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Intramode and Fermi Relaxation in CO_2, Their Influence on Multiple-Pass, Short-Pulse Energy Extraction

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Abstract. Analytical, experimental and numerical results concerning the influence of intramode and Fermi relaxation on multiple-pass, nanosecond-pulse energy extraction are presented. Multiple-pass energy extraction experiments show satisfactory agreement with the analytical and numerical calculations which predict a significant increase in extracted energy. In three passes, an amount of 9.7 J/l was extracted at an efficiency of 4.3%, These values are taken with respect to the volume of the beam inside the amplifier. In a single pass only 3.5 J/l was extracted.

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In the past, many theoretical and experimental studies concerning the amplification of nanosecond CO_2 pulses have been performed [1, 2]. These studies emphasize on the rotational kinetics of the CO_2 molecules in the laser gas and indicate that multiline/multiband operation of CO_2-laser amplifiers significantly increases the energy extraction from these amplifiers.

If it is assumed that only rotational relaxation plays a role in the amplification process for nanosecond pulses, then the maximum amount of energy that can be extracted by multiline energy extraction, denoted as $E_{\text{available}}$, is given by

$$E_{\text{available}} = \tfrac{1}{2} h\nu \Delta L = g_0 E_s^{\infty} L\,, \tag{1}$$

where ν is the frequency of oscillation, Δ the total vibrational population inversion density, g_0 the small-signal gain, E_s^{∞} the saturation energy density for all rotational levels and L the length of the amplifier.

If the energy extraction occurs in two vibrational bands, i.e. in the 00°1–10°0 band as well as in the 00°1–02°0 band, then $E^*_{\text{available}}$ is given by

$$E^*_{\text{available}} = \tfrac{2}{3} h\nu \Delta L\,. \tag{2}$$

In (1 and 2), E_s^{∞} is given by

$$E_s^{\infty} = \frac{h\nu}{2\sigma P(J)}\,, \tag{3}$$

where σ is the cross-section for stimulated emission and $P(J)$ the partition fraction of the transition with rotational quantum number J.

This maximum amount of energy that can be extracted is still small compared to long-pulse energy extraction. This can be found as follows. Using recent results of Nevdakh [3], σ can be estimated to be approximately $\sigma = 1.3 \times 10^{-18}\,\text{cm}^2$ for a CO_2:N_2:He$\,=1:1:3$ mixture at atmospheric pressure. At an ambient temperature of $T = 400$ K the value of $P(J)$ is approximately 0.065 for the $P(20)$ transition, this yields a value of $E_s^{\infty} \approx 110\,\text{mJ/cm}^2$. If a particular laser amplifier has a small-signal gain $g_0 = 4\%\ \text{cm}^{-1}$, then a maximum amount of 4.4 J/l can be extracted by multiline operation. This is not a very high value and it is evident that still a lot of energy is left behind in the laser gas. Part of this energy, stored in the asymmetric stretch mode of CO_2 as well as in the vibration of N_2 can be extracted in a multiple-pass amplifier as a result of the fact that various relaxation processes result in a recovery of the gain between successive passes of the pulse. In this paper we will show the advantageous effects on energy extraction that can be expected if the intramode relaxation (this is the relaxation to thermal equilibrium within a vibrational mode) and the Fermi relaxation have time to occur. Together with the rotational relaxation these are assumed to be the most rapid processes. The analytical results as well as some numerical calculations will be compared with experimental results on energy extraction from a multiple-pass TEA CO_2 laser amplifier. It will be shown that multiple-pass amplification is a very important technique for increasing the short-pulse energy extraction efficiency.

1. Theory

1.1. *Intramode Vibrational Relaxation*

The effect of vibrational energy transfer on the amplification of a laser pulse has been dealt with by Harrach [4]. In his analysis he adopted the vibrational-temperature model. We will here slightly modify and extend his analysis. The text will emphasize on the differences and extensions. Details concerning the necessary assumptions and method of calculation can be found in [Ref. 4, Sect. III]. The purpose of this treatment is to calculate the amount of energy that can be extracted from a medium, starting from a known situation of population inversion. Obviously, the energy extraction process stops when the population inversion is reduced to zero. Therefore, in this analysis it is not important whether the pulse consists of more than one rotational line, as this would only increase the speed of the process leaving the final situation unchanged.

