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# CONSISTENCY AND COMPOSITE NUMERAIRES IN JOINT PRODUCTION INPUT–OUTPUT ANALYSIS; AN APPLICATION OF IDEAS OF T. L. SAATY

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**Abstract**—In this paper we show that Saaty consistency analysis can be applied to problems in joint production input–output analysis. We discuss problems of merging use and make matrices into one matrix of input coefficients, either of commodity × commodity or industry × industry type. The Saaty approach we propose is shown to point to the U.N.'s commodity-technology method as the most appropriate among a number of competing constructs. Some evidence regarding the empirical relevance of the U.N.'s method is discussed.

## 1. INTRODUCTION

T. L. Saaty's work is about coordination and consistency. Given a certain configuration of aims, physical environments, technological or institutional parameters and the like, the Analytic Hierarchy Process (AHP) will coordinate a (possibly great) number of potentially conflicting goals and subgoals. The end result is a listing of priorities which provides a "best" solution in a well-defined sense. Many economic application areas have already been discussed in the literature: locational analysis, investment decision-making, staffing problems, commodity-price projections etc. [1].

However, at the macro-economic level applications have been relatively rare. (For the few existing examples, see Refs [2, 3].) This is rather amazing as the economic literature abounds with references to an "invisible hand", going back to Adam Smith, supposedly coordinating consumer and producer expectations and aspirations, their budget restrictions, production possibilities etc. A substantial part of present-day mathematical economics is devoted to deriving conditions underlying this postulated market clearing mechanism. At present, dynamized versions can be shown to reach stable results which subsequently can be analysed for their optimum properties, such as Pareto efficiency.

It is especially in this area that connections to the Saaty theory are lacking. For certain economic or econometric models based on simultaneous equation regression techniques this may be understandable. Probably substantial rearrangement would be required here to get a satisfactory connecting theory. However, in Leontief input–output analysis the situation is quite different. First of all, the mathematical framework of the Leontief model is rather similar to Saaty's. The basic framework, for example, consists of well-defined relations between square matrices of input and output coefficients, all relevant magnitudes are positive etc. The basic equilibrium equations are

$$\begin{aligned} \mathbf{x} &= T\mathbf{x} + \mathbf{f}, \\ L &= \mathbf{l}\mathbf{x}, \end{aligned} \tag{1}$$

with the accompanying price equation

$$\mathbf{p} = \mathbf{p}T + w\mathbf{l}, \tag{2}$$

where $T$ is the matrix of input coefficients (with Frobenius eigenvalue $< 1$), $\mathbf{f}$ and $\mathbf{x}$ are the (column) vectors of exogenous final demands and required total outputs, $\mathbf{l}$ and $\mathbf{p}$ are the (row) vectors of direct labour input coefficients and equilibrium prices, $L$ is total employment and $w$ is the exogenously determined wage rate. In the model, equilibrium prices are proportional to the quantities of embodied labour, its only primary (i.e. non-produced) factor. $\mathbf{l}^*$, the (row) vector of these quantities of embodied labour, is calculated via equation (3) below, which has an obvious interpretation in terms of the labour required to produce the goods required to produce the goods required to... etc.:

$$\mathbf{l}^* = \mathbf{l}(I - T)^{-1}. \tag{3}$$

Also, total production $\mathbf{x}$ can be viewed in terms of the powers of $T$:

$$\mathbf{x} = (I - T)^{-1}\mathbf{f}. \tag{4}$$

In an earlier paper [4], we discussed properties of the above model in the light of Saaty's approach. Among other things, we showed that the economy's state of equilibrium can be expressed in terms of a matrix of rank 1 of a particular type of input coefficients. Rescaling (to obtain diagonal elements equal to unity) then gives a Saaty reciprocal matrix [4, p. 175].

However, input–output models come in two types. Next to the well-known single-product type, models allowing for multiple products have to be distinguished. The reason must be found in technological advancements and increasing statistical difficulties in allotting outputs to industries supposedly producing a single homogeneous output. To keep track of these developments, the U.N. introduced the so-called use–make framework [5], in which the economy's technological structure is represented by *two* matrices. Each commodity is listed in terms of the industries it is an input to (the use matrix), and each industry is listed as to the commodities it produces (the make matrix). Differences with the original single-product scheme are substantial; for example, the number of commodities need not be equal to the number of industries.

