Novel microwave photonic fractional Hilbert transformer using a ring resonator-based optical all-pass filter

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Abstract: We propose and demonstrate a novel wideband microwave photonic fractional Hilbert transformer implemented using a ring resonator-based optical all-pass filter. The full programmability of the ring resonator allows variable and arbitrary fractional order of the Hilbert transformer. The performance analysis in both frequency and time domain validates that the proposed implementation provides a good approximation to an ideal fractional Hilbert transform. This is also experimentally verified by an electrical S21 response characterization performed on a waveguide realization of a ring resonator. The waveguide-based structure allows the proposed Hilbert transformer to be integrated together with other building blocks on a photonic integrated circuit to create various system-level functionalities for on-chip microwave photonic signal processors. As an example, a circuit consisting of a splitter and a ring resonator has been realized which can perform on-chip phase control of microwave signals generated by means of optical heterodyning, and simultaneous generation of in-phase and quadrature microwave signals for a wide frequency range. For these functionalities, this simple and on-chip solution is considered to be practical, particularly when operating together with a dual-frequency laser. To our best knowledge, this is the first time on-chip demonstration where ring resonators are employed to perform phase control functionalities for optical generation of microwave signals by means of optical heterodyning.

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References and links

1. Introduction

Microwave photonic (MWP) signal processing leverages the advantageous properties of optical devices to realize functionalities performing RF signal processing, so that the constructed systems are commonly associated with the features such as large RF instantaneous bandwidth, RF frequency transparency, light weight, low signal propagation loss, and electromagnetic interference immunity [1], [2]. This technique enables functionalities which are difficult, if not impossible, to realize using only electronics. In the past few years, the advancing of microwave photonics has also exhibited a trend of on-chip realization of various RF signal processing functionalities [3]-[9], enabling on-chip complex MWP signal processors. This improves the system with respect to compactness, robustness, stability, and fabrication cost, especially when the chip is fabricated using CMOS process-compatible equipment.

To enrich the applications of MWP signal processing, several recent investigations have been conducted on photonic implementations of a wideband Hilbert transformer (HT), which is a fundamental operator used for numerous signal processing functionalities in radar, communication, and modern instrumentation systems [10]. However, most of the previously proposed and/or demonstrated photonic HTs use discrete and bulky components such as a fiber bragg grating and circulator [11]-[14], an incoherent delay-line filter operating with multiple lasers [15], [16], and an optical signal processing device comprising a two-dimensional array of liquid-crystal-on-silicon pixels [17]. These approaches manifest the disadvantages of fabrication difficulty, high system complexity and high cost, and/or lack of tunability. In [18], a theoretical study was reported of designing photonic HTs using tunable...
waveguide-based finite-impulse response (FIR) filters. However, in practice, FIR filters require multiple splitting and combining couplers as well as length-varying delay paths, which may be a drawback in terms of the complexity and footprint dimensions of a photonic integrated circuit (PIC). In comparison, infinite-impulse response (IIR) filters are generally more advantageous for their simpler structure and better performance.

In this paper, we propose and demonstrate a simple and space-efficient waveguide implementation of an MWP fractional Hilbert transformer (FHT, defined as the generalized concept of HT and operating as an arbitrary-value phase shifter [19]) using a particular type of IIR filter, namely ring resonator-based all-pass filters. The full programmability of the optical ring resonators (ORRs) allows variable and arbitrary fractional order of the FHT. In addition to that, the waveguide-based structure allows the FHT to be integrated together with other functional building blocks on a photonic integrated circuit (PIC) to create various system-level functionalities for on-chip MWP signal processors. In Section 2, the principle of the proposed FHT is described, followed by a performance analysis in both frequency and time domain. Section 3 presents the electrical S21 response characterization performed on a waveguide realization of the proposed FHT. Section 4 demonstrates two system-level functionalities achieved using the FHT, namely on-chip phase control of microwave signals generated by means of optical heterodyning, and simultaneous generation of in-phase and quadrature microwave signals. The conclusions of this paper are formulated in Section 5.