The Boltzmann factors r, s, q, are defined as

$$r = \exp(-h\nu_3/kT_3)\,, \tag{4}$$

$$s = \exp(-h\nu_2/kT_2)\,, \tag{5}$$

$$q = \exp(-h\nu_1/kT_1)\,, \tag{6}$$

where ν_3, ν_2, ν_1, T_3, T_2, T_1 are the fundamental frequencies and the temperatures of the asymmetric stretch, bending, and symmetric stretch mode, respectively. It is assumed that the rotational relaxation as well as the intramode relaxation are so fast that the vibrational modes can be described in terms of temperatures, before and during the interaction of a laser pulse with the medium. The total vibrational energy E_T^{vib} per unit volume in the CO_2-laser gas is simply given in terms of the Boltzmann factors

$$E_T^{\text{vib}} = N_{CO_2}\left(\frac{h\nu_1 q}{1-q} + \frac{2h\nu_2 s}{1-s} + \frac{h\nu_3 r}{1-r}\right). \tag{7}$$

The partition function Q is given by

$$Q(r, s, q) = [(1-r)\,(1-s)^2\,(1-q)]^{-1}\,. \tag{8}$$

It can be shown that, to a good approximation, the initial value of the population inversion density for a P-branch transition in the 00°1–10°0 band is given by

$$\delta^J(10.6) = f(J)\,N_{CO_2}\,\frac{r_0 - q_0}{Q(r_0, s_0, q_0)}\,, \tag{9}$$

where $f(J)$ is a function of the rotational quantum number.

Now in order to obtain an upper limit on the results for energy extraction, it is assumed that the amplifier input pulse has a duration that is long compared to both the rotational relaxation time and the intramode vibrational relaxation time. The pulse transfers energy from the ν_3 mode of a CO_2 molecule to the ν_1 mode by means of stimulated emission between the energy levels $u = (J-1;\,00^\circ 1)$ and $l = (J;\,10^\circ 0)$. Immediately following the passage of the pulse, the modes are characterized by the final Boltzmann factors r_f, s_f, and q_f. The total vibrational energy per unit volume in the final state is given by (7) with the values r_f, s_f, and q_f. These values are determined in the following way. First, the driving force of stimulated emission can only change the Boltzmann factors to the point where

$$r_f = q_f \tag{10}$$

since the population inversion density is then zero, (9). Secondly, conservation of energy requires

$$E_T^{\text{vib}}(r_0, s_0, q_0) - E_T^{\text{vib}}(r_f, s_f, q_f) = \Delta E_{vv}\,, \tag{11}$$

where ΔE_{vv} is the energy per unit volume, added to the radiation pulse.

Finally, the energy added to the pulse is a fraction $(\nu_3 - \nu_1)/\nu_3$ of the energy lost from the ν_3 mode

$$\begin{aligned}\Delta E_{vv} &= \frac{\nu_3 - \nu_1}{\nu_3}\,\Delta E_3 \\ &= hN_{CO_2}(\nu_3 - \nu_1)\left(\frac{r_0}{1-r_0} - \frac{r_f}{1-r_f}\right).\end{aligned} \tag{12}$$

By solving (11 and 12) simultaneously, and using (10) we obtain values for r_f and q_f. One important assumption has to be made here, i.e. the coupling between the symmetric stretch and the bending mode is neglected at this stage of the analysis, so there is no energy transfer between these modes. This means that $s_0 = s_f$ in (11). The result is

$$r_f = q_f = \frac{d_0}{2 + d_0} \tag{13}$$

with

$$d_0 = \frac{q_0}{1-q_0} + \frac{r_0}{1-r_0}\,. \tag{14}$$

The maximum amount of energy that can be extracted by the pulse when intramode processes have time to occur is given by ΔE_{vv} in (12) using the r_f value from (13). The comparison between this energy and the maximum extractable energy per unit volume due to rotational relaxation ΔE_{rr} [4] alone yields

$$\Delta E_{vv}/\Delta E_{rr} = \gamma\,, \tag{15}$$

where the constant γ is given by

$$\gamma = \frac{2Q(r_0, s_0, q_0)}{r_0 - q_0}\,\frac{v_3 - v_1}{v}\left(\frac{r_0}{1-r_0} - \frac{r_f}{1-r_f}\right). \tag{16}$$

Thus, intramode relaxation increases the maximum amount of energy that can be extracted with a factor γ, where r_f is given by (13). For a gain medium with $T_1 = T_2 = 400$ K, and $T_3 = 1500$ K we find $\gamma = 1.5$.

Of course, it is also possible to have a laser operating in the 9.6 µm band. In this case the lower laser level is in the 02°0 vibrational band. The initial value of the population inversion density for a P-branch transition is similar to (9) given by

$$\delta^J(9.6) = f(\mathrm{J})\,N_{CO_2}\,\frac{r_0 - s_0^2}{Q(r_0, s_0, q_0)}\,. \tag{17}$$

If Fermi relaxation is again neglected, then the Boltzmann factors can be changed to the point where $r_f = s_f^2$. Further we have $q_0 = q_f$. Equations similar to (11 and 12) can now be derived. Solving these equations simultaneously yields

$$s_f = \sqrt{r_f} = \{[4\,e_0(2+e_0)+1]^{1/2} - 1\}/(4+2e_0) \tag{18}$$

with

$$e_0 = \frac{r_0}{1-r_0} + \frac{s_0}{1-s_0}\,. \tag{19}$$

In order to derive a value for γ (15 and 16) can be used if we assume that $q_0 \approx s_0^2$ and $v_1 \approx 2v_2$. In this case the value of r_f given by (18) has to be used. The result for a gain medium with $T_1 = T_2 = 400$ K and $T_3 = 1500$ K is $\gamma = 2.5$. This value is appreciably higher than the value that was obtained when the laser operated in the 10.6 µm band. The reason is the much higher energy contents of the bending mode with respect to the symmetric stretch mode (about a factor of 10 at $T_1 = T_2 = 400$ K). The increase in energy in this vibrational mode can therefore flow into a large reservoir by means of the intramode relaxation.

