INFINITE DIMENSIONAL PARAMETER IDENTIFICATION FOR STOCHASTIC PARABOLIC SYSTEMS

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Abstract: The infinite dimensional parameter estimation for stochastic heat diffusion equations is considered using the method of sieves. The consistency property is also studied for the long run data.

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1. Introduction

The identification of infinite dimensional parameters for stochastic distributed parameter systems becomes important in a wide variety of physical phenomena: stochastic control (Balakrishnan, 1975) and large flexible structures (Curtain and Kotelenez, 1987). In contrast with the identification problem for a finite dimensional unknown parameter vector, it may be impossible to construct an algorithm for generating directly an estimate for an infinite dimensional parameter vector. One possible way to construct a practical estimator is to project the infinite dimensional parameter vector to a finite dimensional subspace. For consistency, one must then check whether the finite dimensional estimate converges to the true one as the number of samples and also the dimension of the projected subspace increase without bound.

In this paper, by using the method of sieves, originally proposed by Grenander (1981), an identification algorithm for identifying the unknown diffusion coefficient of a heat equation is proposed, based on a certain log likelihood type cost function. Nguyen en Pham (1982) used this technique for studying the estimation of time-dependent parameters for lumped systems based on independent observations, and recently Bagchi (1985) succeeded in applying this method to the identification problem for a hereditary system with distributed delay. Here we apply the method proposed in Bagchi (1985) to identify spatially dependent parameters for stochastic parabolic systems.

2. Stochastic parabolic systems

We set $G \subset \mathbb{R}^n$, an open bounded domain with a regular boundary Γ , $T =]0, t_f[$, and we work with the following Hilbert spaces:

$L^2(G)$ is the space of square integrable functions on G with the norm $|\cdot|$ and inner product $(\cdot, \cdot)$. (We also use $|\cdot|$ as the norm in $\mathbb{R}^n$.)

$H^m(G)$ is the m th order Sobolev Space, i.e. the space of functions ϕ such that

$$\phi \in L^2(G), \quad D^\alpha \phi \in L^2(G) \quad \forall \alpha, |\alpha| \leq m$$

($\alpha = (\alpha_1, \dots, \alpha_n)$, α_i natural numbers, $|\alpha| = \alpha_1 + \dots + \alpha_n$ and $D^\alpha \phi = (\partial^{\alpha_1} \dots \partial^{\alpha_n} / \partial x_1^{\alpha_1} \dots \partial x_n^{\alpha_n}) \phi$).

$$H_0^m(G) = \{ \phi \mid \phi \in H^m(G), D^\alpha \phi = 0 \text{ and } |\alpha| \leq m-1 \}.$$

$H^{-m}(G)$ is the dual of $H_0^m(G)$.

Define

$$V = H_0^1(G) \subset H = L^2(G) \subset V' = H^{-1}(G)$$

where we assume that the injection of $V \rightarrow H$ is compact (Adams, 1975, p. 144). Define the linear operator $A(a^0) \in L(V; V')$ such that

$$\langle A(a^0) \phi_1, \phi_2 \rangle = \sum_{i=1}^n \int_G a^0(x) \frac{\partial \phi_1}{\partial x_i} \frac{\partial \phi_2}{\partial x_i} dx \quad (2.1)$$

where

$$(C1) \quad 0 < \alpha \leq a^0(x) \leq \beta \quad \forall x \in \bar{G}$$

and

$$(C2) \quad a^0 \in C^2(G).$$

From (C2), the operator $A(a^0)$ with range restricted to H defines an unbounded operator which is still denoted by $A(a^0)$ with the domain $D = H_0^1 \cap H^2$.

Let w be a H -valued Brownian motion process with the incremental covariance $Q \in L_1(H; H)$ where $L_1(H; H)$ denotes the space of trace class operators.