2. Principle and performance analysis

2.1 Implementation of an FHT using waveguide-based optical filters

When $\tilde{x}(t)$ is the temporal complex envelop of an input optical signal $\tilde{x}_o(t)=\tilde{x}(t)\cdot\exp(j2\pi f_o t)$ to a conventional Hilbert transformer (HT) ($f_o$ is the frequency of the optical carrier), the corresponding envelop $\tilde{y}(t)$ of the output signal $\tilde{y}_o(t)=\tilde{y}(t)\cdot\exp(j2\pi f_o t)$ can be expressed by

$$\tilde{y}(t) = \text{P.V.} \left[ \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\tilde{x}(\tau)}{t - \tau} d\tau \right] = \tilde{x}(t) \ast \frac{1}{\pi t}$$

where P.V. means principle value [10]. Thus, the HT can be implemented using a linear optical filter giving an impulse response $\tilde{h}_{HT}(\tau)=\tilde{h}_{HT}(t)\cdot\exp(j2\pi f_o t)$ with an envelope $\tilde{h}_{HT}(t)=1/(\pi t)$ and a corresponding baseband frequency response $H_{HT}(f)=-\text{sgn}(f)$, where $f$ is the frequency variable. When a conventional HT is generalized into an FHT [19] and the FIR and IIR filter approaches in [20] are used to design waveguide-based filters that approximate the theoretical FHT, the frequency-periodic version of the FHT frequency response is used as the design target specification and is expressed by

$$H_{\text{FHT}}(\Omega) = \left| H_{\text{HT}}(\Omega) \right| e^{j\varphi(\Omega)} = \begin{cases} e^{-j\rho}, & 0 \leq \Omega < \pi \\ e^{j\rho}, & -\pi \leq \Omega < 0 \end{cases}$$

where $\Omega = 2\pi f T$ represents the angular frequency normalized to the defined unit delay $T$ of the filter, $\rho = \rho_{\pi/2}$ and $\rho$ indicates the fractional order. Eq. (2) signifies that an FHT is an ideal phase shifter to the input signals. A FHT becomes the conventional HT when $\rho=1$ ($H_{\text{FHT}}(\Omega) = -\text{sgn}(\Omega)$), and it provides no modification to the input signals when $\rho=0$ ($H_{\text{FHT}}(\Omega) = 1$). Moreover, $H_{\text{FHT}}(\Omega)$ is periodic with $\rho$, since $H_{\text{FHT}}(\Omega)_{\rho=0} = H_{\text{FHT}}(\Omega)_{\rho=\pi/2}$ holds. Take the inverse Fourier transform of $H_{\text{FHT}}(\Omega)$, the corresponding discrete-time impulse response is given by

$$h_{\text{FHT}}(nT) = \begin{cases} \cos(\rho), & n = 0 \\ \sin(\rho) \frac{2\sin^2(n\pi/2)}{n\pi}, & n \neq 0 \end{cases}$$

It can be seen that $h_{\text{FHT}}(nT)$ is zero when $n$ is a nonzero even integer, and its negative taps have opposite-valued coefficients to the corresponding positive taps, forming an antisympometric profile with respect to $n=0$. An illustration of the frequency and impulse response of an ideal FHT with $\rho=1$ is depicted in Fig. 1. Apparently, a theoretical FHT is not realizable
using FIR and IIR filter implementations, since \( h_{\text{FHT}}(nT) \) in Eq. (3) is non-causal and has critical taps in negative time. To solve this problem, one can approximate the modified FHT specification \( H_{\text{FHT,mod}}(\Omega) \) instead, which is different than the theoretical \( H_{\text{FHT}}(\Omega) \) by an additional delay term \( e^{jn_0\Omega} \) with \( n_0 \) being a prescribed delay (normalized to the defined unit delay). This is expressed as below:

\[
H_{\text{FHT,mod}}(\Omega) = H_{\text{FHT}}(\Omega)e^{jn_0\Omega} = \begin{cases} 
  e^{-j\phi} \cdot e^{-jn_0\Omega} & 0 \leq \Omega < \pi \\
  e^{j\phi} \cdot e^{-jn_0\Omega} & -\pi \leq \Omega < 0 
\end{cases}
\]  

(4)

As illustrated in Fig. 1, the additional delay shifts the impulse response such that the critical taps have their presence in the positive time and truncation can be applied to obtain causality.

\[G_{\text{a},N}(z) = \frac{a_N + a_{N-1}z^{-1} + \ldots + a_1z^{-N+1} + z^{-N}}{1 + a_1z^{-1} + \ldots + a_{N-1}z^{-N+1} + a_Nz^{-N}}\]  

(6)

where \( a_1, \ldots, a_N \) are the filter coefficients. Then, the goal is to optimize those filter coefficients such that the frequency response of the filter \( G_{\text{a},N}(\Omega) \) approximates \( H_{\text{FHT,mod}}(\Omega) \) to the most. For instance, this can be achieved using the least square error method as described in [21].

