High sensitivity magnetic flux sensors with direct voltage readout: Double Relaxation Oscillation SQUIDS

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Abstract—The experimental sensitivity of Double Relaxation Oscillation SQUIDS (DROSs) has been compared with theory and with the results obtained by numerical simulations. The experimental sensitivity ranges from 60 to 13h, where h is Planck's constant, for relaxation frequencies from 0.4 up to 10 GHz. For low frequencies the DROS characteristics can be explained by thermal noise on the critical currents. For high frequencies, the voltage-flux characteristics and the sensitivity are limited by the plasma frequency. The cross-over frequency is at 2 GHz, which is about 2 % of the plasma frequency of the DROSs.

I. INTRODUCTION

A Double Relaxation Oscillation SQUID converts a magnetic flux to a voltage with a transfer coefficient from flux to voltage that is typically one or two orders of magnitude larger than in comparable standard type dc SQUIDS [1]. The large transfer coefficient facilitates direct voltage readout by a room-temperature dc amplifier. Recently, it has been demonstrated experimentally that the sensitivity of a DROS with direct readout can be of the same order of magnitude as the sensitivity of a comparable standard type dc SQUID with complicated readout electronics based on flux modulation and impedance matching [2]. In this paper, the maximum sensitivity that can be obtained with a DROS is investigated.

II. THEORY

A DROS consists of two hysteretic dc SQUIDS in series shunted by an inductor L_{sh} and a resistor R_{sh} in series and biased by a dc bias current I_b (see Fig. 1). At bias currents larger than the critical current of the SQUIDS, relaxation oscillations with frequency f_r can occur. These oscillations will be stable if the product $I_b R_{sh}$ is not too large [3]. The maximum value of $I_b R_{sh}$ can be obtained from:

$$\left(\frac{I_b}{2I_0}\right)^2 \beta_c^* \leq 1, \quad (1)$$

where $2I_0$ is the maximum critical current of the (symmetric) dc SQUID and β_c^* is the effective McCumber parameter:

$$\beta_c^* = \frac{2\pi(2I_0)R_{sh}^2 C_{sq}}{\phi_0}, \quad (2)$$

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where C_{sq} is the SQUID capacitance. The relaxation frequency f_r depends mainly on the L-R shunt circuit and the bias current. Typically, f_r is of the order of $(L_{sh}/R_{sh})^{-1}$. The resistor R_d shown in Fig. 1 provides damping of LC resonances between the capacitance of the SQUIDS and the shunt inductance. These resonances may deteriorate the sensitivity of a DROS. For proper damping the damping parameter D, given by

$$D = \frac{L_{sh}}{4C_{sq}R_d^2}, \quad (3)$$

has to be equal to or larger than 1. The lower limit of R_d is set by the condition that the damped SQUIDS have to be hysteretic.

A DROS can be regarded as a critical current comparator. Of the two SQUIDS, only the SQUID with the smallest critical current will oscillate while the other SQUID remains in the zero-voltage state. The average voltage across the oscillating SQUID will be non-zero. So, $V(I_{c1} < I_{c2}) = 0$ and $V(I_{c1} > I_{c2}) = V_c \neq 0$, where the modulation width V_c is typically equal to $0.5 \dots 0.75(I_b R_{sh})$. The optimum bias point of a DROS is at $I_{c1}(\phi) = I_{c2}(\phi_{ref})$ which corresponds to $V(\phi) = V_c/2$. The transfer coefficient $\delta V/\delta \phi$ at this point is determined by the total spread $\Delta I_{c1} + \Delta I_{c2}$ in the critical currents of the SQUIDS:

$$\frac{\delta V}{\delta \phi} = \frac{V_c}{(\Delta I_{c1} + \Delta I_{c2})(\delta I_{c1}/\delta \phi)^{-1}}. \quad (4)$$