Finally it is noted that (under the assumption that initially $T_1 = T_2$) the maximum extractable energy due to rotational relaxation alone is about 10% higher in the 9.6 µm band as a result of the higher quantum size hv.

1.2. Fermi Relaxation

The analysis given in the previous subsection can easily be extended to include the coupling between the symmetric stretch and the bending mode. The derivation of the results for this case can entirely be found in [4]. The results are (for this case we have $s^2 = q$):

$$s_f = \sqrt{q_f} = \sqrt{r_f} = \{[4(c_0+3)c_0+1]^{1/2} - 1\}/[2(c_0+3)] \tag{20}$$

with

$$c_0 = \frac{r_0}{1-r_0} + \frac{s_0^2}{1-s_0^2} + \frac{s_0}{1-s_0}\,. \tag{21}$$

The result is different from (13 and 14) because in the present case the assumption that $s_0 = s_f$ was dropped. Equations (20, 21) are the equations given by Harrach [4]. In his publication he stated that these are the results for the case where all relaxations except rotational and intramode relaxations are neglected. In our opinion, Fermi relaxation was implicitly incorporated in his calculations because of his assumption that $s^2 = q$ during the amplification process.

Equation (12), combined with the value of r_f given by (20 and 21) yields the maximum amount of energy that can be extracted when both the intramode and the Fermi relaxation are taken into account. γ is again calculated with (16) with $q_0 = s_0^2$ and the value of r_f given by (20 and 21). The result for a gain medium with $T_1 = T_2 = 400$ K, and $T_3 = 1500$ K is $\gamma = 2.6$. This result clearly shows the increase in energy extraction that can be expected when Fermi relaxation processes have time to occur.

The analysis presented here assumes, in fact, that the laser pulse is long enough for the various relaxation processes to reach equilibrium. In a multiple-pass amplifier for nanosecond laser pulses, the medium relaxes to an equilibrium situation during the round-trip time interval of the laser pulse. Therefore the situation of intramode and Fermi equilibrium at the end of the amplification can be reached with this technique. The present calculations can be used to estimate the maximum amount of energy that can be extracted from a multiple-pass amplifier system if only rotational, intramode, and Fermi relaxation are taken into account.

2. Experiments

2.1. An Experimental Configuration for Multiple-Pass Pulse Amplification

In order to experimentally investigate the amplification of nanosecond pulses in a multiple-pass amplifier, the configuration shown in Fig. 1 was realized. This configuration was originally proposed by Aver'yanov [5]. A nanosecond pulse, generated by a multiline laser system [6, 7], is directed through the $2 \times 2 \times 40$ cm TEA amplifier several times. The characteristics of the input pulse, such as pulse shape, energy, and spectral contents are determined before the pulse enters the medium for the first time. At the output end, only the energy of the pulse is determined.

The aim of our experiments on multiple-pass amplification is to investigate the influence of the round-trip time and the number of passes through the amplifier on the extracted energy.

Therefore, in our set-up it should be possible to vary the round-trip and in successive passes, the beam should use the same medium as much as possible. Due to the geometry of our amplifier as well as the requirement that the beam should use the same medium in each successive pass, the maximum number of passes in our experiments was restricted to three. In order to have the same time interval for the relaxation processes to occur for every part of the medium, the pulse was directed to the input end of the amplifier instead of directing it immediately backwards into the amplifier. The shortest round-trip time that could be realized was 10 ns. The energy extraction experiments were also performed with a round-trip time of 25 ns, which was achieved by employing additional mirrors.

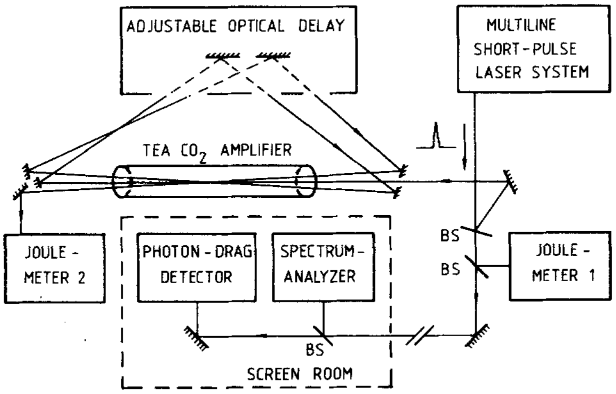

Fig. 1. Optical configuration for energy-extraction experiments on the multiple-pass amplifier. The situation for output energy measurements after three passes is shown