2.2 Optical ring resonator

A schematic of the waveguide implementation of an ORR is depicted in Fig. 2(a), where an ORR is achieved by bridging one output of an Mach-Zehnder interferometer (MZI) to the nearest input, such that a closed ring path is obtained in one arm of the MZI. The MZI couples a portion of the light passing one arm over to the other according to its power coupling coefficient. This corresponds to an IIR as depicted in Fig. 2(b). In the impulse response, the time interval of the taps is determined by the roundtrip duration of the light traveling in the ring path, and it is defined as the unit delay of the ORR. Corresponding to the impulse response, the \( z \)-transform of an ORR is described by

\[
H_{\text{ORR}}(z) = \frac{c + rz^{-1}e^{j\theta}}{1 + crz^{-1}e^{j\theta}}
\]  

(7)
where \( c = (1 - \kappa)^{1/2} \) with \( \kappa \) being the power coupling coefficient, \( r \) indicates the roundtrip amplitude transmission coefficient which in practice is determined by the roundtrip loss, and \( \theta \) represents an additional roundtrip phase shift which can be introduced intentionally to achieve a shift of the resonance frequency of the ORR [20]. This signifies that an ideal lossless ORR is a first-order all-pass filter, and a cascade of \( N \) ORRs can therefore be used to implement \( G_{aN}(z) \) in Eq. (6).

2.3 Frequency-domain analysis

Based on Eq. (7), the magnitude and phase response of an ORR can be expressed by

\[
|H_{\text{ORR}}(\Omega)| = \sqrt{\frac{c^2 + r^2 - 2c r \cos(\Omega)}{1 + cr^2 - 2c r \cos(\Omega)}} \quad \text{and} \quad (8)
\]

\[
\Psi_{\text{ORR}}(\Omega) = \arctan \left( \frac{r \sin(\Omega)}{c - r \cos(\Omega)} \right) - \left( \frac{c r \sin(\Omega)}{1 - c r \cos(\Omega)} \right), \quad \text{respectively}, \quad (9)
\]

where the frequency shift effect of \( \theta \) is neglected for simplicity. As an all-pass filter, these frequency responses are able to approximate to the modified FHT specification \( H_{\text{FHT,mod}}(\Omega) \) in Eq. (4). This is illustrated in Fig. 3, where \( r = 0.97 \) (for a typical roundtrip loss of 0.12 dB) is used for the calculation. Evidently, by varying the power coupling coefficient \( \kappa \), different values of the fractional order \( \rho \) are achieved. In principle, the phase response of an ORR has a constant span of \( 2\pi \) across one frequency period, or in other words one free spectral range (FSR) [20]. Using this as a condition for Eq. (4), a relation \( n_0 = 1 - \rho/2 \) results, which is a property of the proposed FHT and can be used to determine the additional delay \( n_0 \) for a given value of \( \rho \).

Fig. 3. (a)-(e) calculated phase responses of an ORR which are optimized to fit modified FHT specification for different values of fractional order \( \rho \); (f) the corresponding power transmissions of the ORR with \( r = 0.97 \) (inset: close-by view around resonance frequency).
As described in Eq. (9), the phase response of an ORR is not a perfect linear function of frequency. This means that the proposed FHT has a limitation in the operation bandwidth, which depends on the defined phase ripple tolerance. This bandwidth property is numerically presented in Fig. 4 for different values of $\rho$.

Recall the signal processing principles explained in Section 2.1. It can be understood that the proposed FHT functions with intensity-modulated optical signals and requires the optical carrier frequency to be aligned with a resonance frequency of the ORR. However, the power transfer of the ORR depicted in Fig. 3(f) demonstrates a strong notch effect at the resonance frequency when $\rho$ approaches 2, corresponding to a high-Q state of the ORR. This will result in strong optical carrier suppression, making direct optical detection inapplicable. To solve this problem, one possible approach is to apply coherent optical detection instead, using an optical carrier reininsertion circuitry as explained in [6]. However, this requires the conservation of the optical phase relation between the modulated and unmodulated light, which increases the realization difficulty of the system. Alternatively, a more practical approach is to implement the FHT using a cascade of multiple ORRs, instead of using a single ORR as explained above. Being a linear system, a cascade of $N$ ORRs can be described by