For analytical purposes, it is useful to have a condensation of the two matrices to only one "pure" input–output matrix. Here many problems arise, reflected in the fact that two basic schemes exist (plus some mixed forms). In their highly influential work, the U.N. suggested both the so-called "industry-technology method", and the "commodity-technology method". Both methods have their drawbacks; at present the literature still seems undecided as to which method should be preferred. (See U.N. [5], Gigantes [6], Armstrong [7], Flaschel [8], Lal [9], Ten Raa *et al.* [10] and Rainer [11]; for a recent survey, see Stone [12].) Also from the strict price-theoretic point of view (for the main part consisting of students of the labour theory of value and related issues), it was difficult to obtain a satisfactory theory. Possible non-negativity of fundamental magnitudes became a big issue here. (See especially Abraham-Frois and Berrebi [13, Chaps 3 and 4] for fundamentals in this area.)

In this paper we hope to show that also in the complicated and increasingly important joint production systems the Saaty approach may suggest solutions to longstanding problems. Below we shall first present the established joint production model, and state its problems. Then Saaty's approach will be introduced in Section 3. At the end of the paper, some empirical evidence will be discussed.

## 2. JOINT PRODUCTION SYSTEMS

As is well-known, joint production open systems are usually formulated in the form

$$\begin{aligned} S\mathbf{x} &= T\mathbf{x} + \mathbf{f}, \\ L &= \mathbf{l}\mathbf{x} \end{aligned} \tag{5}$$

and

$$\mathbf{p}S = \mathbf{p}T + w\mathbf{l}, \tag{6}$$

with $S$ the new matrix of output coefficients, and both $S$ and T of the dimension commodity × industry.† The model (5, 6) has generated many interpretational difficulties, as may be seen by solving for **x**, resp. **p**:

$$\mathbf{x} = (S - T)^{-1} \tag{7}$$

and

$$\mathbf{p} = w\mathbf{l}(S - T)^{-1}. \tag{8}$$

We easily see that for certain specific matrices $S$ and $T$, elements of **x** or **p** may become negative. We also observe that in these models, a breakdown of vectors **x** in equation (3) or **p** in equation (4) in terms of successive production layers is not straightforward. Therefore, this may question the labour theory of value approach underlying most of input–output economics [14, 15].

Empirically, a quite different approach has been followed by directly imposing specific theoretical structures. Two basic types of specifications have been proposed in this respect, plus a number of hybrid forms. We may mention as "basic" methods the industry-technology model and the commodity-technology model and, as well-known hybrids, the mixed-technology model and the by-product-technology model. To enhance understanding of the subsequent parts, we shall discuss the two main forms briefly below. (For further insight, we refer to the aforementioned literature.)

The industry-technology model rests on the twin assumptions (1) that each industry $j$ has the same input requirements for each unit of output (measured in value terms) and (2) the presence of fixed commodity market shares of industries. In formula form, with $\mathbf{S'e}$ the vector of commodity outputs and $S'$ the corresponding make matrix, we have

$$K = T(\widehat{\mathbf{S'e}})^{-1}S'(\widehat{\mathbf{Se}})^{-1}, \tag{9}$$

where $(\hat{\cdot})$ denotes diagonalization and $K$ the commodity × commodity input coefficient matrix implied by this model. The method's great advantage is that matrices $K$ are always non-negative. On the other hand, a severe drawback is that it is based on the assumption of fixed market shares, which seems unlikely in real-world situations. (See Ten Raa *et al.* [10].)

To cope with the problems of subsidiary production, the U.N. also has recommended a procedure based on the assumption of a unique input structure for each particular commodity. Denoting the—to be derived—input column for good $j$ by $K_{.j}$, the U.N. assumes that the economy is such that

$$T_{.j} = s_{1j}K_{.1} + s_{2j}K_{.2} + \cdots + s_{nj}K_{.n}, \tag{10}$$

where $T_{.j}$ is the $j$th column of the (observed) input matrix $T$ and $s_{ij}$ is the $i$th element of the (observed) $j$th column of $S$. For all sectors together, this gives

$$T = KS. \tag{11}$$

Assuming that $S$ is non-singular, we derive from the above

$$K = TS^{-1}, \tag{12}$$

where $K$ is the implied commodity × commodity matrix of input coefficients. The U.N. seems quite confident of the method [5, p. 39]. However, critics have pointed out that matrix $K$ in equation (12) may contain negative elements, thus compromising its economic interpretation. For an extended

† That is, $s_{ij}$ is the output of good $i$ by industry $j$ operating at a certain well-defined level of operation; and $t_{ij}$ is the input of good $i$ required by industry $j$ at the same level of operation. For connections of model (5, 6) with the U.N.'s framework, see Stone [12].

discussion of both methods (and some other constructs), we refer here to the U.N. [5] and other references.