Table 1. The beam radius r_0, the beam area, and the small signal gain of the pulse during pass number n inside the multiple-pass amplifier

PASS NUMBER N	ROUND-TRIP TIME [ns]	BEAM RADIUS (r_o) [mm]	BEAM AREA [cm^2]	SMALL SIGNAL GAIN [% /cm]
1	10	4.2 ± 0.3	0.56 ± 0.08	3.2 ± 0.3
2	10	5.1 ± 0.5	0.82 ± 0.16	3.0 ± 0.3
3	10	4.7 ± 0.4	0.69 ± 0.12	2.9 ± 0.3
1	25	4.2 ± 0.3	0.56 ± 0.08	3.2 ± 0.3
2	25	4.4 ± 0.4	0.61 ± 0.11	2.9 ± 0.3
3	25	5.5 ± 0.5	0.95 ± 0.17	2.9 ± 0.3

Of course, it is only possible to show the advantageous effect of intramode and Fermi relaxation if the input pulse saturates the medium in the first pass through the amplifier. We could easily achieve pulses of 100–130 mJ and these input pulses were sufficiently intense to achieve saturation. In the experiments, great care was taken to create a collimated, Gaussian beam inside the amplifier. The intensity profile of this beam is given by

$$I(r) = I_0 \exp(-2r^2/r_0^2), \tag{22}$$

where r_0 is the beam radius, defined as the radius at which the intensity has decreased to I_0/e^2.

It was found that the beam radius r_0 could not be larger than 6 mm, without interference fringes occurring. As a result of the propagation distance of the pulse, the beam size is somewhat different for the successive passes through the amplifier despite the use of correcting mirrors (Table 1).

During successive passes, the pulse passes the amplifier at different angles. The differences are only small, being about 2°. However, as a result of this small deviation, at the second and third pass the pulse encounters a small amount of laser gas which was not affected by the first pass. Another effect of this deviation is the fact that the pulse encounters medium with a slightly lower small-signal gain. The amplifier has a gain profile that is more or less uniform in the direction of the electric field but drops in the perpendicular direction. At our operating conditions ($V = 42.5$ kV, energy input 36 J) the FWHM value is about 23 mm. The small-signal gain, averaged over the beam path is given in Table 1.

The mirrors that are used in the experimental set-up do not have a 100% reflectivity. As we are interested in the energy that is extracted from the medium by the laser pulse, it is necessary to correct the experimental data for the losses caused by the mirrors and the NaCl Brewster windows. These corrections are applied to the experimental data but it should be clear that this decreases the accuracy when determining the influence of the number of passes through the amplifier as well as the influence of the round-trip time on the energy extraction.

2.2. *Results of Multiple-Pass Energy Extraction*

Using the set-up described in the previous subsection, energy extraction experiments were performed under various conditions. The number of passes was varied from one to three and the round-trip time was either 10 or 25 ns. Finally, we performed some experiments for different gas mixtures. All the experiments were carried out for a range of input energies of the incoming laser pulse. The pulse duration was between 0.9 and 1.2 ns.

In order to obtain an impression of the reliability and the reproducibility of the experiments we present the uncorrected data for one experiment, as well as the final results when the data are corrected for the losses during the round trips of the pulse (Fig. 2). In all the experiments the output energy and the input energy of the pulse were measured. The extracted energy can easily be derived from these experiments. The data are shown in Fig. 2a. The final results, corrected for the losses during the round trips, are shown in Fig. 2b. At the abscis, the input energy is converted into a corresponding input energy density by means of dividing the input energy by the beam area (given in Table 1) or the average beam area in the case of multiple-pass energy extraction. At the ordinate the extracted energy density is denoted as an efficiency. This efficiency is related to the amount of energy that is available in the 00°1–10°0 transition (defined as $E_{\text{available}}$). The value of $E_{\text{available}}$ is dependent on the gain and is calculated using (1). This value is given in the figure. Therefore a 100% efficiency means an energy extraction of $E_{\text{available}}$ mJ/cm^2. These transformations in the presentation of the data are performed in order to present the results in more general terms, independent of beam area and small-signal gain.

In order to be able to compare the effect of various parameters on the energy extraction from an amplifier, the results are summarized in tabular form. For this purpose, we determined the extracted energy density for all experiments at a fixed value of the input energy density by using the curves, as shown in Fig. 2b. These curves are fitted to the corrected data and the results are presented in Table 2. The column ϱ_{extr} gives the extracted energy in terms of J/l. In this case the volume is related to the volume of the beam, which is defined as $\pi r_0^2 L$, with L the length of the amplifier. The column η_b denotes the laser efficiency, related to the beam volume. In the case of multiple-pass amplification, the average beam volume was taken. The other columns in Table 2 are self-explanatory.

