HYSTERESIS LOSSES IN HOLLOW SUPERCONDUCTORS

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Abstract

Flux penetration into a hollow superconducting filament in a time-varying transverse magnetic field is determined numerically. The magnetization of the filaments is calculated for field variations below and above the penetration field of the filament. The influence of the inner radius of the superconducting filament on the magnetization and the hysteresis losses in the filament is shown.

The critical current density is taken to be constant during the external field cycle or it depends within the superconducting filament on the local magnetic field, which is the sum of the externally applied field and the field induced by the local screening currents. Calculations with the theory presented here are compared with experimental results.

Introduction

The penetration of a transverse time-varying magnetic field into a solid filament has been determined by several authors^{1,2,3}. In all cases the position of the moving boundary between saturated and non-saturated regions inside the filament is numerically calculated using the fact that in the interior region the variation of the external field is cancelled by the field variation caused by the screening currents outside the moving boundary. Knowing the current distribution inside the filament the magnetic moment can be calculated and hence the hysteresis losses during one cycle of the external magnetic field. In all these models the critical current density inside the filament is taken to be constant during a cycle of the external applied field.

Hartmann^{4,5} has developed a numerical method by which the current distribution in a filament of arbitrary shape in a time-varying transverse field can be calculated. In this model it is optional to include the dependence of the critical current density within the superconducting filament on the local magnetic field. The local field is the sum of the externally applied field and the field induced by the screening currents in other parts of the same superconducting filament.

A simple model for the calculation of hysteresis losses in hollow superconducting niobium-tin filaments has been proposed by Eikelboom⁶. Good agreement was found between experimental results and theoretical calculations.

In this paper a full treatment is given of field penetration and hysteresis losses in hollow superconducting filaments in a variable field superimposed on a constant bias field; the fields are parallel to each other and transverse to the wire axis.

Analytical approximations to the numerically calculated magnetization as a function of the magnetic field are given. On basis of the found relation between the magnetization and the applied field expressions are derived for the hysteresis losses. The influence of the field dependence of the critical current density on the hysteresis losses is discussed.

Magnetization measurements have been done with two types

of niobium-tin wires with hollow filaments at bias fields up to 7 T and pulse field variations up to 1 T. From these measurements the penetration field of the filaments as a function of the bias field can be determined and by means of the theory presented here the critical current density can be deduced. The critical current density can also be determined by means of the measured hysteresis losses and the given expressions for these losses. The results of these two methods are compared.

Penetration with constant critical current density

The penetration field B_p of a superconducting filament is defined to be the external field variation at which the last point of the superconductor is penetrated by the flux in an initially current-free filament. The penetration field thus defined is equal to the field in this last point induced by the screening currents in the filament. For a solid filament it is given by

$$B_p = \frac{2}{\pi} \mu_0 J_c R_2 \quad (1)$$

R_2 being the outer radius of the filament. The field variation necessary to penetrate a filament which is not free of current, for example after one initial cycle of the external field, is then equal to $2B_p$. In a hollow filament the penetration is equal to that in a solid filament as long as the moving boundary does not reach the inner radius of the filament. We define B_{p1} to be the external field variation at which the moving boundary in a current-free hollow superconducting filament reaches the inner radius R_1 in point A in fig. 1. B_{p1} can be calculated numerically.

Fig. 1 Cross section of hollow filament with contour corresponding to $B_a = B_{p1}$

The following formula was found to fit the numerically computed values of B_{p1} as a function of $\rho = R_1/R_2$ within one percent

$$B_{p1}(\rho) = [1.000 - 1.395 \rho + 0.579 \rho^2 - 0.184 \rho^3] B_p \quad (2)$$

The moving boundary has been approximated by an ellipse⁶. The field variation in this model at which the short axis of the ellipse is equal to the inner diameter of the hollow filament is found to be an approximation within one percent of the numerically found B_{p1} .

The penetration field of a hollow superconducting

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filament, called B_{p2} , at which point B in fig. 1 is reached by the moving boundary, is smaller than B_p and can be calculated analytically on basis of the above given definition of the penetration field

$$B_{p2}(\rho) = \left[(1-\rho^2) \ln \frac{1+\rho}{1-\rho} + 2\rho - 2\rho^2 \right] \frac{B_p}{4\rho} \quad (3)$$

It is convenient to define a dimension-less normalized field β as being the ratio of the magnetic field and the penetration field B_p of a solid filament. In fig. 2 the relation between $\beta_1 = B_p/B_p$, $\beta_2 = B_{p2}/B_p$ and ρ is shown.

