35)$$

Using the values from Tables 1 and 2, the maximum current is on the order of tens of microamperes; the maximum voltage drop over R_s is on the order of millivolts and therefore negligible (note that this is also valid for the electromagnetic comb drive, because, in that case, $R_s \approx 0$). Because charging is relatively slow, the current is so small that dissipation in R_s can be neglected. The required energy equals

$$E_2 = \int_0^{t_{\text{hw}}} V_{\text{max}} i \, dt = V_{\text{max}}^2 \left[C_+(x_{\text{hw}}) - C_+(x_0) + \frac{t_{\text{hw}}}{R_p} \right]. \quad (36)$$

This energy E_2 minus the dissipative term is equal to the sum of the extra stored energies of the electric field in the comb drive, the potential energy in the springs and the kinetic energy of the moving mass, as can easily be confirmed using (5).

E_3 Discard energy in C_+ , charge C_- .

$$E_3 = \frac{1}{2} V_{\max}^2 C_-(x_{\text{hw}}).$$

E_4 Keep voltage constant on C_- .

$$E_4 = \max \left(V_{\max}^2 \left[C_-(x_1) - C_-(x_{\text{hw}}) + \frac{t_{\text{seek}} - t_{\text{hw}}}{R_p} \right], 0 \right). \quad (37)$$

Note that because the difference $C_-(x_1) - C_-(x_{\text{hw}})$ is negative, the total required energy including dissipation through R_p can be negative. Therefore, E_4 has an explicit lower limit of zero, because it is assumed that the excess energy is not regained.

E_5 Charge C_+ or C_- to the equilibrium voltage $V_{\text{eq}}(x_1)$. If $x_1 < 0$, C_- should be charged to $V_{\text{eq}}(x_1)$, but because C_- is already charged at $V_{\max}$, C_- has to be discharged which does not consume energy. For positive x_1 , C_+ must be charged from zero.

$$E_5 = \begin{cases} 0 & \text{if } x_1 \leq 0, \\ \frac{1}{2} V_{\text{eq}}^2(x_1) C_+(x_1) & \text{if } x_1 > 0. \end{cases}$$

The total seek energy equals

$$E_{\text{seek}}(x_0, x_1, v_0, v_1) = E_1 + E_2 + E_3 + E_4 + E_5. \quad (38)$$

6.3.2. Electrodynamic actuator

Seeking from x_0 to x_1 using an electrodynamic scanner can be subdivided in five steps similar to the steps for a comb-drive scanner. The charging steps are slightly different, because an electrodynamic scanner has one transducer (the coil) whereas a comb-drive scanner has two transducers (one comb drive for positive, and another for negative forces). First, (1) the coil is charged to $+i_{\max}$, then (2) the sled is accelerated while the current is kept constant, (3) at x_{hw} , the coil is discharged and subsequently charged to $-i_{\max}$, (4) the sled is decelerated while the current is kept constant, and finally (5) at x_1 , the coil is charged to $i_{\text{eq}}(x_1)$. Because the mechanical resonance is much slower than the electrical resonance ($\sqrt{mk^{-1}} \gg \frac{L}{R}$), the charging and discharging of the coil are much faster than the movement of the sled. Therefore, the displacement x is considered constant during steps 1 and 3.

The energy cost of a seek operation is the sum of five terms corresponding to the five steps:

E_1 Charge L .

$$E_1 = \frac{1}{2} L i_{\max}^2.$$

E_2 Keep current constant through the coil. The required supply voltage V is equal to the electromotive voltage generated by the coil plus the voltage across the series resistance R_{coil} ,

$$V = \frac{d\lambda}{dt} + i_{\max} R_{\text{coil}}. \quad (39)$$

The required energy equals

$$E_2 = \int_0^{t_{\text{hw}}} i_{\max} V dt = i_{\max} n (x_{\text{hw}} - x_0) + i_{\max}^2 R_{\text{coil}} t_{\text{hw}}. \quad (40)$$

This energy E_2 minus the dissipative term is equal to the sum of the extra stored energies of the springs and the kinetic energy of the moving mass, as can easily be confirmed from integration of the net force on the mass,

$$\Delta E_{\text{kin}} = \int_{x_0}^{x_{\text{hw}}} i_{\max} n - kx dx = i_{\max} n (x_{\text{hw}} - x_0) - \frac{1}{2} k (x_{\text{hw}}^2 - x_0^2).$$

E_3 Charge coil with opposite current $-i_{\max}$.

$$E_3 = \frac{1}{2} L i_{\max}^2.$$

Table 2: Actuator design specifications that meet the requirements of Table 1.