Consider the following stochastic diffusion equations:

$$(u(t), \phi) + \int_0^t \langle A(a^0)u(s), \phi \rangle ds = (u_0, \phi) + (w(t), \phi) \quad \forall \phi \in V. \quad (2.2)$$

Theorem 1 (Bensoussan, 1971, p. 189; Paradoux, 1975, p. 48). *Under (C1), (C2) and*

$$(C3) \quad u_0 \in L^2(\Omega; H),$$

there exists a unique solution $u(t)$ of (2.2) such that

$$u \in L^2(\Omega; C(T; H) \cap L^2(T; V)). \quad \square \quad (2.3)$$

Theorem 2 (Mizohata, 1973, Theorem 5.5, Chapter 5; Tanabe, 1979, Theorem 1.4, Chapter 3). *Under (C1) and (C2), $-A$ generates strongly continuous, contraction semigroup T_t such that*

$$dT_t/dt = -AT_t, \quad T_0 = I, \quad |T_t \phi|^2 \leq C e^{-\alpha t} |\phi|^2 \quad \text{for some } C > 0, \alpha > 0, \phi \in H. \quad \square$$

Define the auto-correlation operator

$$\Lambda(t, s) = E\{u(t) \otimes u(s)\}, \quad (2.4)$$

where

$$\phi_1 \otimes \phi_2 = \phi_1(\phi_2, \cdot).$$

For $\phi \in H$ and noting that $Q = \sum_{i=1}^{\infty} q_i e_i \otimes e_i$ (e_i ; orthonormal basis in H), we have

$$\begin{aligned} E\{u(t)(u(s), \phi)\} &= T_t E\{u_0(u_0, T_s^* \phi)\} + \sum_{i=1}^{\infty} \int_0^{t \wedge s} T_{t-\tau} e_i (e_i, T_{s-\tau}^* \phi) q_i \, d\tau \\ &= T_t E\{u_0 \otimes u_0\} T_s^* \phi + \int_0^{t \wedge s} T_{t-\tau} Q T_{s-\tau}^* \phi \, d\tau, \end{aligned}$$

so that

$$\Lambda(t, s) = T_t E\{u_0 \otimes u_0\} T_s^* + \int_0^{t \wedge s} T_{t-\tau} Q T_{s-\tau}^* \, d\tau. \quad (2.5)$$

We are interested in the stationary system state and, therefore, we choose the initial condition u_0 to be

$$\int_{-\infty}^0 T_{-\tau} \, dw(\tau). \quad (2.6)$$

The auto-correlation $\Lambda(t, s)$ then becomes

$$\Lambda(t-s) = \Lambda(t, s) = \int_{-\infty}^{t \wedge s} T_{t-\tau} Q T_{s-\tau}^* \, d\tau. \quad (2.7)$$

Theorem 3.

$$\int_0^{\infty} \|\Lambda(\tau)\|_{L_2(V'; V)}^2 \, d\tau < \infty \quad (2.8)$$

where $L_2(\cdot; \cdot)$ denotes the space of Hilbert-Schmidt operators. $\square$

This result can be easily established using the property of T_t . For more details, refer to Aihara and Sunahara (1989).

3. Parameter estimation procedure

Noting that the incremental covariance Q of the Brownian motion process w is a trace class operator, it is not clear how a likelihood function may be defined for the infinite dimensional evolution equation (2.2). Hence, instead of introducing the log-likelihood function as the cost, we consider

$$\begin{aligned} L(a) &= - \int_0^{t_f} \sum_{i=1}^n \left(a(x) \frac{\partial u(t)}{\partial x_i}, \frac{\partial}{\partial x_i} (-\Delta)^{-1} du(t) \right) \\ &\quad - \frac{1}{2} \int_0^{t_f} \sum_{i,j=1}^n \left(a(x) \frac{\partial u(t)}{\partial x_i}, \frac{\partial}{\partial x_i} \left(\frac{\partial}{\partial x_j} (-\Delta)^{-1} \right)^* a(x) \frac{\partial u(t)}{\partial x_j} \right) dt \end{aligned} \quad (3.1)$$