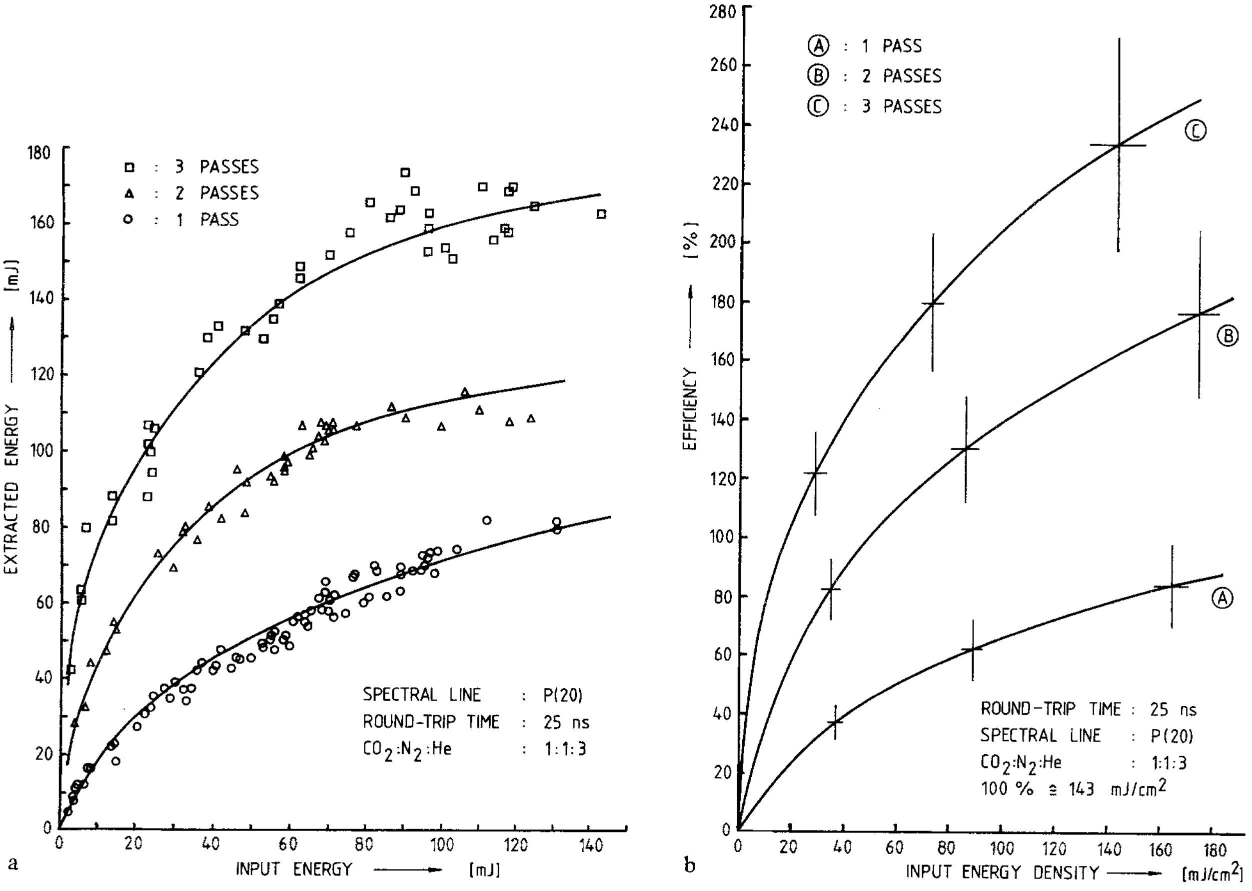

Fig. 2. (a) Extracted energy versus the input energy for a different number of passes through the amplifier. (b) The efficiency related to $E_{\text{available}}$ versus the input energy density. The curves are fitted to the data, corrected for the losses during the round trips. $E_{\text{available}}$ is shown in the figure

Table 2. Results of energy extraction experiments on the 10.6 μm $P(20)$ transition from a multiple-pass amplifier for a fixed value of the input energy density (150 mJ/cm^2). The gas mixture, the round-trip time and the number of passes were varied