Fig. 1 Schematic overview of a DROS

The voltage noise spectral density at $V(\phi) = V_c/2$ is equal to $V_n = 0.5V_c/\sqrt{f_r}$, so the flux noise spectral density ϕ_n equals

$$\phi_n = \frac{V_c}{2\sqrt{f_r}} \left(\frac{\delta V}{\delta \phi} \right)^{-1} = \frac{\Delta I_{c1} + \Delta I_{c2}}{2\sqrt{f_r}} \left(\frac{\delta I_{c1}}{\delta \phi} \right)^{-1}. \quad (5)$$

At relaxation frequencies well below the plasma frequency, the spread in the critical current is determined by thermal noise [4] and is given by

$$\Delta I_c \approx 0.25(2I_0)^{1/3}, \quad (6)$$

with ΔI_c and I_0 in μA for commonly used SQUID parameters [1]. At these frequencies the maximum sensitivity ϵ_{th} and the flux-to-voltage transfer coefficient for a DROS with $\beta_L = 2L_{sq}I_0/\phi_0 = 1$ can be obtained from eqs. (4) - (6):

$$\frac{\epsilon_{th}}{h} = \frac{64}{f_r I_0^{1/3}} \quad (7)$$

and

$$\frac{\delta V}{\delta \phi} = 35 I_0^{2/3} \frac{V_c}{\phi_0}, \quad (8)$$

where $h = 6.63 \cdot 10^{-34}$ Js is Planck's constant, f_r is in GHz and I_0 in μA . For higher frequencies the spread in the critical currents increases due to the interference between the plasma oscillations and the relaxation oscillations in the DROS and eqs. (6)-(8) are no longer valid. This high frequency regime is the subject of investigation of this paper.

III. EXPERIMENTAL RESULTS

The DROSs are based on $2 \times 2 \mu m^2$ Nb/ AlO_x tunnel junctions with a critical current I_0 of 35 μA and a capacitance of 0.25 pF each. The two dc SQUIDs are identical with a SQUID inductance around 30 pH and a half-turn input coil. In table I, the parameters of the DROSs are listed. DROSs A1-A3 are designed with a timeconstant $\tau_{sh} = L_{sh}/R_{sh}$ of about 1 ns, DROSs B1-B3 have $\tau_{sh} \approx 0.1$ ns. The design considerations of the fabricated DROSs are discussed in [2].

TABLE I
parameters of the fabricated DROSs

#	L_{sh} [nH]	R_{sh} [Ω]	R_d [Ω]	D
A1	3.0	2.3	12	10
A2	1.7	1.8	9	10
A3	0.9	0.9	7	10
B1	0.3	1.4	12	1
B2	0.17	1.2	9	1
B3	0.09	0.7	7	1

Fig. 2 Micrograph of a DROS of type B3

A micrograph of a DROS of type B3 is depicted in fig. 2. This DROS is shunted by two symmetric L-R circuits to minimize the self-coupling to the SQUID holes. The damping resistors are located below the top Nb wiring layer connecting the two SQUIDs.

The DROS output voltage was pre-amplified by a low-noise dc amplifier with an input noise of 1.5 nV/ $\sqrt{Hz}$ at 1 kHz. The intrinsic flux noise was obtained by subtracting the voltage noise of the amplifier from the measured voltage noise spectral density. The DROSs were shielded by a Nb tube.

Experimental characteristics of the DROSs are listed in table II. The product $(I_b R_{sh})_{opt}$ and the transfer coefficient $\delta V/\delta \phi$ are determined at the bias current with optimum sensitivity. For the DROSs A1-A3 the maximum intrinsic sensitivity ϵ_{intr} ranges from 18h to 60h, depending on the L-R shunt circuit. The maximum sensitivity of DROSs B1-B3, however, is independent on the values of L_{sh} and R_{sh} and equals 13-14h, which corresponds to a flux noise spectral density of 0.36 $\mu \phi_0/\sqrt{Hz}$ (@ 1 kHz). In these DROSs, the transfer coefficient is significantly smaller than in DROSs A1-A3. This indicates that in these SQUIDs the spread in the critical currents and therefore the transfer coefficient and sensitivity are no longer determined by thermal noise, but by the dynamics of the relaxation oscillations.