where

$$\Delta = \sum_{i=1}^n \partial^2(\cdot) / \partial x_i^2, \quad (3.2a)$$

and from Lions (1969, p. 195), $(-\Delta)^{-1}$ is defined by

$$(-\Delta)^{-1} = \sum_{k=1}^{\infty} (1/\lambda_k) \psi_k \otimes \psi_k, \quad (3.2b)$$

where $\{\psi_i\}$ is a special orthonormal basis in H , which satisfies

$$-\Delta\psi_k = \lambda_k\psi_k, \quad \psi_k \in V, \quad (3.2c)$$

and then

$$\frac{\partial}{\partial x_i} \left(\frac{\partial}{\partial x_j} (-\Delta)^{-1} \right)^* = \sum_{k=1}^{\infty} (1/\lambda_k) \frac{\partial}{\partial x_i} \psi_k \otimes \frac{\partial}{\partial x_j} \psi_k. \quad (3.2d)$$

Suppose that $\{\phi_i\}$ is an orthonormal basis in H with values in H^1 . We approximate the unknown parameter $a(x)$ by

$$a^k(x) = \sum_{i=1}^k a_i \phi_i(x). \quad (3.3)$$

Using this approximation, the cost functional $L(a)$ defined by (3.1) can be expressed by

$$\begin{aligned} L(a^k) = & - \int_0^{t_f} \sum_{i=1}^n \left(a^k \frac{\partial u(t)}{\partial x_i}, \frac{\partial}{\partial x_i} (-\Delta)^{-1} du(t) \right) \\ & - \frac{1}{2} \int_0^{t_f} \sum_{i,j=1}^n \left(a^k \frac{\partial u(t)}{\partial x_i}, \frac{\partial}{\partial x_i} \left(\frac{\partial}{\partial x_j} (-\Delta)^{-1} \right)^* a^k \frac{\partial u(t)}{\partial x_j} \right) dt. \end{aligned} \quad (3.4)$$

The estimate of a^0 (true value) is then determined by maximizing (3.4) with respect to $\{a_1, a_2, \dots, a_k\}$ and the estimate becomes

$$\hat{a}_k^{t_f} = \sum_{i=1}^k \hat{a}_i(t_f) \phi_i(x) \quad (3.5)$$

where $\hat{a}_i(t_f)$ satisfies, by defining $\hat{a}(t_f) = \{\hat{a}_1(t_f), \hat{a}_2(t_f), \dots, \hat{a}_k(t_f)\}'$,

$$F^{(t_f)} \hat{a}(t_f) = g^{(t_f)} \quad (3.6)$$

and where the matrix $F^{(t_f)}$ and vector $g^{(t_f)}$ have components

$$f_{ij}^{(t_f)} = \frac{1}{t_f} \int_0^{t_f} \sum_{p,m=1}^n \left(\phi_i \frac{\partial u(t)}{\partial x_p}, \frac{\partial}{\partial x_p} \left(\frac{\partial}{\partial x_m} (-\Delta)^{-1} \right)^* \phi_j \frac{\partial u(t)}{\partial x_m} \right) dt \quad (3.7)$$

and

$$g_j^{(t_f)} = - \frac{1}{t_f} \int_0^{t_f} \sum_{p=1}^n \left(\phi_j \frac{\partial u(t)}{\partial x_p}, \frac{\partial}{\partial x_p} (-\Delta)^{-1} du(t) \right), \quad (3.8)$$

respectively.

Remark. $(-\Delta)^{-1}$ can be numerically realized by using the finite difference and/or finite element methods. We do not get into this question in the present paper.