## 3. A SAATY APPROACH TO PROBLEMS IN JOINT PRODUCTION LEONTIEF ECONOMICS

First of all, let us rewrite the joint production system in a form analogous to the closed model form we introduced in Ref. [4]. We obtain

$$\left[\begin{array}{c|c} S & 0 \\ \hline 0 & 1 \end{array}\right]\left[\begin{array}{c} \mathbf{x} \\ \hline L \end{array}\right] = \left[\begin{array}{c|c} T & \frac{\mathbf{f}}{L} \\ \hline \mathbf{1} & 0 \end{array}\right]\left[\begin{array}{c} \mathbf{x} \\ \hline L \end{array}\right] \tag{13}$$

as a compact expression for the real output system, and a corresponding expression for the price system:

$$[\mathbf{p} \mid w]\left[\begin{array}{c|c} S & 0 \\ \hline 0 & 1 \end{array}\right] = [\mathbf{p} \mid w]\left[\begin{array}{c|c} T & \frac{\mathbf{f}}{L} \\ \hline L & 0 \end{array}\right]. \tag{14}$$

Let us also, in obvious notation, simplify the above relations to

$$B\mathbf{z} = A\mathbf{z} \tag{15}$$

and

$$\mathbf{y}B = \mathbf{y}A. \tag{16}$$

We immediately see that the system is more complicated than the earlier treated single-product system. In fact, all we know is that $\mathbf{z}$ is a right-hand eigenvector of matrices $A^{-1}B$ or $B^{-1}A$, and that $\mathbf{y}$ is a left-hand eigenvector of matrices $BA^{-1}$ or $AB^{-1}$, both corresponding to a unit eigenvalue. There is no general qualitative statement available regarding the elements of matrices like $B^{-1}A$, $(B^{-1}A)^2$, $(B^{-1}A)^3$ etc.† Thus, an analysis of relations (15) and (16) in terms of the amounts of the quantities of commodities and labour embodied in each good, as in the single-product case, is not straightforwardly possible.

Nevertheless, additional assumptions regarding the economy might help us out. Here, however, as described in Sections 1 and 2, economic theorizing offers a variety of options. Nevertheless, as we hope to show, also here Saaty may force a solution. In Ref. [4], we have discussed relations between the AHP and Leontief's system of indirect production layers. We developed the concept of the total production vector as a composite numeraire, and discussed the relations between this numeraire and Saaty's concept of a priority vector. Now, let us assume that also in joint production systems prices and commodities are coordinated by Smith's "invisible hand", taking into account the amounts of all commodities (labour included) embodied in each good. We then, in fact, postulate the existence of a reciprocal matrix $Q$ of rank 1:

$$Q = \begin{bmatrix} 1 & q_{12} & \cdots & q_{1n} \\ 1/q_{12} & 1 & \cdots & q_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ 1/q_{1n} & 1/q_{2n} & \cdots & 1 \end{bmatrix},$$

where $q_{ij}$ is to be interpreted as the "weight" of commodity $i$ *vis-à-vis* commodity $j$, in terms of the

† See especially Abraham-Frois and Berrebi [13, Chaps 3 and 4] on problems associated with efforts to develop a value-theoretic theory regarding such systems.

number of composite numeraires embodied in it (but rescaled to unit elements $q_{ii}$; cf. matrix $K$ in Ref. [4, p. 175]).

Now, if indeed real-world coordination would be governed by this, rescaling to *unit levels of production* would result in a rank 1 matrix $Q'$ of the form

$$Q' = (\mathbf{Bz} \cdot \mathbf{y}), \tag{17}$$

with $\mathbf{y} \cdot B\mathbf{z} = 1$, where prices $\mathbf{y}$ are seen to reflect the number of composite numeraires (scalar multiples of total output vector $B\mathbf{z}$) embodied in each product.† Following the procedure of Ref. [4], we interpret $Q'$ as the infinite power of an—as yet unknown—matrix $M$ (i.e. $Q' = M^{\infty}$). But this means that this matrix must have right- and left-hand Frobenius eigenvectors $B\mathbf{z}$ and $\mathbf{y}$, resp. So

$$B\mathbf{z} = (M)B\mathbf{z} \tag{18}$$

and

$$\mathbf{y} = \mathbf{y}M. \tag{19}$$

This, however, is nothing but (an extended version of) the U.N.'s commodity-technology assumption we discussed in Section 2.‡ Thus, via $MB = A$, we can calculate the unknown (primitive) matrix $M$.§