<i>Electrostatic comb-drive</i>	Initial capacitance	$C(0)$	15 pF
	Maximum voltage	$V_{\max}$	150 V
	Series resistance	R_s	100 Ω
	Parallel resistance	R_p	100 M Ω
<i>Electrodynamic</i>	Inductance	L	25 μ H
	Maximum current	$i_{\max}$	10 mA
	Force constant	n	0.5 N A ⁻¹
	Series resistance	R_{coil}	30 Ω
	Parallel resistance	R_{par}	∞

E_4 Keep current constant through the coil. The energy generated by the coil may be more than the dissipated energy in R_{coil} . Therefore, E_4 has an explicit lower limit of zero, because it is assumed that the excess energy cannot be regained.

$$E_4 = \max \left(-i_{\max} n (x_1 - x_{\text{hw}}) + i_{\max}^2 R_{\text{coil}} (t_{\text{seek}} - t_{\text{hw}}), 0 \right). \quad (41)$$

E_5 Charge the coil to the equilibrium current $i_{\text{eq}}(x_1)$. If $x_1 < 0$, the coil is already charged at $-i_{\max}$ and only needs discharging to reach $i_{\text{eq}}(x_1)$, which does not consume energy. For positive x_1 , the coil must be charged from zero current.

$$E_5 = \begin{cases} 0 & \text{if } x_1 \leq 0, \\ \frac{1}{2} L i_{\text{eq}}^2(x_1) & \text{if } x_1 > 0. \end{cases}$$

The total seek energy equals

$$E_{\text{seek}}(x_0, x_1, v_0, v_1) = E_1 + E_2 + E_3 + E_4 + E_5. \quad (42)$$

7. Results for theoretical designs

To compare the actuator types quantitatively, a design that meets the requirements specified in Table 1 is made for each actuator type. The characteristics of the theoretical devices are listed in Table 2. The specification of the available force at $x = 0$ is used together with a maximum current or maximum voltage to determine the inductance or capacitance of the actuator. The initial capacitance $C(0)$ of the comb-drive includes a typical bond-pad capacitance on the order of 1 pF [33]. The inductance L of the electrodynamic actuator is estimated from the coil dimensions in the design by IBM [14]. Other aspects of the designs are based on private communication with Mark A. Lantz, IBM Research - Zurich, and Ref. [10].

7.1. Seek time

The time for a seek from $x_0 = 0$ to x_1 with an initial velocity $v_0 = 0$ and a final velocity $v_1 = v_{\text{write}}$ is plotted in Figure 5 for the two actuator types. Because the desired final velocity v_1 in this case is positive (towards positive direction), the graph is asymmetric. In case of a seek to a negative location x_1 , extra time is needed to reverse direction at the end of the seek. Note that the scan table has to overshoot to a more negative position than x_1 to be able to accelerate and reach the positive v_1 at x_1 .

Although the electrodynamic available force is quite different from the comb-drive available force, the electrodynamic seek performance is roughly the same, but slightly faster: the higher deceleration force compensates for the lower acceleration force compared with the comb drive.

A seek time well within 2 ms is obtained for most seeks. Note that, in this case starting at rest from zero, negative seeks take 0.8 ms longer, because of the desired positive writing velocity at the end of the seek, which requires a turn around.

Figure 5: The seek time for seeking from $x_0 = 0$ to x_1 , with an initial velocity $v_0 = 0$ and a final velocity $v_1 = v_{\text{write}} = 15 \text{ mm s}^{-1}$. The asymmetry stems from v_1 being positive.

Figure 6: The dissipated power versus displacement in equilibrium, p_{stay} .

Figure 7: The energy cost of moving the scan table at read velocity, v_{read} , starting from $x_0 = -50\mu\text{m}$, versus the end displacement x_1 . The energy cost of reads from different x_0 can be obtained by subtracting the value at x_0 from the value at x_1 .

7.2. Stay energy

Fig. 6 shows the dissipated power for equilibrium displacements. The electrodynamic actuator requires about the same power as the electrostatic comb-drive when $|x| < 5\mu\text{m}$, but it consumes an order of magnitude more energy when $|x| > 30\mu\text{m}$.

7.3. Read/write energy

Fig. 7 shows the energy cost of read operations starting at $x_0 = -50\mu\text{m}$ with velocity v_{read} , as a function of the end displacement x_1 . Because

$$E_{\text{read}}(x_b, x_c, v) = E_{\text{read}}(x_a, x_c, v) - E_{\text{read}}(x_a, x_b, v),$$

the energy cost for a general read operation from x_0 to x_1 at velocity v can be obtained from Figure 7 by subtracting the value at x_0 from the value at x_1 . E_{read} scales approximately inversely proportional with the velocity,

$$E_{\text{read}}(x_0, x_1, v_a) \approx \frac{v_b}{v_a} E_{\text{read}}(x_0, x_1, v_b);$$

therefore, write operations ($v = v_{\text{write}}$) consume approximately 15 times less energy.

