Input–output decoupling of Hamiltonian systems: The linear case

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Abstract: In this note we give necessary and sufficient conditions for a linear Hamiltonian system to be input–output decouplable by Hamiltonian feedback, i.e. feedback that preserves the Hamiltonian structure. In a second paper we treat the same problem for nonlinear Hamiltonian systems.

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1. Introduction

A physically well-motivated subclass of the set of all linear systems

$$\begin{aligned}\dot{x} &= Ax + Bu, \\ y &= Cx,\end{aligned}\tag{1.1}$$

consists of the *Hamiltonian* systems. Such a Hamiltonian system is defined as follows. The state space X is even dimensional, say $X = \mathbb{R}^{2n}$ and is endowed with a *symplectic form*

$$J = \begin{pmatrix} 0 & -I_n \\ I_n & 0 \end{pmatrix}.$$

Furthermore the matrices A , B and C satisfy

$$\begin{aligned}A^T J + JA &= 0 \quad (A \text{ is Hamiltonian}), \\ B_j^T &= C.\end{aligned}\tag{1.2}$$

It follows that necessarily the number of inputs equals the number of outputs, say $u, y \in \mathbb{R}^m$. Without loss of generality we may assume that B is injective, or equivalently, C is surjective. From an input–output point of view a Hamiltonian system is fully characterized by the condition $G(t) = -G^T(-t)$, or $H(s) = H^T(-s)$, where G resp. H

is the impulse response resp. transfer matrix of (1.1), see e.g. [1,7,8]. Many physical systems are of this Hamiltonian form, for example linear conservative mechanical systems. In a second paper [11] we shall consider *nonlinear* Hamiltonian systems (cf. [8]), but we mention already that the linearization of a nonlinear Hamiltonian system (e.g. a robot manipulator) also exhibits this linear Hamiltonian structure (1.2).

In this paper we wish to tackle the input–output decoupling problem for linear Hamiltonian systems; that is we seek for a feedback $u = Fx + Gv$ for the Hamiltonian systems (1.1), such that in the modified system

$$\begin{aligned}\dot{x} &= (A + BF)x + BGv, \\ y &= Cx,\end{aligned}\tag{1.3}$$

with $v = (v_1, \dots, v_m)^T$, $y = (y_1, \dots, y_m)^T$, the input v_i does not affect the output y_j for $j \neq i$. Moreover we require the input v_i to ‘control’ the output y_i , $i = 1, \dots, m$. Necessary and sufficient conditions for the solvability of this problem are well known, cf. [2,5]. However in this paper we shall add the extra requirement that the system (1.3) after feedback is again Hamiltonian. This restricts the allowed feedback in the following way.

Proposition 1.1. *Suppose (1.1) defines a Hamiltonian system. The system (1.3) is again Hamiltonian if and only if*

- (a) $G = I_m$,
- (b) $F = HC$ for some (m, m) -matrix H with $H = H^T$.

Proof (see [7,8]). The triple $(A + BF, BG, C)$ has to satisfy (1.2), i.e.

$$(A + BF)^T J + J(A + BF) = 0, \quad (BG)^T J = C.$$

From the last identity we obtain $G = I_m$. The first equation yields

$$F^T B^T J + JBF = F^T C - C^T F = 0.$$

Therefore the symplectic form

$$\begin{pmatrix} 0 & -I_m \\ I_m & 0 \end{pmatrix}$$

on $\mathbb{R}^{2m}$ is zero restricted to the subspace $\text{Im}(\begin{smallmatrix} C \\ F \end{smallmatrix})$, and this implies that

$$\dim \text{Im} \begin{pmatrix} C \\ F \end{pmatrix} \leq m.$$

Since $\text{rank } C = m$ it follows $F = HC$ for some (m, m) -matrix H . Now from $F^T C - C F^T = 0$ this yields $C^T H^T C - C^T H C = 0$ or $H = H^T$. $\square$

Such a feedback $u = HCx + v = Hy + v$, with $H = H^T$, which leaves the Hamiltonian structure invariant, will be called a *Hamiltonian* feedback. From a physical viewpoint such a feedback adds a 'potential energy' $\frac{1}{2}y^T Hy$ to the internal energy of the system, while leaving the input channels invariant (i.e. no precompensation factor G). Now the *Hamiltonian input-output decoupling problem* can be formulated as follows: Consider the Hamiltonian system (1.1). Find, if possible, an output feedback $u = Hy$, with $H = H^T$, such that in

$$\begin{aligned} \dot{x} &= (A + BHC)x + Bu, \\ y &= Cx, \end{aligned} \quad (1.4)$$

the transfer functions from u_i to y_j are zero for $j \neq i$ and nonzero for $i = j$, $i, j = 1, \dots, m$.