$$H_{C,N}(\Omega) = \prod_{n=1}^{N} H_{\text{ORR},n}(\Omega) = \prod_{n=1}^{N} \left| H_{\text{ORR},n}(\Omega) \right| e^{\sum_{n=1}^{N} \varphi_{\text{ORR},n}(\Omega)}$$

where $|H_{C,N}(\Omega)| = \prod_{n=1}^{N} |H_{\text{ORR},n}(\Omega)|$ and $\varphi_{C,N}(\Omega) = \sum_{n=1}^{N} \varphi_{\text{ORR},n}(\Omega)$. This signifies that a desired fractional order $\rho$ can be obtained by letting each ORR in the cascade provide only a fraction of $\rho$, such that the high-Q state of ORR is avoided. Consequently, a significant reduction of the optical carrier suppression will be achieved. To demonstrate this, the frequency responses of a cascade of two ORRs and those of a single ORR are compared in Fig. 5 for $\rho = 2$ and $r = 0.97$.

**Fig. 4.** FHT operation bandwidth related to phase ripple tolerance for different values of $\rho$.

**Fig. 5.** (a) phase responses and (b) corresponding power transmissions of a cascade of two ORRs (black) and a single ORR (grey) for $\rho = 2$ and $r = 0.97$.

### 2.4 Time-domain analysis

To further validate the proposed FHT, the temporal responses of an ORR to two types of test signals, namely sinusoidal signals and transform-limited first-order Hermit-Gaussian pulses [10], were simulated. The ORR was defined with a roundtrip duration (unit delay) of 66.67 ps,
corresponding to a FSR of 15 GHz. To match this specification, two frequencies of 5 GHz and 10 GHz were used for the sinusoidal signals; the input pulses were derived from a transform-limited Gaussian pulse with a full-with half-maximum (FWHM) duration of 100 ps, corresponding to a FWHM bandwidth of 4 GHz. In the simulation, intensity modulation and direct direction were considered. Fig. 6 presents the resulting temporal responses of the ORR versus those of a theoretical FHT. Here, the loss and delay effects of the ORR were removed so as to have a clear comparison of the waveforms. Evidently, a good agreement in terms of the temporal response results. This proves that an FHT can be implemented using ORRs.

Fig. 6. Simulated temporal responses of an ORR with an FSR of 15 GHz versus those of a theoretical FHT: (a) sinusoidal signals of 5 and 10 GHz are used as the inputs; (b)-(d) first-order Hermit-Gaussian pulses derived from a transform-limited Gaussian pulse with FWHM duration of 100 ps are used as the inputs for different values of $\rho$ (the loss and delay effects of the ORR are removed from the simulation results).

3. Device characterization

To demonstrate the performance of the proposed FHT, a number of ORRs were fabricated in TriPleX™ waveguide technology, a proprietary technology of LioniX B.V. [22]. In this technology, the waveguides are fabricated using CMOS process-compatible equipments, and recently a very low propagation loss of 0.1 dB/cm and simultaneously a small bend radius of 70 µm have been characterized [23]. This waveguide consists of two vertically-stacked parallel strips of Si$_3$N$_4$ with a thickness of 170 nm, forming an “=” shape in its cross-section. Between the two strips is an intermediate layer of SiO$_2$ with a thickness of 500 nm, which is also the material of surrounding cladding. The use of this double-strip geometry increases the effective index of the optical mode as compared to a single strip geometry, thus increasing the confinement of the mode and thereby reducing the bend loss. The width of the strips is optimized to result in a high effective index of the mode while the waveguide only supports a single (TE) mode at a wavelength around 1550 nm. This waveguide is considered to be an enabling platform technology for the realization of low loss, compact on-chip MWP signal processors. The ORRs used for the experiments of this paper have the layout as depicted in Fig. 2(a). The roundtrip length and the FSR are 13 mm and 15 GHz, respectively. In both the ring path and the lower arm of the MZI, an optical phase shifter is implemented using a
resistor-based heater placed on top of a length of the waveguide. Therefore, the ORRs are programmable with tunability in both the resonance frequency and the power coupling coefficient by means of the thermo-optical tuning mechanism.