The electrostatic comb drive consumes one order of magnitude less energy than the electrodynamic actuator during read/write operations.

7.4. Seek energy

The energy cost of a seek operation starting at rest and ending at write speed at displacement x_1 is shown in Figure 8 for the three actuator types investigated. Note that seeking in the negative direction costs more energy, because the desired positive final velocity requires a turnaround. The electrostatic comb-drive consumes the least amount of energy. Averaging over 1000 searches where x_0 , x_1 , v_0 , and v_1 are randomly picked, shows that the electrodynamic actuator requires 5.8 times more energy than the electrostatic comb drive [29]. For very short seeks, the energy cost is approximately equal for both actuator types.

8. Probe-storage system study

We simulated the activity of a probe storage device serving in a computer system to compare the energy consumption of the two actuator types under a realistic workload. The positions and velocities of the actuators are affected by the file system, depending on the location of the data requested by the user. The probe storage device receives requests

Figure 8: The energy cost of seeking from $x_0 = 0$ to x_1 , with an initial velocity $v_0 = 0$ and a final velocity $v_1 = v_{\text{write}}$. Seeking in the negative direction costs more energy, because the desired positive final velocity requires a turnaround.

Table 3: Simulation result: energy consumption in milli Joules for different file systems.

		1k-ext2	1k-ext3	4k-ext2	4k-ext3
Electrodynamic	Seek	32.7	39.4	31.5	43.0
	Shutdown	13.7	19.8	12.9	18.5
	R/W	6.8	7.0	7.1	9.8
	Total	53.2	66.2	51.5	71.3
Comb drive	Seek	13.2	14.7	13.0	16.6
	Shutdown	2.4	3.7	2.2	3.1
	R/W	0.8	0.9	0.9	1.1
	Total	16.4	19.3	16.1	20.8

from the computer system to read or write data at a certain location indicated by a logical block address. The mapping of this logical address to a physical location on the storage medium is decided by the probe storage device. In the results presented here, we have assumed a mapping in which each probe has its own probe field, with each probe field being a grid of columns and rows, in which the cells are sequentially numbered along the fast scanning axis starting from one corner. Note that further study is needed to optimize the mapping specifically for a probe-storage device; for example, a serpentine pattern or an Archimedean spiral [34] may be preferable. The simulated device has 1024 probes that all are active for every request.

We use the DiskSim simulator [35]. We incorporated our models of the comb drive and electrodynamic actuators in DiskSim. To drive the simulator, we collected file system traces of the usage of a flash card plugged into a PDA. Typical user scenarios were played on the PDA, for which we captured the storage requests incurred. Two key parameters are varied for the simulated storage devices: the file system and the basic storage unit, called block. We chose two file systems, `ext2` and `ext3`, and two block sizes, 1 kB and 4 kB. The ‘Stay’ mode is not used in the simulation; the device performs an immediate ‘Shutdown’ (a low-power seek to the zero position) after a read/write operation has finished. The simulator calculates the total energy consumed and the average request response time. The response time is the time between the arrival of a request and the time all data requested is delivered. This is split in the time required for the ‘Seek’ phase and for the ‘Read/Write’ phase. The simulation method is described in detail in Ref. [21].

Table 4: Simulation result: response times in milliseconds for different file systems.

		1k-ext2	1k-ext3	4k-ext2	4k-ext3
Electrodynamic	R/W	4.69	3.67	5.30	4.58
	Seek	1.34	1.35	1.28	1.37
Comb drive	R/W	4.57	3.75	5.16	4.53
	Seek	1.52	1.60	1.49	1.60

8.1. Simulation results

Table 3 lists the simulation results of the energy consumption of the simulated probe storage devices. Across the four file-system settings, we find that the electrostatic actuators are as much as three times more energy-efficient than the electrodynamic actuators.

The energy figures are split into three components corresponding to the distinct operation modes Read/Write, Seek, and Shutdown. From Table 3, we see that the Seek mode has the largest share. This is because maximum ‘thrust’ and hence maximum actuator power are applied to reduce the time to satisfy a request. The electrodynamic scanner requires roughly 2.5 times more energy for seeking than the comb-drive scanner.