2. Geometric and Hamiltonian preliminaries

Recall the following notions from the geometric approach to linear system theory, cf. [9,10]. A subspace $V \subset X$ is called (A, B) -invariant for the — not necessarily Hamiltonian — system (1.1) if

$$AV \subset V + \text{Im } B,$$

or equivalently, if there exists a matrix F such that $(A + BF)V \subset V$. The maximal (A, B) -invariant subspace in $\text{Ker } C$ (which exists) is denoted by $V^*(A, B, C)$. A subspace $R \subset X$ is called a controllability subspace of (1.1) if R is (A, B) -invariant and the eigenvalues of $A + BF|_R$ can arbitrarily be assigned by using an appropriate feedback F which leaves R invariant: $(A + BF)R \subset R$. The maximal controllability subspace contained in $\text{Ker } C$ (which also exists) is denoted by $R^*(A, B, C)$. Furthermore a subspace $S \subset X$ is

called (C, A) -invariant if

$$A(S \cap \text{Ker } C) \subset S,$$

or equivalently, if there exists a matrix K such that $(A + KC)S \subset S$. The minimal (C, A) -invariant subspace containing $\text{Im } B$ (which exists) will be denoted as $S^*(A, B, C)$. We will now derive some elementary facts on (A, B) - and (C, A) -invariant subspaces of a Hamiltonian system, see also [7]. Therefore assume that the state space X equals $\mathbb{R}^{2n}$ and is endowed with a symplectic form J . For a linear subspace $V \subset X$ the orthogonal complement of V with respect to J , denoted as $V^\perp$, is given by

$$V^\perp = \{x \in \mathbb{R}^{2n} \mid x^T J v = 0, \forall v \in V\}.$$

Proposition 2.1. *Let (A, B, C) be a Hamiltonian system. Then*

(a) (A, B) is controllable $\Leftrightarrow (C, A)$ is observable.

(b) $V \subset \mathbb{R}^{2n}$ is (A, B) -invariant if and only if $V^\perp$ is (C, A) -invariant.

(c) $V^*(A, B, C) = (S^*(A, B, C))^\perp$.

(d) $R^*(A, B, C) = V^*(A, B, C) \cap (V^*(A, B, C))^\perp$.

Proof. (a) Check the rank conditions for controllability and observability by using (1.2).

(b) Suppose $V \subset \mathbb{R}^{2n}$ satisfies

$$AV \subset V + \text{Im } B.$$

Then

$$V^\perp \cap (\text{Im } B)^\perp \subset (AV)^\perp.$$

Now

$$\begin{aligned} x \in (AV)^\perp &\Leftrightarrow x^T J A v = 0, \forall v \in V \\ &\Leftrightarrow x^T A^T J v = 0, \forall v \in V \\ &\Leftrightarrow A x \in V^\perp. \end{aligned}$$

Hence $(AV)^\perp = A^{-1}(V^\perp)$ (here $A^{-1}W := \{x \in \mathbb{R}^{2n} \mid Ax \in W\}$ for a subspace W). Furthermore, since $B^T J = C$ we have $(\text{Im } B)^\perp = \text{Ker } C$. So

$$V^\perp \cap \text{Ker } C \subset A^{-1}(V^\perp)$$

$$\text{or } A(V^\perp \cap \text{Ker } C) \subset V^\perp.$$

(c) Follows directly from (b). (In fact it is easily seen that the subspaces V^i and S^i in the V^* and S^* algorithm, cf. [9,10], are dual in the sense that $V^i = (S^i)^\perp$.)