An analysis of prices and quantities in terms of production layers is now easily obtained. We have

$$\begin{aligned} B\mathbf{z} &= MB\mathbf{z} \\ &= M^2 B\mathbf{z} \end{aligned} \tag{20}$$

etc. It is easily seen that we may continue this procedure to obtain expressions containing the higher powers of $M$. Taking limits, we obtain the expression

$$B\mathbf{z} = M^{\infty} B\mathbf{z}. \tag{21}$$

For the price relation the situation is similar, we obtain

$$\mathbf{y} = \mathbf{y}\mathbf{M}^{\infty}. \tag{22}$$

Analogous to vector $(1/\alpha)\mathbf{z}$ in Ref. [4], vector $B\mathbf{z}$ can be interpreted as the system's (standardized) composite numeraire. Like in Section 2, positive labour values are associated with the scheme, and recorded in the last row of $M^{\infty}$.¶

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† Note that proportions of vectors $\mathbf{y}$ and $B\mathbf{z}$ are known from equations (15) and (16).

‡ The commodity-technology method postulates that $B\mathbf{z} = A\mathbf{z}$ can be replaced by $B\mathbf{z} = MB\mathbf{z}$, and $\mathbf{y}B = \mathbf{y}A$ by $\mathbf{y}B = \mathbf{y}MB$; we see that equation (19) can be written as $\mathbf{y}B = \mathbf{y}MB$.

§ For the vector of labour coefficients $\mathbf{l}$ in equation (13) this implies $\mathbf{l} = \mathbf{l}_m B$, with $\mathbf{l}_m$ the vector of labour coefficients associated with $M$.

¶ Note, for example, also that Toker [14] and Hollander [15] link avoiding indeterminacy of labour contents in the multi-product case to bringing in extra conditions into the system. However, the problem of deciding on which one(s) is not addressed in their work.

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# APPENDIX

Above we have concluded that Saaty's analysis points to the U.N.'s commodity-technology assumption as an appropriate additional condition. This implies that in empirical investigations, if matrix $M$ should have negative elements, these should be small in magnitude and in number. Numerical exercises are rather exceptional, however. Of the more elaborate ones, we may mention Armstrong [7], Lal [9], Ten Raa *et al.* [10] and Rainer [11]. Consensus seems to be that indeed only a small proportion of the elements of the appropriate matrices is negative, but that a theoretical justification of the method itself is still lacking. In view of the new theoretical background we have proposed in this paper, additional empirical research regarding the occurrence of negative entries certainly seems worthwhile. As a contribution to such research, we have calculated matrix $TS^{-1}$ for Canada (Table A1), using the data presented by Ten Raa *et al.* [10]. Of $43^2 = 1849$ entries, only 64, i.e. 3.46%, were found to be negative. More than half of these (34) were very small, i.e. $<0.0025$ in absolute magnitude. Of the remaining negative entries, 5 were relatively large (0.067, 0.050, 0.047, 0.031 and 0.027), 5 were in the interval [0.015, 0.0185] and the remaining 20 in the interval [0.0025–0.0105].

Very recently, Rainer [11] presented the outcomes of an exercise for Austrian data for 1976, in the form of a 175 × 175 input–output table. Rainer's results are very much in line with ours. For example, it appears that the inputs of 32 commodities do not show any negative value at all. For about half of the commodities, the input structure shows no negative values, or else values which amount to $<1\%$ of the respective total inputs. Overall it is found that in value terms 1.4% of total intermediate use is negatives. Of these, the negative values in the inputs of just 24 commodities comprise 60.1% of total negatives; for the 46 commodities with the highest negatives this share reaches 80.1%.

In interpreting these numbers, we should keep in mind that these exercises can only be qualified as a very first effort; for a full verdict the obtained matrices should receive very careful additional re-examination. Especially in view of the limited amount of information that normally is available to estimate each individual coefficient (the use of inter- or extrapolation techniques, errors in sampling and sampling methods, classification problems etc.) any conclusions should be drawn with great care. Nevertheless, the Saaty-based approach we have suggested in this paper seems to pass the acid test of confrontation with reality.