TABLE II
experimental characteristics

DROS #	$(I_b R_{sh})_{opt}$ [μV]	$\delta V/\delta \phi$ [mV/ ϕ_0]	ϕ_n [$\mu \phi_0/\sqrt{Hz}$]	ϵ_{intr} [in h]
A1	149	4.4	0.76	60
A2	137	7.0	0.52	29
A3	118	3.8	0.40	18
B1	117	2.7	0.36	13
B2	102	2.6	0.36	14
B3	101	2.0	0.36	13

IV. NUMERICAL SIMULATIONS

The relaxation oscillations have been simulated numerically with the help of the junction simulation program JSIM [5]. The tunnel junctions were modelled with a critical current I_0 of 35 μA , a capacitance of 0.25 pF, and a constant subgap resistance. The DROSs were simulated with a SQUID inductance of 30 pH and a bias current according to the optimum value of $I_b R_{sh}$ listed in table II. The reference flux is equal to $0.25\phi_0$. Thermal noise was not included in the simulations. Both the relaxation frequencies and the voltage-flux characteristics have been determined by the simulations.

A simulation of the current I_1 through the SQUIDs (see fig. 1) is shown in fig. 3a for a DROS of type A2. The current is oscillating between 33 μA and the critical current of about 58 μA . The relaxation frequency is 1 GHz. At this frequency, the extra spread in the critical current induced by the dynamic behaviour of the SQUIDs is very small, which leads to a high value of the transfer coefficient $\delta V/\delta\phi$ in the calculated $V-\phi$ curve shown in fig. 3b. The transfer of 12 mV/ ϕ_0 obtained from the simulations is much larger than the value of 3 mV/ ϕ_0 which can be obtained from eq. (8), using $V_c = 0.6(I_b R_{sh})_{\text{opt}}$. So, at 1 GHz the transfer and sensitivity of these DROSs is determined by thermal noise on the critical current. In fig. 4,

the simulation results for a DROS of type B2 with a relaxation frequency of 6 GHz are shown. At this frequency, there is a significant spread ΔI_c in the critical current. This is due to the interference of the plasma oscillations with the relaxation oscillations. Due to this spread, the coefficient $\delta V/\delta\phi$ is much lower than in DROS A2, as can be seen in fig. 4b. The interference between the plasma oscillations and the relaxation oscillations can best be explained by the 3-dimensional washboard model of the potential of a dc SQUID. The tilt of the washboard is proportional to the current through the SQUID, which, in a DROS, is oscillating as shown in fig. 3a and 4a. With decreasing current, the slope of the washboard is reduced until the running particle is trapped in one of the local minima of the washboard: the SQUID has switched from the voltage state to the zero-voltage state. After trapping, the particle oscillates in the potential well with the plasma frequency. These oscillations are damped by the damping resistor R_d and the subgap resistance of the junctions. At this time, the current through the SQUID and consequently the tilt of the washboard will increase again, until the local minimum has disappeared and the particle will start to run down the washboard: the SQUID has switched to the voltage state. At high relaxation frequencies, the current through the SQUIDs increases very rapidly and the plasma oscillations after trapping have not been damped completely at the time the current has reached

Fig. 3a Simulated relaxation oscillations in DROS A2

Fig. 4a Simulated relaxation oscillations in DROS B2

Fig. 3b Simulated $V-\phi$ curve of DROS A2

Fig. 4b Simulated $V-\phi$ curve of DROS B2

the critical current of the SQUID. Due to these plasma oscillations the SQUID may switch to the voltage state at a current different from the expected critical current. This effect causes the spread in the critical current observed in fig. 4a. The plasma frequency of the simulated SQUIDs is 104 GHz, which is only one order of magnitude larger than the relaxation frequency in DROS B2.