(d) Use the fact that

$$R^*(A, B, C) = V^*(A, B, C) \cap S^*(A, B, C),$$

cf. [4] and (c). $\square$

3. Main theorem

Let (A, B, C) be a Hamiltonian system on $(\mathbb{R}^{2n}, J)$. A subspace $V \subset \mathbb{R}^{2n}$ is called *symplectic* if $V^\perp \cap V = 0$. We denote by b_i the i -th column of the matrix B and by c_i the i -th row of the matrix C , $i = 1, \dots, m$. Furthermore $\hat{B}_i$ is the $(2n, m-1)$ -matrix derived from B by deleting the i -th column b_i , $i = 1, \dots, m$. Similarly $\hat{C}_i$ is the $(m-1, 2n)$ -matrix resulting from C by deleting c_i , $i = 1, \dots, m$. Our main theorem is now as follows.

Theorem 3.1. *Let (A, B, C) be a minimal Hamiltonian system with $V^*(A, B, C) = 0$. Then the Hamiltonian input-output decoupling problem can be solved if and only if the following two conditions are satisfied:*

- (a) $b_i \in V^*(A, B, \hat{C}_i)$, $i = 1, \dots, m$,
- (b) $V^*(A, B, c_i) = (V^*(A, B, \hat{C}_i))^\perp$, $i = 1, \dots, m$.

Proof. For simplicity we will write

$$V_i^* = V^*(A, B, \hat{C}_i) \quad \text{and} \quad \hat{V}_i^* = V^*(A, B, c_i).$$

(If) Since $b_i \in V_i^*$ it follows that

$$V_i^* = V^*(A, B, \hat{C}_i) = V^*(A, \hat{B}_i, \hat{C}_i)$$

and, since $\hat{V}_i^* = \sum_{j \neq i} V_j^*$, that

$$\hat{V}_i^* = V^*(A, B, c_i) = V^*(A, b_i, c_i).$$

Now the triple (A, b_i, c_i) is again a Hamiltonian system (with scalar input and output) and therefore we can apply Proposition 2.1 to conclude that

$$R^*(A, b_i, c_i) = \hat{V}_i^* \cap (\hat{V}_i^*)^\perp.$$

Now suppose that

$$\hat{R}_i^* := R^*(A, b_i, c_i) \neq 0.$$

Then we have $b_i \in \hat{R}_i^*$ and hence

$$A\hat{R}_i^* \subset \hat{R}_i^* + b_i = \hat{R}_i^*$$

and also by definition $\hat{R}_i^* \subset \text{Ker } c_i$. Furthermore $\hat{R}_i^* \subset (\hat{V}_i^*)^\perp = V_i^*$ and so $\hat{R}_i^* \subset \text{Ker } \hat{C}_i$. Therefore

$\hat{R}_i^*$ is A -invariant and is contained in $\text{Ker } C$. Hence if $\hat{R}_i^* \neq 0$ we obtain a contradiction with the observability of (A, B, C) . It follows that $\hat{V}_i^* \cap (\hat{V}_i^*)^\perp = 0$ and hence $\hat{V}_i^*$ is a symplectic subspace of $(\mathbb{R}^{2n}, J)$ for all $i = 1, \dots, m$. Since $V_i^* = (\hat{V}_i^*)^\perp$ this also yields that V_i^* is symplectic for all $i = 1, \dots, m$. Moreover

$$0 = \hat{V}_i^* \cap (\hat{V}_i^*)^\perp = \hat{V}_i^* \cap V_i^* = \sum_{j \neq i} V_j^* \cap V_i^*$$

implies that the subspaces $V_1^*, \dots, V_m^*$ are *independent* (cf. [10]). This yields that there exists an $(m, 2n)$ -matrix F such that

$$(A + BF)V_i^* \subset V_i^*, \quad i = 1, \dots, m$$

(this also follows from the 'noninteracting condition' $b_i \in V_i^*$). Moreover

$$\text{Im } B \subset \sum_{i=1}^m V_i^*, \quad A \left(\sum_{i=1}^m V_i^* \right) \subset \sum_{i=1}^m V_i^*$$

and hence by controllability $\mathbb{R}^{2n} = V_1^* \oplus \dots \oplus V_m^*$. Furthermore because $V_i^* = (\hat{V}_i^*)^\perp$ and $\hat{V}_i^*$ is (A, B) -invariant we obtain by Proposition 2.1 that V_i^* is also (C, A) -invariant, $i = 1, \dots, m$. Therefore there exists an (m, m) -matrix H such that

$$(A + BHC)V_i^* \subset V_i^*, \quad i = 1, \dots, m.$$

The last thing we have to show is that we may take H to be a symmetric matrix.