The second component is due to Shutdown, where the moving medium is brought back to rest. Effectively this mode is a reverse seek that is performed at low speed to reduce the energy cost. In this mode, the comb-drive scanner consumes between 5.4 and 6 times less energy than the electrodynamic scanner.

The third energy component is due to read/write energy. Here the difference in consumed energy is between 7.8 and 8.9 times. The contribution of the read/write energy to the total consumed energy is relatively small. The reason for the large difference between the energy of Seek as compared to that of Read/Write is two-fold. Firstly, a higher power (maximum current) is dissipated during seeking to attain short seek times. Secondly, the actuators travel only short distances to read/write data because of the large number of probes used simultaneously, which reduces the per-probe number of bits.

With regard to timing performance, both actuators show comparable response times. They have an average response time of 6 ms as shown in Table 4. The seek time is the same for both file systems because they share a similar data layout. A future study may look into the influence of the data layout on the seek time and optimize it specifically for probe storage. The seek time is approximately 0.2 ms less for the electrodynamic scanner than the comb-drive scanner.

9. Discussion

The seek performances of the electrostatic comb drive and the electrodynamic actuator are approximately equal, because their design is such that their maximum acceleration at $x = 0$ is equal. Because the performance is similar, the energy cost comparison provides a measure of the relative energy efficiency of these two actuator types.

The electrostatic comb drive is the most energy efficient for all three operation modes. The dissipation by R_p or R_{coil} dominates the energy cost of staying put and moving at read speed. Therefore, a good figure-of-merit for relatively slow movements is the dissipated power at maximum current or voltage, $i_{max}^2 R_{coil}$ for the magnetic scanner and $V_{max}^2 R_p^{-1}$ for the electrostatic comb drive. This figure-of-merit is 13 times smaller for the comb-drive scanner than for the electrodynamic scanner, corresponding to the difference in p_{stay} and E_{read} seen in Figures 6 and 7.

Realistic use of the nanopositioner in a probe-storage system results in many high-acceleration seek operations. Because of the high parallelization of probes in such a system, the scanner moves slowly (read/write) only for brief periods, and seek operations dominate the energy consumption. The energy cost difference for seek operations is less than the difference for the other operations. The dissipation by R_p or R_{coil} is an important factor, but the significant contribution from the energy cost of charging the capacitance or coil reduces the difference between the actuator types.

The ext2 file system is about 20-30% more energy efficient than ext3 because of the extra overhead caused by the file-system journalling in ext3 [20]. The extra energy is used to protect file-system consistency (no corrupt metadata) because ext3 writes the file-system-related metadata blocks twice for extra robustness against power failure. The block size does not have much influence on the energy used to read or write user data, because the user data is

typically much larger than a single block (audio and video). Increasing the block size mainly affects the metadata reading and writing (single blocks), and because the metadata is written twice in `ext3` we see an increase in energy consumed for an increased block size.

Electrostatic actuators are noticeably more energy efficient than their electrodynamic counterparts. This assumes that the efficiency of the driving electronics is similar for both actuator types. Note that the actuator power is only one part of the total energy consumption of a device. In a probe storage device the total read/write energy consumption may overshadow the energy consumed by the actuation system. The per-bit read/write energy can be as much as 1 nJ [36], but the fundamental limit for stable storage is many orders of magnitude lower.

The analytical equations can be used to optimize the use of the actuation system. For example, in a probe storage device, the energy consumption may be reduced by a less aggressive ‘shutdown’ policy where the scanner stays put for some time after a request before returning to its rest position instead of the currently assumed instant ‘shutdown’. The brief wait period can potentially reduce the seek distance between requests, reducing the currently large seek energy consumption. This reduction of seek energy cost would come at an energy cost of staying put, but because the energy consumption of comb drives is low when staying put, the less aggressive ‘shutdown’ policy can result in a lower net energy consumption.

10. Conclusion

The energy cost and seek time equations for comb-drive scanners and electrodynamic scanners are summarized in Table 5. The energy consumption and seek time were successfully simulated for probe storage devices with a comb-drive scanner design and an electrodynamic scanner design using these equations and a measured trace of file-system requests of a flash card in a PDA. Because the scanners are designed to have the same maximum acceleration at $x = 0$, their performances are approximately equal, resulting in a fair comparison of energy consumption. The probe-storage system simulations show that for a realistic workload most actuation energy is spent on high-acceleration seek operations. For the parameters used (Tables 1 and 2), the comb drives consume a factor of 3.3 less energy than the electrodynamic actuators.