For the RF response measurement, an optical carrier generated by a CW laser (EM4-253-80-057) was externally modulated using a Mach-Zehnder intensity modulator (Avanex PowerLog FA-20), where the modulating RF signal was generated by a vector network analyzer (Agilent NA5230A PNA-L). The frequency of the optical carrier was aligned to a resonance frequency of the ORR according to the device operation principle. Since the waveguide was designed only for TE polarization [23], polarization maintaining fibers were used to feed the correct-polarized light into the chip. After passing through the ORR, the modulated optical signal is fed to a photodetector (Discovery semiconductor DSC710) for direct optical detection, and the detected RF signal is measured by the vector network analyzer to characterize the frequency response of the ORR. The measurement reference was defined by calibrating the $S_{21}$ response to the state where the ORR is decoupled ($\kappa = 0$). This removes the effects of the other components in the setup from the measurement results. As expected, the $S_{21}$ measurements of the ORR are in good agreement with the calculations in Fig. 3, approximating flat magnitude responses and linear phase responses in the bandwidth of interest. In Fig. 7(a), the measured phase responses for different values of the power coupling coefficient $\kappa$ are depicted and are fitted to the design target specifications described in Eq. (4). In Fig. 7(b), the resulting FHT phase shifts are demonstrated by means of removing the delay effect (linear phase) from the measured phase responses as explained in Eq. (5). Evidently, a wideband FHT is achieved, which is characterized by an operation bandwidth from 3 GHz to 12 GHz and a maximum phase ripple of 5 degree.

![Fig. 7](imageurl)

**Fig. 7.** (a) measured RF phase responses of a waveguide realization of an ORR for different values of power coupling coefficient $\kappa$ and the curve-fittings to the target FHT specifications; (b) demonstration of the resulting FHT phase shift by removing the delay effect (linear phase) from the measured phase responses.

4. System-level functionality demonstration

4.1 Functionality description

Beside the conventional signal processing schemes employing FHT [10], an alternative application example of the proposed FHT can be found in the context of optical generation of microwave signals by means of optical heterodyning, namely the generation of microwave signals by means of the beating of two CW optical signals at a photodetector [2]. In principle, the phase of the generated microwave signal $\psi_{RF}(f_{RF})$ is equal to the phase difference between the two beating optical signals $\Delta \psi_o(f_1, f_2)$ with $f_1$ and $f_2$ denoting the frequencies of the two beating optical signals. Here, the proposed FHT can be employed before the detector to introduce an additional phase difference $\Delta \psi_{ORR}(f_1, f_2)$ in $\Delta \psi_o(f_1, f_2)$, which serves as a simple and on-chip solution to provide the phase control functionality on $\Delta \psi_o(f_1, f_2)$, or equivalently, the phase control functionality on $\psi_{RF}(f_{RF})$, since $\psi_{RF}(f_{RF}) = \Delta \psi_o(f_1, f_2) + \Delta \psi_{ORR}(f_1, f_2)$. 
The principle of using the proposed FHT to achieve this phase control functionality is illustrated in Fig. 8. It is shown that by varying the fractional order ρ of the FHT (implemented by tuning the power coupling coefficient κ of the ORR), ΔΨORR(f1, f2) is varied accordingly; and by shifting the FHT phase response to different operation positions (implemented by tuning the additional roundtrip phase shift θ of the ORR), a full 2π changing range of ΔΨORR(f1, f2) can be achieved. However, to allow the variability of ΔΨORR(f1, f2), the frequency period of the FHT, or equivalently, the FSR of the ORR ΔFSR must fulfill the condition n·ΔFSR ≠ Δfo = f2 − f1 = fRF with n being an positive integer. The reason for this is that when n·ΔFSR = Δfo, ΔΨORR(f1, f2) becomes a constant ΔΨORR(f1, f2) = 2πΔfo/ΔFSR = n·2π which is independent of κ and θ of the ORR. In this case, the FHT loses its capability to modify ΔΨORR(f1, f2) and therefore is unable to perform the phase control functionality. Moreover, it is preferable that the ΔFSR of the ORR is such that both beating optical signals can be accommodated in the FHT operation bandwidth. This way, the possible strong amplitude suppression around the resonance frequency of the ORR can be avoided.