$CO_2:N_2:He$	ROUND-TRIP TIME [ns]	NUMBER OF PASSES	AVERAGE BEAM AREA [cm^2]	INPUT ENERGY [mJ]	EXTRACTED ENERGY [mJ]	EXTRACTED ENERGY DENSITY [mJ/cm^2]	EFFICIENCY related to $E_{available}$ [%]	ρ_{extr} [J/l]	EFFICIENCY η_b [%]
1:1:3	–	1	0.56±0.08	84±12	67±5	119±26	83±19	3.0±0.7	1.3±0.3
1:1:3	10	2	0.69±0.12	104±18	133±8	193±45	135±31	4.8±1.1	2.1±0.5
1:1:3	10	3	0.69±0.12	104±18	197±9	286±63	200±45	7.1±1.6	3.1±0.7
1:1:3	25	2	0.59±0.10	89±15	143±17	242±55	181±41	6.1±1.7	2.7±0.6
1:1:3	25	3	0.71±0.12	107±18	242±28	341±72	238±50	8.5±2.4	3.8±1.1
2:1:5	–	1	0.56±0.08	84±12	57±4	102±22	84±18	2.5±0.4	1.1±0.2
2:1:5	25	2	0.59±0.10	89±15	120±15	203±46	166±37	5.1±1.5	2.3±0.7
2:1:5	25	3	0.71±0.12	107±18	203±25	286±61	234±50	7.2±2.1	3.2±0.9
1:1:2	–	1	0.56±0.08	84±12	79±6	141±31	87±18	3.5±0.8	1.6±0.3
1:1:2	25	2	0.59±0.10	89±15	173±18	293±63	182±39	7.3±2.0	3.2±0.9
1:1:2	25	3	0.71±0.12	107±18	274±30	386±81	240±50	9.7±2.7	4.3±1.2

2.3. *Evaluation of the Experimental Results*

In this subsection we will discuss the influence of various parameters on the energy extraction from, and the efficiency of, our multiple-pass amplifier. However, before we do this some remarks should be made with respect to the values of the efficiency related to $E_{available}$. The expected (theoretical) value of this efficiency is about 50% for single-line and single-pass energy extraction (Appendix A). As can be seen from Table 2, our values are about 85%. There are several reasons which may cause this discrepancy.

First, the theoretical values are calculated for a uniform beam profile, while in the experiments the beam has a Gaussian shape. Therefore, the experimental value of this efficiency depends on the value that is taken for the beam area.

Second, in the theoretical calculations it is assumed that only the energy initially stored in the vibrational population inversion is available on a nanosecond time scale. Recently, it has been shown, however, that the intramode relaxation time constant is very rapid (< 1 ns) [8]. Therefore this relaxation has effect on the extractable energy. Some calculations concerning this effect are presented in Appendix A.

Third, the calculations do not take into account the effect of the so-called radial radiation transport [9]. It has been shown by Ernst that this phenomenon results in an increase in energy extraction by a Gaussian beam with respect to a uniform beam. The theoretical maximum of the increase is 100%. This value is found when the gain profile, induced by the radiation is described by a quadratic equation. Actually, higher-order terms should be taken into account. These decrease the effect as there is no radial radiation transport far from the axis since the gain profile is uniform in that region. A rigorous treatment would be necessary in order to obtain accurate quantitative results concerning this effect. This treatment should also take into account the transient character of a pulse passing an amplifier.

Finally, the absolute values of the energy measurements depend on the calibration of the detectors and as absolute calibration of these energy detectors is difficult this may result in an inaccuracy of about 10 to 20%. This inaccuracy is not included in the errors given in Table 2 as it does not influence the values of the measured energies relative to each other (all experiments were performed using the same Gentec ED-500 and ED-100 pyroelectric detectors).

a) Effect of the Round-Trip Time. Using a $CO_2:N_2:He = 1:1:3$ gas mixture and the $P(20)$ transition, the energy extraction was investigated for a round-trip time of 10 and 25 ns. It can be seen from Table 2 that in the case of 2 and 3 passes through the amplifier, more energy was extracted in the case of a round-trip time of 25 ns. This is in agreement with our expectations because during 25 ns, relaxation processes have more time to occur. In [7, 8] we report on experiments concerning the evolution of the gain in a CO_2 laser amplifier in order to determine values for the Fermi and intramode relaxation time constants. Knowledge of these values is important for the proper design of a multiple-pass amplifier. We found a (Fermi) relaxation time constant for the $P(20)$ transition of 17 ns. It was found, however, that the difference in the gain of the laser medium at the moment the pulse reenters the medium is only a few percent for a round-trip time of either 10 or 25 ns, see also the calculation in Fig. 3. As a result of the fact that the beam areas are slightly different for the various round-trip times and the fact that the correction for the losses is also somewhat different it is difficult to obtain quantitative results for the advantageous effect of increasing the round-trip time from 10 to 25 ns, as the disturbing effects are too large. The results in [8] indicate that 25 ns is a suitable choice for the round-trip time in a multiple-pass amplifier.

b) Effect of the Number of Passes and the Laser Mixture. It is evident that multiple-pass amplification substantially increases the energy extraction efficiency from short-pulse laser amplifiers. In three passes, the extracted energy clearly exceeds $E_{available}$. The results indicate, in agreement with the calculations presented in the next section, that an increase of the number of passes would lead to a further increase of the extracted energy.