High-acceleration movements require fast charging of the actuator capacitance or inductance, resulting in a large energy consumption. For the parameters used, seek operations with the electrostatic comb-drives cost roughly 2.5 times less energy than with the electrodynamic actuators. The energy consumption during slow movements is dominated by dissipation in the leakage resistor R_p parallel to the comb drive or in the series resistance R_{coil} of the coils. The electrostatic comb drive requires an order of magnitude less energy for standing still at a certain displacement and for moving at constant read or write speed than the electrodynamic actuator.

The equations for the energy consumption of both actuator types can be exploited to optimize the actuation system itself and its usage for a specific application. For probe data storage, interesting future research targets include optimization of the data layout on the medium and optimization of the ‘shutdown’ policy.

11. Acknowledgments

This work was supported by the Dutch Foundation for Science and Technology STW, applied science division of NWO and the technology program of the Ministry of Economic Affairs under project number TES.06369.

Table 5: Summary of energy cost and seek time for an electrostatic comb-drive scanner with shark-fin-shaped optimized fingers [11] and an electrodynamic scanner. When $x_0 > x_1$, mirror transform $\mathcal{M}$ must be applied first (see (2)).

	Optim' comb-drive	Electrodynamic actuator
$E_{\text{stay}}(x, t)$	$= \frac{R_p + R_s}{R_p^2} V_{\text{eq}}^2(x) \cdot t$	$= R_{\text{coil}}^2 \text{eq}(x) \cdot t$
$f(x) = \frac{R_p + R_s}{R_p^2} \frac{V_{\text{eq}}^2}{v} \left[x - \frac{F_{\text{wall},0}}{k} \ln \left(\frac{k x + F_{\text{wall},0}}{F_{\text{wall},0}} \right) \right]$	$= f(x_0) - f(x_1)$	$= \frac{R_{\text{coil}} k^2 (x_1^3 - x_0^3)}{3 v \eta^2}$
$E_{\text{read}}(x_0, x_1, v), x_0 < x_1 \leq 0$	$= f(x_0) - f(x_1)$	$= \frac{kx_1^2 + L_{\text{eq}}^2(x_1)}{2} + \frac{R_{\text{coil}} k^2 (x_1^3 - x_0^3)}{3 v \eta^2}$
$E_{\text{read}}(x_0, x_1, v), x_0 \leq 0 < x_1$	$= \frac{kx_1^2 + C_+(x_1) V_{\text{eq}}^2(x_1)}{2} + f(x_0) + f(x_1)$	$= \frac{kx_1^2 + L_{\text{eq}}^2(x_1)}{2} - \frac{kx_0^2 + L_{\text{eq}}^2(x_0)}{2}$
$E_{\text{read}}(x_0, x_1, v), 0 < x_0 < x_1$	$= \frac{kx_1^2 + C_+(x_1) V_{\text{eq}}^2(x_1)}{2} - \frac{kx_0^2 + C_+(x_0) V_{\text{eq}}^2(x_0)}{2}$ $+ f(x_1) - f(x_0)$	$= \frac{R_{\text{coil}} k^2 (x_1^3 - x_0^3)}{3 v \eta^2}$
$x_{\text{hw}} = \frac{x_0 + x_1}{2} + \frac{v_1^2 - v_0^2}{4a}$	$x_{\text{hw}} = \frac{x_0 + x_1}{2} + \frac{v_1^2 - v_0^2}{4a}$	$x_{\text{hw}} = \frac{x_0 + x_1}{2} + \frac{v_1^2 - v_0^2}{4a} + \frac{k}{4 F_{\text{wall},0}} (x_1^2 - x_0^2)$
$t_{\text{seek}}(x_0, x_1, v_0, v_1)$	$= 2 \sqrt{\frac{v_0^2 + v_1^2}{2a^2} + \frac{x_1 - x_0}{a}} - \frac{v_0 + v_1}{a}$	$= \sqrt{\frac{\pi}{k}} \left[\arcsin \left(\frac{x_{\text{hw}} - x^*}{\sqrt{(x_0 - x^*)^2 + (v_0 \sqrt{\frac{\pi}{k}})^2}} \right) - \arctan \left(\frac{x_0 - x^*}{v_0 \sqrt{\frac{\pi}{k}}} \right) + \pi_0 \right]$ $+ \arcsin \left(\frac{-x_{\text{hw}} - x^*}{\sqrt{(x_1 + x^*)^2 + (v_1 \sqrt{\frac{\pi}{k}})^2}} \right) + \arctan \left(\frac{\frac{x_1 + x^*}{v_1 \sqrt{\frac{\pi}{k}}}} \right) + \pi_1]$ $\pi_i = \begin{cases} \pi & \text{if } v_i < 0 \\ 0 & \text{if } v_i \geq 0 \end{cases}$
$E_{\text{seek}}(x_0, x_1, v_0, v_1)$	$= V_{\text{max}}^2 \left[C_+(x_{\text{hw}}) - \frac{1}{2} C_+(x_0) + \frac{1}{2} C_-(x_{\text{hw}}) \right]$ $+ V_{\text{max}}^2 \frac{I_{\text{hw}}}{R_p}$ $+ \max \left(0, V_{\text{max}}^2 \left[C_-(x_1) - C_-(x_{\text{hw}}) + \frac{t_{\text{seek}} - I_{\text{hw}}}{R_p} \right] \right)$ $+ \begin{cases} 0 & \text{if } x_1 \leq 0 \\ \frac{1}{2} V_{\text{eq}}^2(x_1) C_+(x_1) & \text{if } x_1 > 0 \end{cases}$	$= L_{\text{max}}^2$ $+ i_{\text{max}} n (x_{\text{hw}} - x_0) + i_{\text{max}}^2 R_{\text{coil}} I_{\text{hw}}$ $+ \max \left(0, -i_{\text{max}} n (x_1 - x_{\text{hw}}) + i_{\text{max}}^2 R_{\text{coil}} (t_{\text{seek}} - I_{\text{hw}}) \right)$ $+ \begin{cases} 0 & \text{if } x_1 \leq 0 \\ \frac{1}{2} L_{\text{eq}}^2(x_1) & \text{if } x_1 > 0 \end{cases}$