Fig. 2 Relation between the normalized fields β_1 and β_2 and the ratio ρ between inner and outer radius of the hollow filament

The magnetization of a fully penetrated filament is given by

$$M_{sol} = -\frac{4}{3\pi} J_c R_2 = -\frac{2}{3} \frac{B_p}{\mu_0} \quad (4)$$

Good analytical approximations for the magnetization of a solid filament as a function of the external field variation have been found by other authors^{1,2}. With $\beta = B/B_p$ we find in our numerical model the following relation³ between the magnetization and β

$$M_I(\beta) = -\left[1.998\beta - 2.161\beta^2 + 1.006\beta^3 - 0.176\beta^4 \right] \frac{B_p}{\mu_0} \quad (5)$$

The magnetization of a fully penetrated hollow filament is smaller than that of a solid one and is calculated to be

$$M_{hol} = [1 - \rho^3] M_{sol} \quad (6)$$

For each value of ρ we can distinguish three regions:

- I : $B_a \leq B_{p1}(\rho)$
- II : $B_{p1}(\rho) \leq B_a \leq B_{p2}(\rho)$
- III : $B_{p2}(\rho) \leq B_a$

The magnetization for region I is given by (5) and for region III by (6). For region II an approximation accurate within one percent for $\rho < 0.8$ to numerically found values is given by

$$M_{II}(\beta) = M_{hol} + \frac{aB_p}{\mu_0} (\beta_2 - \beta)^n, \quad (7)$$

a and n being a function of ρ

Expressions for the coefficients a and n are easily found by means of the requirement that the magnetization and its derivative are continuous in $\beta = \beta_1$. Fig. 3 shows the magnetization M as a function of β . The dashed curve shows the magnetization for $\beta = \beta_2$ for $\rho = 0.1$.

Fig. 3. The magnetization as a function of $\beta = B/B_p$ for $\rho = 0.0, 0.4, 0.5, 0.6, 0.7, 0.8$. The dashed line shows the magnetization as a function of $\beta_2(\rho)$

Hysteresis losses per unit volume per cycle of the external field are given by the integral $Q_h = \oint M(B_a) dB_a$ and have been calculated using the above expressions for the magnetization. In a filament which is no longer current-free after an initial cycle an inwardly moving boundary should be regarded as a current layer of $2J_c$ superimposed on the original screening currents and the maximal variation in the magnetization is then reached at $\beta = 2\beta_2$. We define ΔB as the field variation during one field cycle. Hysteresis losses $Q_h(\beta)$ with $\beta = \Delta B/B_p$ are given by

$$\begin{aligned} Q_h(\beta) &= Q_1(\beta) \text{ for } \beta \leq 2\beta_1, \\ Q_h(\beta) &= Q_2(\beta) \text{ for } 2\beta_1 \leq \beta \leq 2\beta_2, \\ Q_h(\beta) &= Q_3(\beta) \text{ for } \beta \geq 2\beta_2 \end{aligned}$$

with Q_1 , Q_2 and Q_3 defined as follows:

$$Q_1(\beta) = \frac{B_p^2}{\mu_0} [0.360\beta^3 - 0.126\beta^4 + 0.013\beta^5] \quad (8a)$$

$$Q_2(\beta) = Q_1(2\beta_1) + \frac{8aB_p^2}{\mu_0(n+1)} (\beta_2 - \beta/2)^{n+1} + 2B_p \beta [M_{II}(\beta/2) - 2M_{hol}] + K \quad (8b)$$

$$Q_3(\beta) = Q_2(2\beta_2) - 2B_p M_{hol} (\beta - 2\beta_2) \quad (8c)$$

$$\text{with } K = -\frac{8aB_p^2}{\mu_0(n+1)} (\beta_2 - \beta_1)^{n+1} - 4B_p \beta_1 [M_{II}(\beta_1) - 2M_{hol}] \quad (8d)$$

We can define the loss factor $\Gamma(\beta)$ as follows:

$$Q_h(\beta) = \frac{(\Delta B)^2}{2\mu_0} \Gamma(\beta) \quad (9)$$

In fig. 4 $\Gamma(\beta)$ is shown as function of $\beta = \Delta B/B_p$.

Fig. 4 Loss factor $\Gamma(\beta)$ for hysteresis losses in hollow filaments with $\rho = 0.0, 0.4, 0.5, 0.6, 0.7, 0.8$

It is seen that for $2\beta_1 \leq \beta \leq 2\beta_2$ losses in hollow filaments are higher than in solid filaments, whereas for β sufficiently greater than $2\beta_2$ they are smaller.