Since V_i^* is symplectic there exists a symplectic basis (q_i, p_i) for V_i^* , i.e. a basis

$$(q_i, p_i) = (q_i^1, \dots, q_i^{k_i}, p_i^1, \dots, p_i^{k_i})$$

with $\dim V_i^* = 2k_i$, such that J restricted to V_i^* equals

$$\begin{pmatrix} 0 & -I_{k_i} \\ I_{k_i} & 0 \end{pmatrix}.$$

in this basis. Then because $(V_i^*)^\perp = \sum_{j \neq i} V_j^*$ it follows that $(q_1, \dots, q_n, q_1, \dots, p_n)$ is a symplectic basis for $\mathbb{R}^{2n}$, with

$$J = \begin{pmatrix} 0 & -I_n \\ I_n & 0 \end{pmatrix}.$$

Next we will use the relations

$$(A + BHC)V_i^* \subset V_i^*, \quad i = 1, \dots, m.$$

We have

$$(A + BHC)V_1^* \subset V_1^*, \quad V_1^* \subset \text{Ker } \hat{C}_1, \\ (A + BHC)\hat{V}_1^* \subset \hat{V}_1^*, \quad \hat{V}_1^* \subset \text{Ker } c_1.$$

Hence

$$(JA + JBHC)V_1^* = (-Q - C^T HC)V_1^* \\ \subset JV_1^* = V_1^*$$

and

$$(-Q - C^T HC)\hat{V}_1^* = \hat{V}_1^*,$$

where $Q = -JA$ is symmetric. Writing out in the above basis we get

$$(Q + C^T HC) \begin{pmatrix} * \\ 0 \\ * \\ 0 \end{pmatrix} \subset \begin{pmatrix} * \\ 0 \\ * \\ 0 \end{pmatrix}, \\ (Q + C^T HC) \begin{pmatrix} 0 \\ * \\ 0 \\ * \end{pmatrix} \subset \begin{pmatrix} 0 \\ * \\ 0 \\ * \end{pmatrix},$$

where we have symbolically written

$$V_1^* = \begin{pmatrix} * \\ 0 \\ * \\ 0 \end{pmatrix}, \quad \hat{V}_1^* = \begin{pmatrix} 0 \\ * \\ 0 \\ * \end{pmatrix}.$$

In the same way the matrix C can be expressed in this basis as

$$C = \begin{pmatrix} c_1 \\ \hat{C}_1 \end{pmatrix} = \begin{pmatrix} c_{11} & 0 & c_{13} & 0 \\ 0 & C_{22} & 0 & C_{24} \end{pmatrix}.$$

Furthermore the matrices Q and H will be partitioned according to this basis and we obtain that Q and $C^T HC$ have the following form respectively:

$$\begin{pmatrix} Q_{11} & Q_{12} & Q_{13} & Q_{14} \\ Q_{21} & Q_{22} & Q_{23} & Q_{24} \\ Q_{31} & Q_{32} & Q_{33} & Q_{34} \\ Q_{41} & Q_{42} & Q_{43} & Q_{44} \end{pmatrix}, \\ \begin{pmatrix} c_{11}^T h_{11} c_{11} & c_{11}^T h_{12} c_{22} & c_{11}^T h_{11} c_{13} & c_{11}^T h_{12} c_{24} \\ C_{22}^T h_{21} c_{11} & C_{22}^T H_{22} C_{22} & C_{22}^T h_{21} c_{13} & C_{22}^T H_{22} C_{24} \\ c_{13}^T h_{11} c_{11} & c_{13}^T h_{12} c_{22} & c_{13}^T h_{11} c_{13} & c_{13}^T h_{12} c_{24} \\ C_{24}^T h_{21} c_{11} & C_{24}^T H_{22} C_{22} & C_{24}^T h_{21} c_{13} & C_{24}^T H_{22} C_{24} \end{pmatrix},$$

and so the matrix $Q + C^T HC$ leaves the subspaces

$$\begin{pmatrix} * \\ 0 \\ * \\ 0 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 0 \\ * \\ 0 \\ * \end{pmatrix}$$