*See overleaf for Table A1*

Table A1

| | Agricu | Forest | Fishin | Metmns | Minfls | Nmtmns | Minser | Food | Tobacc | Rubber | Leathr | Textil | Knittn | Clothn |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Agricu | 0.055 | - | -0.001 | - | - | - | - | 0.288 | 0.222 | - | - | - | - | 0.006 |
| Forest | - | 0.110 | - | - | - | - | - | - | - | -0.001 | - | -0.001 | 0.001 | - |
| Fishin | - | - | 0.010 | - | - | - | - | 0.019 | - | - | - | - | - | 0.018 |
| Metmns | - | - | - | 0.003 | - | - | - | - | - | - | - | - | - | - |
| Minfls | - | - | - | - | -0.031 | 0.005 | - | - | - | - | - | - | - | - |
| Nmtmns | - | - | 0.002 | 0.006 | - | -0.001 | 0.006 | - | - | - | - | - | - | - |
| Minser | - | - | - | 0.054 | 0.069 | 0.027 | - | - | - | - | - | - | - | - |
| Food | 0.108 | -0.004 | -0.001 | - | - | -0.004 | - | 0.187 | - | - | 0.051 | - | - | - |
| Tobacc | - | - | - | - | - | - | - | - | 0.234 | - | - | - | - | - |
| Rubber | 0.001 | - | - | - | - | 0.003 | - | 0.005 | 0.002 | 0.037 | 0.135 | 0.013 | -0.008 | 0.002 |
| Leathr | - | - | - | - | - | - | - | - | - | - | 0.159 | - | - | 0.021 |
| Textil | 0.002 | 0.002 | 0.026 | - | - | - | - | 0.001 | 0.013 | 0.059 | 0.002 | 0.337 | 0.444 | 0.342 |
| Knittn | - | - | - | - | - | - | - | - | - | - | - | - | 0.003 | - |
| Clothn | - | - | - | - | - | - | - | - | - | - | 0.005 | 0.019 | 0.030 | 0.065 |
| Wood | - | -0.001 | 0.010 | - | - | 0.002 | - | - | - | - | 0.001 | - | - | - |
| Furnit | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| Paper | - | - | - | - | - | 0.006 | - | 0.031 | 0.095 | 0.019 | 0.014 | 0.013 | 0.008 | 0.005 |
| Printn | - | - | - | - | - | - | - | 0.003 | 0.001 | - | - | - | 0.004 | - |
| Pmetal | - | - | - | 0.013 | 0.006 | -0.003 | 0.039 | - | - | -0.014 | - | -0.006 | 0.004 | - |
| Fameta | 0.004 | 0.008 | 0.004 | 0.002 | 0.001 | -0.007 | - | 0.019 | 0.004 | 0.009 | 0.010 | 0.001 | - | - |
| Machin | 0.009 | 0.007 | 0.014 | 0.011 | 0.008 | 0.023 | 0.017 | - | - | -0.005 | - | -0.001 | - | - |
| Trequi | - | 0.001 | 0.072 | 0.002 | - | - | - | - | - | -0.047 | 0.003 | -0.050 | 0.034 | -0.004 |
| Electr | - | - | 0.038 | - | - | - | 0.012 | - | - | -0.005 | - | -0.002 | 0.001 | - |
| Minpro | - | - | 0.001 | 0.002 | - | -0.002 | 0.017 | 0.009 | - | 0.002 | - | - | - | - |
| Petpro | 0.037 | 0.018 | 0.054 | 0.015 | 0.004 | 0.011 | 0.026 | 0.001 | - | - | 0.001 | 0.003 | - | - |
| Chemic | 0.059 | - | 0.001 | 0.029 | 0.009 | -0.067 | 0.005 | 0.009 | 0.005 | 0.299 | 0.020 | 0.060 | -0.027 | 0.004 |
| Mscman | - | - | 0.014 | - | - | -0.001 | - | - | - | -0.002 | 0.013 | 0.031 | -0.018 | 0.032 |
| Constr | 0.023 | 0.016 | 0.015 | 0.015 | 0.084 | 0.011 | - | 0.002 | 0.003 | 0.001 | 0.002 | 0.003 | - | - |
| Transp | 0.002 | 0.122 | 0.006 | 0.012 | - | 0.016 | 0.035 | 0.007 | 0.001 | - | 0.001 | - | - | - |
| Commun | 0.008 | 0.002 | 0.001 | 0.002 | 0.003 | 0.001 | 0.006 | 0.003 | 0.002 | 0.005 | 0.006 | 0.004 | 0.004 | 0.005 |
| Utilit | 0.016 | - | - | 0.035 | 0.021 | 0.022 | 0.014 | 0.005 | 0.003 | 0.013 | 0.005 | 0.010 | 0.001 | 0.002 |
| Wholsl | 0.031 | 0.006 | 0.030 | 0.013 | 0.006 | 0.017 | 0.015 | 0.020 | 0.007 | 0.014 | 0.026 | 0.037 | -0.007 | 0.034 |
| Retail | 0.004 | - | 0.003 | - | - | 0.001 | 0.001 | - | - | - | - | - | - | - |
| Dwelln | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| Financ | 0.044 | 0.073 | 0.014 | 0.032 | 0.204 | 0.038 | 0.029 | 0.008 | 0.016 | 0.019 | 0.025 | 0.012 | 0.021 | 0.019 |
| Edheal | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| Amusem | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| Busser | 0.002 | 0.005 | 0.001 | 0.034 | 0.084 | 0.016 | 0.017 | 0.006 | 0.010 | 0.012 | 0.013 | 0.007 | 0.005 | 0.008 |
| Accser | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| Perser | 0.002 | 0.038 | 0.015 | 0.008 | 0.002 | 0.023 | 0.013 | 0.004 | 0.002 | 0.003 | 0.013 | 0.002 | 0.004 | 0.001 |
| Trnmgn | 0.012 | 0.002 | 0.010 | 0.007 | 0.001 | -0.001 | 0.009 | 0.022 | 0.009 | 0.011 | 0.013 | 0.011 | 0.003 | 0.005 |
| Operof | 0.026 | 0.158 | 0.017 | 0.155 | 0.057 | 0.171 | 0.153 | 0.020 | 0.013 | 0.042 | 0.024 | 0.026 | 0.013 | 0.008 |
| Proadv | - | 0.002 | - | 0.001 | 0.010 | -0.013 | 0.018 | 0.031 | 0.058 | 0.033 | 0.031 | 0.018 | 0.007 | 0.018 |