Appendix A. Optimal charging of a capacitor

Here we will derive the energy-optimal method of charging a capacitor without and with parallel resistor through a series resistor within a certain period T .

Appendix A.1. Without leakage resistor

To charge an ideal capacitance without parallel leakage resistor R_p through a series resistor R_s , intuitively the optimal method is charging with a constant current. This can be proved as follows.

After transferring the required charge $Q = \int_0^T i \, dt$ to the capacitor, the total dissipated energy equals $E_{\text{diss}} = R_s \int_0^T i^2 \, dt$. The optimal charge method minimizes the integral of the square of the current (the dissipated energy), for a fixed integral of the current (the charge).

The solution to this minimization problem can be found at home. Consider a cross section of a basin filled with water. The waterlevel is flat because that is the state with lowest energy. The cross section has a certain width w , and our calculations are per unit length of the basin (perpendicular to the section). The water height $h(x)$ can have a different value at each position x along the width. The total amount of water in the basin equals $A = \int_0^w h \, dx$. The gravitational potential energy of a column of water of width Δx equals $\frac{\rho g}{2} h^2 \Delta x$. The total potential energy therefore equals $P = \frac{\rho g}{2} \int_0^w h^2 \, dx$. Note that minimizing P with constraint A is equivalent to the optimal charging problem of an ideal capacitor. The solutions are therefore the same: the optimal water level $h(x)$ is constant in the basin (a flat surface); the optimal charge current $i(t)$ is a constant current. Thus, the energy dissipated while charging a capacitance C without leakage within a certain period T up to a voltage V equals $E_{\text{diss}} = R_s C^2 V^2 / T$. Note that one can charge an ideal capacitor with zero dissipated energy if infinite time is available!

Appendix A.2. With leakage resistor

The following calculation focusses on obtaining the optimal curve for the voltage across the capacitance V , from which the charge current can be inferred.

The dissipation power equals the sum of the dissipation in R_s from the capacitor current $C \frac{dV}{dt}$ and the dissipation in R_p and R_s by the current through R_p ,

$$p_{\text{diss}} = R_s C^2 \left(\frac{dV}{dt} \right)^2 + \frac{V^2}{R_p^2} (R_s + R_p). \quad (\text{A.1})$$

Defining

$$a = \sqrt{\frac{1}{R_s C^2} \frac{R_s + R_p}{R_p^2}} \approx \sqrt{\frac{1}{R_p R_s C^2}}, \quad (\text{A.2})$$

we obtain the minimization problem

$$\min_{V(t)} E_{\text{diss}}[V(t)]; \quad E_{\text{diss}} = R_s C^2 \int_0^T \left(\frac{dV}{dt} \right)^2 + a^2 V^2 \, dt, \quad (\text{A.3})$$

with boundary conditions

$$V(t=0) = V_1, \quad (\text{A.4})$$

$$V(t=T) = V_2, \quad (\text{A.5})$$

where the capacitor is pre-charged at V_1 and T is the time at which we want the capacitor to be charged at V_2 .