4. Consistency of the estimate

In order to check the consistency property of the estimate (3.5), the basic equation (3.6) is rewritten as

follows. First, from (3.8) we have

$$\begin{aligned}
 g_j^{(t_f)} &= \frac{1}{t_f} \int_0^{t_f} \sum_{p,m=1}^n \left(\phi_j \frac{\partial u(t)}{\partial x_p}, \frac{\partial}{\partial x_p} \left(\frac{\partial}{\partial x_m} (-\Delta)^{-1} \right)^* a^0(x) \frac{\partial u(t)}{\partial x_m} \right) dt \\
 &\quad - \frac{1}{t_f} \int_0^{t_f} \sum_{p=1}^n \left(\phi_j \frac{\partial u(t)}{\partial x_p}, \frac{\partial}{\partial x_p} (-\Delta)^{-1} dw(t) \right) \\
 &= \sum_{q=1}^k f_{jq}^{(t_f)} a_q^0 + \frac{1}{t_f} \int_0^{t_f} \sum_{p,m=1}^n \left(\phi_j \frac{\partial u(t)}{\partial x_p}, \frac{\partial}{\partial x_p} \left(\frac{\partial}{\partial x_m} (-\Delta)^{-1} \right)^* \right. \\
 &\quad \left. \times \left(a^0(x) - \sum_{q=1}^k a_q^0 \phi_q(x) \right) \frac{\partial u(t)}{\partial x_m} \right) dt \\
 &\quad - \frac{1}{t_f} \int_0^{t_f} \sum_{p=1}^n \left(\phi_j \frac{\partial u(t)}{\partial x_p}, \frac{\partial}{\partial x_p} (-\Delta)^{-1} dw(t) \right), \tag{4.1}
 \end{aligned}$$

where

$$a_q^0 = (a^0, \phi_q). \tag{4.2}$$

Hence, setting $h_j^{(t_f)}$ as the 2nd and 3rd terms of the right hand side of (4.1), (3.6) becomes

$$F^{(t_f)}(\hat{a}_k(t_f) - a^0(k)) = h^{(t_f)} \tag{4.3}$$

where

$$a^0(k) = \{a_1^0, a_2^0, \dots, a_k^0\}', \tag{4.4}$$

and $h_j^{(t_f)}$ denotes the j th component of $h^{(t_f)}$.

Lemma 1. $\exists C_1 > 0$ (independent of i and j) such that

$$E \left\{ \left| f_{ij}^{(t_f)} - E \{ f_{ij}^{(t_f)} \} \right|^2 \right\} \leq C_1 / t_f. \tag{4.5}$$

Proof. From (3.7), it is easy to show that

$$E \left\{ \left| f_{ij}^{(t_f)} - E \{ f_{ij}^{(t_f)} \} \right|^2 \right\} = \frac{1}{t_f^2} \int_0^{t_f} \int_0^{t_f} E \{ X_{ij}(t) X_{ij}(s) \} dt ds \tag{4.6}$$

where

$$\begin{aligned}
 X_{ij}(t) &= \sum_{p,m=1}^n \left(\phi_i \frac{\partial u(t)}{\partial x_p}, \frac{\partial}{\partial x_p} \left(\frac{\partial}{\partial x_m} (-\Delta)^{-1} \right)^* \phi_j \frac{\partial u(t)}{\partial x_m} \right) \\
 &\quad - E \left\{ \sum_{p,m=1}^n \left(\phi_i \frac{\partial u(t)}{\partial x_p}, \frac{\partial}{\partial x_p} \left(\frac{\partial}{\partial x_m} (-\Delta)^{-1} \right)^* \phi_j \frac{\partial u(t)}{\partial x_m} \right) \right\}. \tag{4.7}
 \end{aligned}$$