*—continued*

Table A1—*continued*

| | Wood | Furnit | Paper | Printn | Pmetal | Fameta | Machin | Trequi | Electr | Minpro | Petpro | Chemic | Mscman | Constr |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Agricu | - | - | -0.002 | - | - | - | - | - | - | - | - | -0.001 | 0.001 | - |
| Forest | 0.292 | -0.011 | 0.113 | -0.003 | - | - | - | - | - | - | - | - | -0.006 | - |
| Fishin | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| Metmns | - | - | - | - | 0.276 | -0.013 | -0.001 | - | - | 0.007 | - | 0.001 | 0.017 | - |
| Minfls | - | - | 0.003 | - | 0.029 | - | - | - | - | 0.008 | 0.632 | 0.002 | - | - |
| Nmtmns | - | - | 0.003 | - | 0.004 | - | 0.001 | - | - | 0.064 | - | 0.015 | - | 0.004 |
| Minser | - | - | - | - | - | - | - | - | - | - | - | - | - | 0.025 |
| Food | - | - | 0.002 | - | - | - | - | - | - | - | - | 0.011 | - | - |
| Tobacc | - | - | -0.002 | - | - | - | - | - | - | - | - | - | - | - |
| Rubber | 0.004 | 0.050 | 0.008 | - | - | - | 0.017 | 0.022 | 0.007 | 0.007 | - | 0.008 | 0.032 | 0.013 |
| Leathr | - | - | - | - | - | - | - | - | - | - | - | - | 0.005 | - |
| Textil | - | 0.092 | 0.006 | 0.001 | - | -0.001 | - | 0.003 | - | - | - | - | 0.015 | 0.006 |
| Knittn | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| Clothn | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| Wood | 0.122 | 0.068 | 0.064 | -0.001 | 0.001 | 0.001 | - | 0.003 | 0.001 | 0.002 | - | - | 0.004 | 0.056 |
| Furnit | 0.001 | 0.036 | - | - | - | - | - | - | - | - | - | - | - | - |
| Paper | 0.003 | 0.023 | 0.147 | 0.186 | - | 0.004 | - | 0.001 | 0.008 | 0.022 | - | 0.021 | 0.015 | 0.007 |
| Printn | - | 0.001 | - | 0.064 | - | - | - | - | - | - | - | 0.001 | 0.003 | - |
| Pmetal | 0.001 | 0.026 | 0.004 | 0.001 | 0.173 | 0.285 | 0.094 | 0.053 | 0.121 | 0.007 | - | 0.008 | 0.037 | 0.024 |
| Fameta | 0.010 | 0.074 | 0.004 | - | 0.006 | 0.157 | 0.062 | 0.030 | 0.030 | 0.010 | 0.004 | 0.015 | 0.027 | 0.094 |
| Machin | - | 0.003 | -0.001 | - | 0.007 | -0.008 | 0.223 | 0.013 | 0.008 | - | - | - | -0.001 | 0.009 |
| Trequi | - | - | - | - | - | -0.010 | 0.017 | 0.444 | - | -0.002 | - | - | -0.002 | 0.002 |