Fig. 4a also shows that the return current in DROS B2 ranges from 20 to 33 μ A. This is due to switching noise, discussed in [6]. Switching noise introduces a spread in the relaxation frequency, but does not affect the flux noise of a DROS significantly [1].

The sensitivity ϵ_{sim} of the DROSs has been calculated by eq. (5) using the simulated V - ϕ curves and relaxation frequencies. These values are listed in table III, together with the simulated relaxation frequencies and transfer coefficients $(\delta V/\delta \phi)_{\text{sim}}$. The sensitivities ϵ_{th} in the thermal regime according to eq. (7) are also listed in table III. These values were calculated using the relaxation frequencies obtained from the simulations.

TABLE III
theoretical and simulated characteristics

DROS #	f_r [GHz]	$(\delta V/\delta \phi)_{\text{sim}}$ [mV/ ϕ_0]	ϵ_{sim} [in h]	ϵ_{th} (eq. 7) [in h]
A1	0.4	-	-	49
A2	1.0	12	0.6	20
A3	3.1	2.6	7	6
B1	4.0	1.1	17	5
B2	6.0	0.82	17	3
B3	10	0.85	13	2

Table III shows that the sensitivity at 3 GHz obtained by the simulations equals the sensitivity according to eq. (7). Apparently, the cross-over from the thermal regime to the plasma frequency limited regime is at about 3 GHz. At higher frequencies, the sensitivity obtained from the simulations is about constant at 13-17h, and is in very good agreement with the experimental data in table II. The transfer $\delta V/\delta \phi$ decreases from 3 mV/ ϕ_0 to 0.8 mV/ ϕ_0 if the frequency increases from 3 to 10 GHz. This decrease can also be observed in the experimental results in table II.

The simulated sensitivity and the sensitivities in the thermal regime are depicted in fig. 5 as a function of the relaxation frequency, together with the experimental sensitivities listed in table II. At low frequencies the experimental sensitivity can well be explained by thermal noise on the critical current (eq. (7)). For high frequencies ϵ_{intr} is in good agreement with the simulations, showing that the sensitivity of DROSs at these relaxation frequencies is limited by the plasma frequency. Fig. 5 shows that the cross-over

Fig. 5 Simulated sensitivity (solid line), sensitivity in the thermal regime (dashed line), and experimental sensitivity (crosses) of six different DROSs as a function of the relaxation frequency obtained from the simulations.

relaxation frequency is at 2 GHz, which is 2 % of the plasma frequency of these SQUIDs.

V. CONCLUSIONS

The experimental characteristics of DROSs with different relaxation frequencies have been compared with theory and with characteristics obtained by numerical simulations. At relaxation frequencies below 2 GHz, the sensitivity and flux to voltage transfer coefficient are mainly determined by thermal noise and can well be described by existing theory. For larger frequencies, the sensitivity and the flux-to-voltage transfer coefficient are limited by the plasma oscillations in the SQUIDs.

REFERENCES

- [1] D.J. Adelerhof, H. Nijstad, J. Flokstra, and H. Rogalla, *J. Appl. Phys.* **76**, 3875 (1994).
- [2] D.J. Adelerhof, J. Kawai, G. Uehara, and H. Kado, "High sensitivity double relaxation oscillation SQUIDs", *Appl. Phys. Lett.* in press.
- [3] D.J. Adelerhof, M.J. van Duuren, J. Flokstra, and H. Rogalla, Proceedings "Superconductive Devices and Circuits", SPIE Vol. **2160**, p. 142 (Los Angeles, 1994).
- [4] T.A. Fulton and L.N. Dunkleberger, *Phys. Rev. B* **9**, 7460 (1974).
- [5] JSIM 2.1, developed by E.S. Fang, Univ. of California, Berkeley CA 94720, USA.
- [6] D.J. Adelerhof, J. Flokstra, and H. Rogalla, *Phys. B* **194-196**, 141 (1994).