Using the Euler-Lagrange equation [37, chap. 5], minimization problem (A.3) with boundary conditions (A.4) and (A.5) is equivalent with the differential equation

$$\frac{d^2 V}{dt^2} - a^2 V = 0. \quad (\text{A.6})$$

Figure A.9: Optimal charge voltage waveform, minimizing the energy cost. Charge from $V_1 = 20\text{V}$ to $V_2 = 50\text{V}$ in a certain time $T = \{0.2, 0.05, 0.015\}\text{ms}$. $C = 45\text{pF}$ and $R_s = 100\Omega$.

Note that for infinite R_p , $a = 0$ and the solution for V becomes a linear curve, consistent with Appendix A.1. The solution for V is

$$V = De^{at} + Fe^{-at}, \quad (\text{A.7})$$

$$D + F = V_1, \quad (\text{A.8})$$

$$D = \frac{V_2 - V_1 e^{-at}}{e^{at} - e^{-at}}. \quad (\text{A.9})$$

The optimal charge current equals

$$i = C \frac{dV}{dt} + \frac{V}{R_p} = \sqrt{\frac{1}{R_p R_s}} (De^{at} - Fe^{-at}) + \frac{V}{R_p}. \quad (\text{A.10})$$

Because $R_s \ll R_p$, the optimal supply voltage is approximately equal to voltage V across the capacitor.

Fig. A.9 shows the optimal voltage curves for different charge times T and for two values of R_p . Except for charge times below $50\mu\text{s}$, it is most energy efficient to let the capacitor discharge, before charging it up to the desired voltage; when leaving for holidays, it is better to turn off the heating and fully reheat the house upon returning, instead of maintaining the small steady resupply of heat to compensate for heat loss (leakage) to the outside world.

When V_1 is zero, the solution for V simplifies to

$$V = V_2 \frac{e^{at} - e^{-at}}{e^{aT} - e^{-aT}}. \quad (\text{A.11})$$

The dissipated energy as a function of charge time T then equals

$$E_{\text{diss}} = V_2^2 C \sqrt{\frac{R_s}{R_p}} \frac{e^{2aT} + 1}{e^{2aT} - 1}, \quad (\text{A.12})$$

and is shown in Figure A.10. The dissipated energy no longer tends to zero for large T (as it does for charging without leakage). Charging from 0V for a relatively long charge time T , the dissipated energy equals approximately $CV_2^2 \sqrt{\frac{R_s}{R_p}}$, which in general is very small compared with $\frac{1}{2}CV_2^2$. We conclude that the energy cost of charging a discharged capacitor to V_2 requires just the stored energy $\frac{1}{2}CV_2^2$, because the dissipated energy is very small even when charging within 0.1ms .

Figure A.10: Optimal charge energy dissipated as function of charge time T . Charge to 150 V, $C = 45$ pF and $R_s = 100 \Omega$. Compare with the stored energy $\frac{1}{2}CV^2 = 506$ nJ.

Note that the *square* of the derivative of V is added to the dissipated energy in (A.3). The minimization problem tries to minimize the energy cost also during discharging, meaning that discharging adds to the total energy cost. When the solution for V has a negative derivative, the solution is incorrect, because a better option may be to discharge the capacitor even more before starting to charge it again. Nevertheless, the solution shown is correct for charging from 0 V and indicates that for pre-charged capacitors, already for charge times of less than 10 ns it is energy efficient to discharge the capacitor first. So, regardless of the initial voltage of the capacitor, when the capacitor should be charged up to V_2 in, say, 0.5 ms, the required energy equals $\frac{1}{2}CV_2^2$ to a very good approximation.

Appendix B. Seek time for electrodynamic actuator

The solution given by Hong and Brandt [24] for the seek time for the electrodynamic actuator requires calculation of a turnaround time $t_{\text{turnaround}}$ when either v_0 or v_1 is negative. A table is provided with information on when and how many times $t_{\text{turnaround}}$ must be added to the seek time. However, only an approximate equation for $t_{\text{turnaround}}$ is given. Moreover, the equation in Ref. [24] for the half-way point, y_m , is not valid when $|v_0| \neq |v_1|$. Below, a more general solution is given for $t_{\text{seek,Ltz}} = t_{\text{accel}} + t_{\text{decel}}$, which is valid for all v_0 and v_1 when $x_0 \leq x_1$.