invariant. This yields

$$\begin{aligned} c_{11}^T h_{12} c_{22} + Q_{12} &= 0, \\ C_{22}^T h_{21} c_{11} + Q_{21} &= 0, \\ C_{22}^T h_{21} c_{13} + Q_{23} &= 0, \\ c_{13}^T h_{12} c_{22} + Q_{32} &= 0, \\ c_{11}^T h_{12} c_{24} + Q_{14} &= 0, \\ C_{24}^T h_{21} c_{11} + Q_{41} &= 0, \\ c_{13}^T h_{12} c_{24} + Q_{34} &= 0, \\ C_{24}^T h_{21} c_{13} + Q_{43} &= 0. \end{aligned} \tag{3.1}$$

Now the matrix Q is symmetric, so $q_{12} = q_{21}^T$, $q_{23} = q_{32}^T$, $q_{14} = q_{41}^T$ and $q_{34} = q_{43}^T$. It follows that we may take $h_{21} = h_{12}^T$ in (3.1). Having fixed h_{11} and $h_{12} = h_{21}^T$ we proceed by considering V_2^* and $\hat{V}_2^*$ (or $\hat{V}_2^* \setminus V_1^*$). Continuing up to V_m^* and $\hat{V}_m^*$ in this way we conclude that we may take H satisfying

$$(A + BHC)V_i^* \subset V_i^*, \quad i = 1, \dots, m,$$

to be symmetric. Therefore it follows from $b_i \in V_i^*$, $i = 1, \dots, m$, that $u = Hy + v$ is a Hamiltonian feedback which decouples the system. Clearly the subsystems

$$(A + BHC|V_i^*, b_i, c_i), \quad i = 1, \dots, m,$$

are minimal Hamiltonian systems.

(Only if) It is well known that the input-output decoupling by general state feedback is possible if and only if

$$\text{Im } B = \sum_{i=1}^m \text{Im } B \cap V_i^*,$$

see [5]. Since the precompensation matrix G equals I_m , it follows that $b_i \in V_i^*$, $i = 1, \dots, m$. Because $V^*(A, B, C) = 0$ we must have

$$(A + BHC)V_i^* \subset V_i^*, \quad i = 1, \dots, m,$$

for a certain symmetric H . Consider the subspace $V = V_i^* \cap \hat{V}_i^*$ for an $i \in \{1, \dots, m\}$. Clearly $V \subset \text{Ker } C$, so $(A + BHC)V = AV \subset V$ and thus by the observability of the system $V = 0$. Hence for $i =$

$1, \dots, m,$

$$V_i^* \cap \sum_{j \neq i} V_j^* = 0.$$

Moreover by controllability we have $\sum_{i=1}^m V_i^* = \mathbb{R}^{2n}$. We will now use Proposition 2.1 for showing that condition (b) is satisfied. Since

$$(A + BHC)V_i^* \subset V_i^*$$

the subspace V_i^* is (C, A) -invariant and so $(V_i^*)^\perp$ is (A, B) -invariant. Because

$$(V_i^*)^\perp \subset (\text{Im } b_i)^\perp = \text{Ker } \hat{C}_i$$

we obtain

$$(V_i^*)^\perp \subset \hat{V}_i^*, \quad i = 1, \dots, m.$$

However since

$$\dim(V_i^*)^\perp = 2n - \dim V_i^*$$

and

$$\mathbb{R}^{2n} = V_i^* \oplus \hat{V}_i^*, \quad i = 1, \dots, m,$$

we may conclude $(V_i^*)^\perp = \hat{V}_i^*$ for all $i = 1, \dots, m$. $\square$

Remark. Notice that if (a) and (b) are satisfied V_i^* is not only the maximal (A, B) -invariant subspace contained in $\text{Ker } \hat{C}_i$, but also equal to the minimal (C, A) -invariant subspace containing $\text{Im } b_i$, $i = 1, \dots, m$. Furthermore it follows that V_i^* equals $R^*(A, B, \hat{C}_i)$.