Three laser-gas mixtures were investigated. Besides the $CO_2:N_2:He = 1:1:3$ mixture, which was used in most experiments, we also performed energy extraction measurements with $CO_2:N_2:He = 1:1:2$ and $2:1:5$ mixtures. Table 2 shows that the efficiencies related to $E_{\text{available}}$ are almost the same in these experiments. It can be seen that in self-sustained TEA amplifiers probably the best results are obtained with gas mixtures containing equal amounts of CO_2 and N_2 and containing as little He as is essential to achieve a homogeneous gas discharge.

3. Numerical Calculations on Multiple-Pass Pulse Amplification

In [8] we described a numerical model for the amplification of nanosecond pulses. In that paper the model was used to fit the experimental data on the evolution of the gain with numerical calculations. It is also very well suited for performing numerical calculations on the energy extraction from a multiple-pass amplifier. In order to calculate the energy extraction by the pulse in pass number N, the output of the previous pass is taken as the input pulse. During the amplification of the pulse both rotational relaxation (τ_r) and intramode vibrational relaxation (τ_{vv}) are taken into account. Detailed information concerning this model can be found in [7]. Table 3 shows the input parameters for a typical run. The values of the relaxation constants are taken according to those that were found experimentally [8]. The Fermi relaxation time constant is denoted by τ_{12}, the relaxation time constant for the transfer of energy from N_2 to the asymmetric stretch mode of Co_2 by τ_{43}. Figure 3 shows the evolution of the gain after the injection of the first pulse into the amplifier at the moment the gain has reached its maximum value. The extracted energy can be found in Fig. 4. It can be seen that, though increasing in strength, the pulse extracts less energy in each successive pass as a result of the decreasing gain. The decrease, however, is not very rapid. Figure 4 shows that, if the input pulse is intense enough, the extracted energy exceeds $E_{\text{available}}$ after a few passes. This is in agreement with our experimental observations.

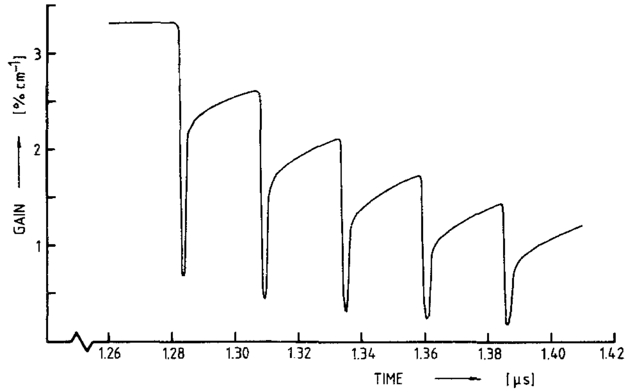

Fig. 3. Calculated evolution of the gain in a multiple-pass amplifier. The input parameters are shown in Table 3

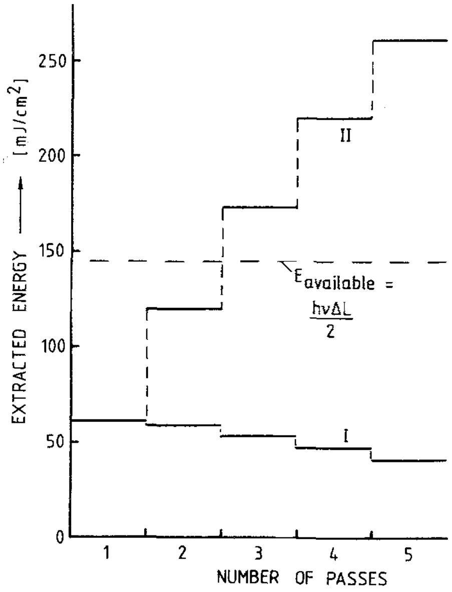

Fig. 4. Calculated extracted energy from the multiple-pass amplifier versus the number of passes for the input parameters listed in Table 3. Curve *I*: Extracted energy in pass number *n*. Curve *II*: Cumulated extracted energy

Table 3. List of important parameters used for the calculation on multiple-pass pulse amplification

Gas mixture $CO_2:N_2:He$	:	1:1:3
Amplifier length	:	40 cm
Input energy into laser gas	:	70 J/l
Energy density of input pulse	:	106 mJ/cm^2
Laser-pulse duration	:	1 ns FWHM
Laser transition	:	10.4 μm P(20)
Number of passes	:	5
Round-trip time	:	26 ns
τ_{vv}	:	1 ns
τ_{12}	:	≈ 17 ns
τ_{43}	:	≈ 260 ns
τ_r	:	≈ 0.16 ns

4. Conclusions

The analytical, experimental and numerical results on multiple-pass pulse amplification show that this technique is an efficient method for short-pulse energy extraction from a CO_2 laser amplifier. A combination