To obtain our solution for the seek time, we require use of the trigonometric identity

$$p \sin \omega t + q \cos \omega t = \sqrt{p^2 + q^2} \sin(\omega t + \phi), \quad (\text{B.1})$$

where

$$\phi = \begin{cases} \arcsin\left(\frac{q}{\sqrt{p^2 + q^2}}\right) & \text{if } p \geq 0, \\ -\pi - \arcsin\left(\frac{q}{\sqrt{p^2 + q^2}}\right) & \text{if } p < 0, \end{cases} \quad (\text{B.2})$$

or equivalently

$$\phi = \arctan\left(\frac{q}{p}\right) + \begin{cases} 0 & \text{if } p \geq 0, \\ \pi & \text{if } p < 0. \end{cases} \quad (\text{B.3})$$

This identity is used to rewrite equations of the form

$$p \sin \omega t + q \cos \omega t = r, \quad (\text{B.4})$$

as

$$\frac{r}{\sqrt{p^2 + q^2}} = \sin(\omega t + \phi), \quad (\text{B.5})$$

which is solved for t by taking the arcsine of both sides.

Note that equation 18 from Ref. [24] is of the same form as (B.4) above. Because v_0 can be negative, the solution for acceleration time t_{accel} has to be split, resulting in

$$t_{\text{accel}} = \begin{cases} \sqrt{\frac{m}{k}} \left[\arcsin \left(\frac{x_{\text{hw}} - x^*}{\sqrt{(x_0 - x^*)^2 + (v_0 \sqrt{\frac{m}{k}})^2}} \right) - \arcsin \left(\frac{x_0 - x^*}{\sqrt{(x_0 - x^*)^2 + (v_0 \sqrt{\frac{m}{k}})^2}} \right) \right] & \text{if } v_0 \geq 0, \\ \sqrt{\frac{m}{k}} \left[\pi + \arcsin \left(\frac{x_{\text{hw}} - x^*}{\sqrt{(x_0 - x^*)^2 + (v_0 \sqrt{\frac{m}{k}})^2}} \right) + \arcsin \left(\frac{x_0 - x^*}{\sqrt{(x_0 - x^*)^2 + (v_0 \sqrt{\frac{m}{k}})^2}} \right) \right] & \text{if } v_0 < 0. \end{cases} \quad (\text{B.6})$$

Note that the maximum equilibrium displacement $x^* = ma_0 k^{-1}$, see (12). The solution for the deceleration time t_{decel} is found by multiplying equation 22 from Ref. [24] by -1 , leading to

$$t_{\text{decel}} = \begin{cases} \sqrt{\frac{m}{k}} \left[\arcsin \left(\frac{x_1 + x^*}{\sqrt{(x_1 + x^*)^2 + (v_1 \sqrt{\frac{m}{k}})^2}} \right) - \arcsin \left(\frac{x_{\text{hw}} + x^*}{\sqrt{(x_1 + x^*)^2 + (v_1 \sqrt{\frac{m}{k}})^2}} \right) \right] & \text{if } v_1 \geq 0, \\ \sqrt{\frac{m}{k}} \left[\pi - \arcsin \left(\frac{x_1 + x^*}{\sqrt{(x_1 + x^*)^2 + (v_1 \sqrt{\frac{m}{k}})^2}} \right) - \arcsin \left(\frac{x_{\text{hw}} + x^*}{\sqrt{(x_1 + x^*)^2 + (v_1 \sqrt{\frac{m}{k}})^2}} \right) \right] & \text{if } v_1 < 0. \end{cases} \quad (\text{B.7})$$

Note that the result is identical to the result of Ref. [24] when $v_0 \geq 0$ and $v_1 \geq 0$ and (20) is used for x_{hw} .

A more compact notation is obtained when (B.3) is used instead of (B.2):

$$t_{\text{accel}} = \sqrt{\frac{m}{k}} \left[\arcsin \left(\frac{x_{\text{hw}} - x^*}{\sqrt{(x_0 - x^*)^2 + (v_0 \sqrt{\frac{m}{k}})^2}} \right) - \arctan \left(\frac{x_0 - x^*}{v_0 \sqrt{\frac{m}{k}}} \right) + \pi_0 \right], \quad (\text{B.8})$$

$$t_{\text{decel}} = \sqrt{\frac{m}{k}} \left[\arcsin \left(\frac{-x_{\text{hw}} - x^*}{\sqrt{(x_1 + x^*)^2 + (v_1 \sqrt{\frac{m}{k}})^2}} \right) + \arctan \left(\frac{x_1 + x^*}{v_1 \sqrt{\frac{m}{k}}} \right) + \pi_1 \right], \quad (\text{B.9})$$

where

$$\pi_i = \begin{cases} 0 & \text{if } v_i \geq 0, \\ \pi & \text{if } v_i < 0. \end{cases} \quad (\text{B.10})$$

For correct calculation of t_{accel} and t_{decel} ,

$$\pi_i + \arctan \left(\frac{1}{v_i} \right) = \frac{\pi}{2} \text{ if } v_i = 0. \quad (\text{B.11})$$

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