| Electr | 0.001 | 0.007 | - | - | - | -0.003 | 0.038 | 0.020 | 0.196 | 0.001 | - | - | 0.005 | 0.041 |
| Minpro | 0.008 | 0.006 | 0.003 | - | 0.010 | 0.009 | 0.002 | 0.008 | 0.010 | 0.137 | - | 0.005 | 0.004 | 0.061 |
| Petpro | 0.003 | 0.001 | 0.019 | - | 0.012 | 0.001 | - | 0.001 | 0.001 | 0.013 | 0.021 | 0.040 | -0.001 | 0.008 |
| Chemic | 0.011 | 0.011 | 0.056 | 0.017 | 0.004 | 0.010 | - | 0.005 | 0.018 | 0.021 | 0.032 | 0.245 | 0.042 | 0.009 |
| Mscman | 0.002 | 0.015 | - | 0.012 | - | - | - | 0.004 | 0.003 | 0.007 | 0.003 | 0.002 | 0.163 | 0.006 |
| Constr | 0.003 | 0.002 | 0.004 | 0.001 | 0.008 | 0.001 | 0.003 | 0.001 | 0.002 | 0.004 | 0.022 | 0.005 | 0.001 | - |
| Transp | - | 0.001 | 0.005 | - | 0.004 | 0.001 | - | 0.003 | 0.001 | 0.005 | 0.078 | 0.007 | - | 0.008 |
| Commun | 0.004 | 0.006 | 0.004 | 0.021 | 0.003 | 0.006 | 0.006 | 0.002 | 0.008 | 0.006 | 0.003 | 0.009 | 0.009 | 0.002 |
| Utilit | 0.014 | 0.006 | 0.041 | 0.003 | 0.025 | 0.006 | 0.004 | 0.003 | 0.005 | 0.030 | 0.010 | 0.032 | -0.001 | - |
| Wholsl | 0.020 | 0.045 | 0.017 | 0.013 | 0.034 | 0.012 | 0.027 | 0.011 | 0.016 | 0.014 | 0.001 | 0.015 | 0.021 | 0.045 |
| Retail | - | - | - | - | - | - | - | - | - | - | - | - | - | 0.006 |
| Dwelln | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| Financ | 0.018 | 0.031 | 0.012 | 0.020 | 0.007 | 0.015 | 0.018 | 0.005 | 0.015 | 0.014 | 0.014 | 0.021 | 0.041 | 0.017 |
| Edheal | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| Amusem | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| Busser | 0.006 | 0.008 | 0.008 | 0.022 | 0.007 | 0.009 | 0.011 | 0.033 | 0.011 | 0.008 | 0.014 | 0.021 | 0.021 | 0.036 |
| Accser | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| Perser | 0.007 | 0.002 | 0.007 | 0.004 | 0.002 | 0.005 | 0.002 | - | 0.002 | 0.008 | 0.001 | 0.011 | 0.005 | 0.016 |
| Trnmgn | 0.021 | 0.014 | 0.034 | 0.014 | 0.037 | 0.015 | 0.013 | 0.011 | 0.009 | 0.036 | 0.008 | 0.025 | 0.007 | 0.015 |
| Operof | 0.062 | 0.018 | 0.050 | 0.019 | 0.054 | 0.031 | 0.019 | 0.024 | 0.020 | 0.071 | 0.017 | 0.042 | 0.013 | 0.016 |
| Proadv | 0.007 | 0.020 | 0.008 | 0.036 | 0.003 | 0.016 | 0.022 | 0.010 | 0.020 | 0.015 | 0.018 | 0.055 | 0.057 | 0.004 |