Penetration with B-dependant critical current density

In reality the critical current density in superconductors such as niobium-tin is a decreasing function of the magnetic field. The general shape of this function in the low field region is taken as given by the so-called Kim-relation

$$J_c(B) = J_{c0} \frac{B_0}{B_0 + |B|} \quad (10)$$

Fig. 5 shows the calculated magnetization of a solid filament (B_p defined with $J_c = J_{c0}$) as a function of β for the cycle of the external field $0 \rightarrow 4B_p \rightarrow (-4B_p) \rightarrow 2B_p \rightarrow 0 \rightarrow 2B_p$. The well known shift of the maximum from the $\beta=0$ axis is observed which is a direct consequence of the dependance of the critical current density on the local field induced by the screening currents in the filament.

Fig. 5 Magnetization of solid filament as function of the magnetic field calculated with critical current density depending on the local field

In the shown magnetization loop of the field variation $0 \rightarrow 2B_p \rightarrow 0$ no symmetry can be seen. The area of this loop corresponds to the hysteresis losses of the cycle.

In situations of a variable field superimposed on a constant bias field where the hysteresis losses are calculated with formulas assuming a constant critical current density it is custom to take the mean field during the cycle $B = B_{stat} + \Delta B/2$ as the effective field which determines the critical current density. The worst case for positive fields for such a procedure is a cycle at low background field such as $0 \rightarrow 2B_p \rightarrow 0$. When the losses in this case calculated numerically with $J_c(B)$ as given by expression (10) are compared with the calculated losses obtained from the model with a constant critical current density it is seen that the difference is less than 5%. In case of a constant non-zero background field the approximation will be even better.

Experiments

Hysteresis losses have been measured in wires with 36 and 192 niobium-tin filaments⁶. These wires are produced by a powder technique in which the superconducting niobium-tin is formed from $NbSn_2$ powder on the inner side of a niobium tube. From the observed magnetization loops the penetration fields B_{p2} can be determined accurately for $B_a \geq 3.5T$. By means of expression (3) the penetration field B_p can be deduced and calculated losses can be compared with experimental results. For the wire with 36 filaments a good fit was found for $R_1 = 27\mu m$ and $R_2 = 39\mu m$, for the wire with 192 filaments for $R_1 = 10\mu m$ and $R_2 = 14\mu m$.

By means of the above given expressions (8) for the hysteresis losses and these values for R_1 and R_2 the critical current density can be determined on basis of the measured losses for all cases of bias field and pulse field variation. Fig. 6 shows the dependance of the critical current density on the magnetic field.

Fig. 6 Dependence of the critical current density on the magnetic field, $\bullet$: 36 filament wire, $+$: 192 filament wire (calculated from losses); solid: calculated on basis of observed penetration field of 192-filament wire

The solid curve is a fit of the critical current density calculated on basis of the observed B_{p2} in case

of the wire with 192 filaments. The dots correspond to results of calculations on basis of measured hysteresis losses in the 36-filament wire, the '+' correspond to the 192-filament wire. Good agreement is found between the values of the critical current densities for the two wires calculated on basis of measured losses. These values are in the low field region somewhat higher than those calculated from the observed penetration field, but in this region both methods are based on less accurate experimental results.

Fig. 7 and 8 show the measured hysteresis losses in the two types of wires.

Fig. 7 Hysteresis losses in 36-filament wire as a function of bias field and pulse field variation

Fig. 8 Hysteresis losses in 192-filament wire as a function of bias field and pulse field variation

The solid curves are calculated with a critical current density as given by the Kim relation (10) with $J_{c0} = 13.4 \times 10^{10} \text{ A/m}^2$ and $B_0 = 0.974$ fitted to the data of fig. 6 for $B \leq 3.5 \text{ T}$. For $B \geq 3.5 \text{ T}$ the critical current density is given by a polynomial fit to the data of fig. 6. The abrupt increase in the hysteresis losses in the 36-filament wire at $B_{\text{stat}} = 3 \text{ T}$ and $\Delta B = 0.5 \text{ T}$, which corresponds to $\beta = 2\beta_1$, is clearly visible in fig. 7.

The fact that a good fit is found for these two wires over a wide range of β by means of one relation between the critical current density and the magnetic field indicates that the theoretical model leads to reliable results.

Conclusion

Hysteresis losses in hollow superconducting filaments can be determined well for all ranges of transverse time-varying fields. These losses can be calculated with the local critical current density within the filament as determined by the local field. For cases where the field variation is not large in comparison with the background field a model with a constant critical current density determined by the mean field during the cycle is a good approximation. Good agreement is found between theoretical calculations and experimental results.

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