We conclude from the above theorem that the Hamiltonian input-output decoupling problem is solvable if and only if the Hamiltonian system $\dot{x} = Ax + Bu$, $y = Cx$ with $V^*(A, B, C) = 0$ can be partitioned into m independent Hamiltonian subsystems after applying Hamiltonian feedback. Furthermore the diagonal elements of the output feedback $u = Hy$ remain to be freely chosen. This freedom can be used to adjust the behaviour of these subsystems. We leave the case $V^*(A, B, C) \neq 0$ for further research (see also [11]).

In a similar way as in Theorem 3.1 we can deal with the Hamiltonian block-decoupling problem, that there is given a partitioning of the outputs

$$y = (y_1, \dots, y_m)^\top = (\underline{y}_1, \dots, \underline{y}_k)^\top$$

into k disjoint groups (and thus also of the inputs

$$u = (u_1, \dots, u_m)^\top = (\underline{u}_1, \dots, \underline{u}_k)^\top)$$

and the question is whether or not we can find a Hamiltonian feedback $u = Hx + v$, $H = H^\top$, such that $\underline{v}_i$ does not affect $\underline{y}_j$, $j \neq i$, and $\underline{v}_i$ controls $\underline{y}_i$, $i = 1, \dots, k$. Writing, as before, B_i for the input matrix corresponding to the inputs $\underline{u}_i$, $\hat{B}_i$ for the input matrix B with deleting B_i and similarly C_i and $\hat{C}_i$ we obtain the following sufficient conditions.

Corollary 3.2. Let (A, B, C) be a minimal Hamiltonian system with $V^*(A, B, C) = 0$. Then the Hamiltonian block-decoupling problem is solvable if the following three conditions hold:

- (a) $\text{Im } B_i \in V^*(A, B, \hat{C}_i)$, $i = 1, \dots, k$,
- (b) $V^*(A, B, C_i) = (V^*(A, B, \hat{C}_i))^\perp$, $i = 1, \dots, k$,
- (c) $R^*(A, B_i, C_i) = 0$, $i = 1, \dots, k$.

Proof. Let

$$V_i^* := V^*(A, B, C_i^*) \quad \text{and} \quad \hat{V}_i^* = V^*(A, B, C_i),$$

$i = 1, \dots, k$. Since $\text{Im } B_i \in V_i^*$ we have $V_i^* = V^*(A, \hat{B}_i, \hat{C}_i)$ and because $\hat{V}_i^* = \sum_{j \neq i} V_j^*$ we obtain $\hat{V}_i^* = V^*(A, B_i, C_i)$. The triple (A, B_i, C_i) being Hamiltonian allows us to apply Proposition 2.1 to conclude

$$0 = R^*(A, B_i, C_i) = \hat{V}_i^* \cap (\hat{V}_i^*)^\perp,$$

$j = 1, \dots, k$. So again, the $\hat{V}_i^*$'s are symplectic subspaces and

$$0 = \hat{V}_i^* \cap (\hat{V}_i^*)^\perp = \sum_{j \neq i} V_j^* \cap V_i^*$$

implies that they are independent. The rest of the proof is now as in Theorem 3.1. $\square$

Remark. Condition (c) of Corollary 3.2 implies that the subsystems (A, B, C_i) are left-invertible, cf. [10], and therefore also right-invertible. In this way the 'geometric' decoupling problem with the prerequisite of output controllability coincides with the transfer function approach of [3].

We will conclude this paper with a very simple example.

Example 3.3. *Time-reversible* Hamiltonian systems have the following additional structure (cf. [6]). There exists a symplectic basis of $\mathbb{R}^{2n}$ such that

$$A = \begin{pmatrix} 0 & P \\ -Q & P \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ B_1 \end{pmatrix}, \quad C = (C_1 \quad 0),$$

with $P = P^T$, $Q = Q^T$ and $B_1^T = C_1$. Considering only the matrix A we know that if $P > 0$ there is a symplectic basis for $\mathbb{R}^{2n}$ such that

$$A = \begin{pmatrix} 0 & I \\ -D & 0 \end{pmatrix}$$

with D a diagonal matrix. Setting $B_1 = I_m = C_1$ ($m = n$), we clearly have a decoupled system consisting of n independent oscillators. However, given $B_1 = C_1 = I_n$, input-output decoupling is possible if and only if P is already diagonal. (Q may be arbitrary since $Q + I_n H I_n$ can be freely assigned by the choice of H .) $\square$

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