![Fig. 8. Illustration of the phase control functionality of the proposed FHT for optical generation of microwave signals using optical heterodyning: (a) with the phase responses of the FHT for θ = 0 and (b) for θ = π (fo FB denotes the resonance frequency of the ORR).](image)

4.2 Experimental demonstration

To demonstrate the phase control functionality described in the previous section, a PIC consisting of a splitter and an ORR with a FSR of 15 GHz was used. The setup for the experiment is depicted in Fig. 9, where two CW lasers (EM4-253-80-057) were frequency-positioned as illustrated in the inset of Fig. 9. By adjusting the frequency spacing between f1 and f2, microwave signals with frequencies from 7 GHz to 10 GHz were generated. For the phase measurement, an oscilloscope (Agilent Infinium 54854A) was employed which has a bandwidth limit of 4 GHz. Due to this bandwidth limit, electrical frequency downconversion was performed before the phase measurement. Here, to guarantee accurate measurement results, a phase-calibrated LO signal was used for the signal mixing.

First, a full 2π phase changing range of a generated microwave signal with a frequency of 9 GHz was demonstrated, where the signal phase ΨRF(fRF) was varied using the principle illustrated in Fig. 8. Fig. 10(a) depicts the generated microwave signal at output 2, where output 1 was used as the phase reference. Further, based on the same principle, the simultaneous generation of in-phase and quadrature microwave signals using both outputs of the splitter circuit was demonstrated. Fig. 10(b) depicts the generated microwave signals of both I and Q channels, where a constant quadrature phase relation is achieved for the signal frequency varying from 7 GHz to 10 GHz. To achieve this functionality, the delay effects (linear phase) of the I and Q channel need to be equalized as illustrated in the inset of Fig. 9, so that the phase difference between the two outputs ΔΨRF,I,Q is only determined by the FHT phase shift ϕ of the ORR with the relation ΔΨRF,I,Q = 2ϕ. In this demonstration, the power coupling coefficient κ of the ORR was set to 0.52 corresponding to a FHT phase shift ϕ = -135° (see Fig. 7(b)). This results in the desired quadrature phase relation ΔΨRF,I,Q = -270°.
equivalently, $\Delta \Psi_{RF,I} = 90^\circ$. In principle, the frequency periodicity of the ORR allows this quadrature phase relation to be available for multiple ranges of frequencies. However, constrained by the speed limitation of the photodetectors in this setup, this functionality was only demonstrated for a maximum frequency of 10 GHz.

Fig. 9. Setup for experimental demonstration of on-chip phase control of microwave signals generated by optical heterodyning, and simultaneous generation of in-phase and quadrature microwave signals (inset: an example of the frequency positioning of the two lasers and the optical phase responses of the two channels where the delay effects of the two channels are equalized).

Fig. 10. Demonstrations of two functionalities achieved using the FHT for optical generation of microwave signals by means of optical heterodyning: (a) microwave phase control with a full $2\pi$ phase changing range and (b) simultaneous generation of in-phase and quadrature microwave signals for a wide frequency range.

5. Conclusions

In this paper, a novel on-chip implementation of a wideband MWP FHT is proposed and demonstrated. The implementation uses an optical all-pass filter based on ORRs, which features simplicity, space efficiency and compatibility for large-scale integration. The full programmability of the ORRs allows variable and arbitrary fractional order of the Hilbert transformer. The performance analysis in both frequency and time domain validates that the proposed implementation provides a good approximation to an ideal FHT. A waveguide realization of an ORR with a FSR of 15 GHz demonstrates a FHT bandwidth from 3 GHz to 12 GHz with a maximum phase ripple of 5 degree. However, this bandwidth can be upscaled by using ORRs with larger FSRs. The waveguide-based structure allows an ORR-based FHT to be integrated with other functional building blocks on a PIC to create various system-level functionalities for on-chip complex MWP signal processors. As an example, a PIC consisting
of a splitter and an ORR was used to demonstrate two functionalities for optical generation of microwave signals by means of optical heterodyning, namely microwave phase control and simultaneous generation of in-phase and quadrature microwave signals for a wide frequency range. For these functionalities, this simple and on-chip solution is considered to be practical, particularly when operating together with a dual-frequency laser. Constrained by the speed limit of the photodetectors in the setup, the functionality demonstration was only performed for the frequency range from 7 GHz to 10 GHz for this paper. However, in principle, the demonstrated functionalities can be available in multiple ranges of frequencies by making use of the frequency periodicity of the device. To our best knowledge, this is the first-time on-chip demonstration where ORRs are employed to perform phase control functionalities for optical generation of microwave signals by means of optical heterodyning.

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