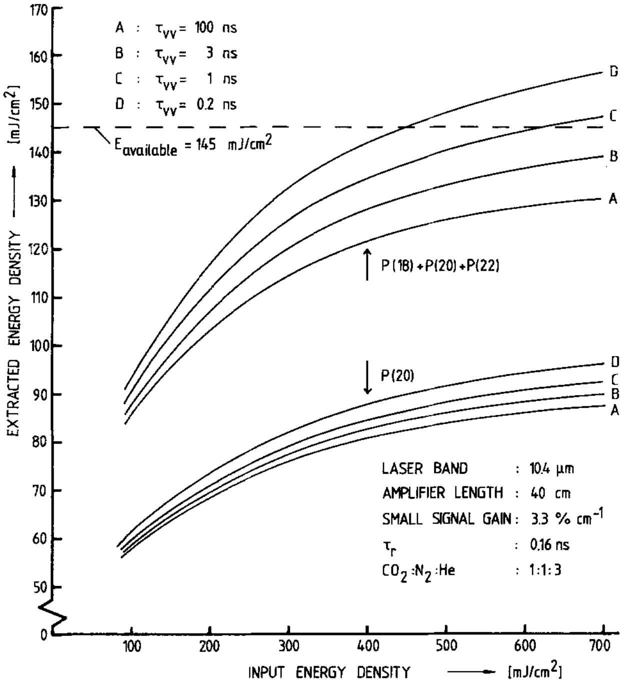

Fig. A.1. Calculated extracted energy density versus the input energy density for various values of the intramode relaxation time constant τ_{vv}. The pulse consisted of either one or three lines in the 10.6 µm band

of this technique with multiline energy extraction will probably give the best results.

The maximum amount of energy was extracted from a $CO_2:N_2:He=1:1:2$ mixture. On the *P*(20) transition we extracted 9.7 J/l using 3 passes with a round-trip time of 25 ns. This extracted energy density is related to the volume of the beam. The corresponding efficiency is 4.3%.

Appendix A

In the present paper, the energy that is extracted from an amplifier is frequently compared with $E_{available}$, where $E_{available}$ is defined as the maximum amount of energy that can be extracted from the 00°1–10°0 transition under the assumption that only rotational relaxation has time to occur.

As the intramode vibrational relaxation time constant τ_{vv} is also believed to be very rapid (about 1 ns), this process has an effect on the energy extraction by a single nanosecond pulse. In order to estimate the influence of the intramode relaxation on the energy extraction by a pulse of 1 ns duration, some calculations are presented using the model described in [7, 8].

The results are shown in Figs. A.1 and 2. It can be seen from Fig. A.1 that the effect of the intramode relaxation on the extracted energy increases when the rotational relaxation time constant of 0.16 ns. atm acts less as a bottleneck, This is the case when energy is extracted on three rotational lines simultaneously as shown in the upper set of curves.

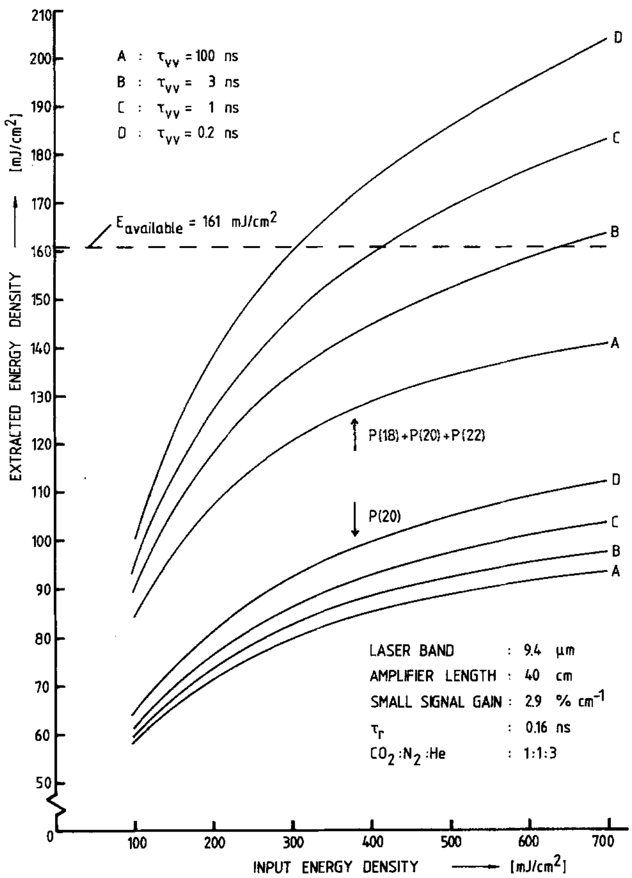

Fig. A.2. Calculated extracted energy density versus the input energy density for various values of the intramode relaxation time constant τ_{vv}. The pulse consisted of either one or three lines in the 9.6 µm band

The numerical calculations also show the more effective intramode relaxation if the laser operates in the 9.6 µm band. This is shown in Fig. A.2. The results indicate that higher values of the extracted energy can be obtained if the laser operates on lines in the 9.6 µm band.

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