—*continued*

Table A1—*continued*

| | Transp | Commun | Utilit | Wholsl | Retail | Dwelln | Financ | Edheal | Amusem | Busser | Accser | Perser | Trnmgn | Operof | Proadv |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Agricu | - | - | - | - | 0.020 | - | - | - | - | - | 0.014 | - | - | 0.005 | - |
| Forest | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| Fishin | - | - | - | - | - | - | - | - | - | - | 0.001 | - | - | - | - |
| Metmns | - | - | 0.002 | - | - | - | - | - | - | - | - | - | - | - | - |
| Minfls | 0.002 | - | 0.033 | - | 0.001 | - | - | - | - | - | - | - | - | - | - |
| Nmtmns | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| Minser | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| Food | - | - | - | - | -0.008 | - | - | - | -0.007 | - | 0.196 | - | - | 0.034 | 0.009 |
| Tobacc | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| Rubber | 0.006 | - | - | 0.002 | 0.001 | - | - | - | - | - | - | 0.002 | - | 0.043 | - |
| Leathr | - | - | - | - | - | - | - | - | - | - | - | - | - | 0.002 | - |
| Textil | - | - | - | - | -0.001 | - | - | 0.004 | - | -0.001 | 0.008 | 0.008 | - | 0.006 | - |
| Knittn | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| Clothn | - | - | - | - | 0.001 | - | - | - | - | - | - | - | - | 0.002 | - |
| Wood | - | - | - | 0.001 | -0.001 | - | - | - | - | -0.001 | - | 0.012 | - | - | - |
| Furnit | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| Paper | - | - | - | 0.008 | 0.016 | - | - | - | - | - | 0.012 | 0.002 | - | 0.036 | - |
| Printn | - | 0.004 | - | 0.001 | 0.001 | - | 0.002 | 0.003 | 0.004 | - | - | - | - | 0.075 | 0.213 |
| Pmetal | - | -0.005 | - | - | - | - | - | - | - | - | - | 0.001 | - | 0.003 | - |
| Fameta | - | -0.001 | - | 0.005 | - | - | - | - | 0.001 | - | - | 0.002 | - | 0.057 | - |
| Machin | - | - | - | - | - | - | - | - | - | - | - | - | - | 0.103 | - |
| Trequi | 0.025 | - | - | - | - | - | - | - | - | - | - | - | - | 0.064 | 0.010 |
| Electr | 0.001 | 0.014 | - | - | - | - | - | - | - | - | - | - | - | 0.042 | - |
| Minpro | - | - | - | - | - | - | - | - | - | - | 0.001 | - | - | 0.004 | - |
| Petpro | 0.036 | 0.001 | 0.018 | 0.011 | 0.004 | - | 0.001 | 0.004 | 0.002 | - | 0.005 | 0.011 | - | - | 0.018 |
| Chemic | - | - | - | -0.002 | -0.006 | - | -0.001 | 0.011 | - | -0.005 | - | 0.041 | - | 0.044 | - |
| Mscman | - | 0.002 | - | -0.002 | -0.003 | - | - | 0.019 | 0.029 | -0.002 | 0.001 | 0.023 | - | 0.032 | 0.014 |
| Constr | 0.030 | 0.017 | 0.032 | 0.001 | 0.004 | 0.138 | 0.035 | 0.002 | 0.007 | - | 0.002 | - | - | - | - |
| Transp | 0.100 | 0.018 | 0.006 | 0.025 | 0.011 | - | 0.001 | 0.002 | 0.003 | 0.001 | 0.003 | 0.004 | 1.000 | - | 0.177 |
| Commun | 0.022 | 0.030 | 0.003 | 0.032 | 0.023 | - | 0.028 | 0.036 | 0.011 | 0.024 | 0.011 | 0.010 | - | - | 0.105 |
| Utilit | 0.006 | 0.002 | 0.008 | 0.005 | 0.018 | - | 0.007 | 0.003 | 0.012 | - | 0.013 | 0.009 | - | - | - |
| Wholsl | 0.011 | - | - | 0.018 | - | - | - | 0.006 | 0.003 | -0.001 | 0.016 | 0.016 | - | 0.113 | 0.013 |
| Retail | 0.005 | 0.006 | - | 0.002 | - | - | - | - | 0.042 | - | 0.008 | 0.003 | - | 0.042 | 0.013 |
| Dwelln | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| Financ | 0.030 | 0.010 | 0.018 | 0.060 | 0.085 | 0.016 | 0.119 | 0.043 | 0.099 | 0.053 | 0.055 | 0.049 | - | - | - |
| Edheal | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| Amusem | - | 0.015 | - | - | - | - | - | - | 0.065 | - | 0.009 | - | - | - | 0.006 |
| Busser | 0.013 | 0.018 | 0.012 | 0.033 | 0.020 | - | 0.041 | 0.017 | 0.014 | 0.060 | 0.013 | 0.006 | - | - | 0.094 |
| Accser | 0.004 | - | - | - | - | - | - | - | - | - | - | - | - | - | 0.175 |
| Perser | 0.027 | 0.004 | 0.004 | 0.008 | 0.005 | - | 0.006 | 0.015 | 0.020 | 0.013 | 0.009 | 0.014 | - | 0.175 | 0.020 |
| Trnmgn | 0.003 | - | 0.004 | 0.001 | - | - | - | 0.001 | - | - | 0.007 | 0.004 | - | 0.019 | 0.004 |
| Operof | 0.038 | 0.016 | 0.012 | 0.024 | 0.016 | - | 0.032 | 0.050 | 0.072 | 0.043 | 0.026 | 0.078 | - | - | - |
| Proadv | 0.014 | 0.012 | 0.003 | 0.016 | 0.040 | - | 0.024 | 0.011 | 0.076 | 0.031 | 0.020 | 0.016 | - | - | - |