Noting that

$$\begin{aligned}
 E\{X_{ij}(t)X_{ij}(s)\} &= E\left\{\sum_{p,m=1}^n\left(\phi_i\frac{\partial u(t)}{\partial x_p},\frac{\partial}{\partial x_p}\left(\frac{\partial}{\partial x_m}(-\Delta)^{-1}\right)^*\phi_j\frac{\partial u(t)}{\partial x_m}\right)\right. \\
 &\quad \times \sum_{p,m=1}^n\left(\phi_i\frac{\partial u(s)}{\partial x_p},\frac{\partial}{\partial x_p}\left(\frac{\partial}{\partial x_m}(-\Delta)^{-1}\right)^*\phi_j\frac{\partial u(s)}{\partial x_m}\right)\Bigg\} \\
 &\quad - E\left\{\sum_{p,m=1}^n\left(\phi_i\frac{\partial u(t)}{\partial x_p},\frac{\partial}{\partial x_p}\left(\frac{\partial}{\partial x_m}(-\Delta)^{-1}\right)^*\phi_j\frac{\partial u(t)}{\partial x_m}\right)\right\} \\
 &\quad \times E\left\{\sum_{p,m=1}^n\left(\phi_i\frac{\partial u(s)}{\partial x_p},\frac{\partial}{\partial x_p}\left(\frac{\partial}{\partial x_m}(-\Delta)^{-1}\right)^*\phi_j\frac{\partial u(s)}{\partial x_m}\right)\right\} \quad (4.8)
 \end{aligned}$$

and using the Gaussian property of the state u , we get

$$\begin{aligned}
 E\{X_{ij}(t)X_{ij}(s)\} &= 2\sum_{p,m=1}^n\sum_{q,r=1}^n E\left\{\left(\phi_i\frac{\partial u(t)}{\partial x_p}, E\left\{\frac{\partial}{\partial x_p}\left(\frac{\partial}{\partial x_m}(-\Delta)^{-1}\right)^*\phi_j\frac{\partial u(t)}{\partial x_m}\otimes\phi_i\frac{\partial u(s)}{\partial x_q}\right\}\right.\right. \\
 &\quad \left.\left.\times\frac{\partial}{\partial x_q}\left(\frac{\partial}{\partial x_r}(-\Delta)^{-1}\right)^*\phi_j\frac{\partial u(s)}{\partial x_r}\right)\right\}\Bigg\} \\
 &= 2\sum_{p,m=1}^n\sum_{q,r=1}^n\left[\phi_i\frac{\partial}{\partial x_p}\left(\frac{\partial}{\partial x_q}\left(\frac{\partial}{\partial x_r}(-\Delta)^{-1}\right)^*\phi_j\frac{\partial\Lambda(t,s)}{\partial x_r}\right)^*,\right. \\
 &\quad \left.\left(\frac{\partial}{\partial x_p}\left(\frac{\partial}{\partial x_m}(-\Delta)^{-1}\right)^*\phi_j\frac{\partial}{\partial x_m}\left(\phi_i\frac{\partial\Lambda(t,s)}{\partial x_q}\right)^*\right)\right], \quad (4.9)
 \end{aligned}$$

where $[\cdot, \cdot]$ denotes the Hilbert–Schmidt inner product. From (3.2d), we have

$$\begin{aligned}
 &\left|\frac{\partial}{\partial x_p}\left(\frac{\partial}{\partial x_m}(-\Delta)^{-1}\right)^*\xi\right|^2 \\
 &\leq \sum_{p,m=1}^n\sum_{k,l=1}^{\infty}\frac{1}{\lambda_k\lambda_l}\left(\frac{\partial\psi_k}{\partial x_p},\frac{\partial\psi_l}{\partial x_p}\right)\left(\frac{\partial\psi_k}{\partial x_m},\xi\right)\left(\frac{\partial\psi_l}{\partial x_m},\xi\right) \\
 &= \sum_{m=1}^n\sum_{k=1}^{\infty}\frac{1}{\lambda_k}\left(\frac{\partial\psi_k}{\partial x_m},\xi\right)^2 \quad (\text{from (3.2c)}) \\
 &= \sum_{m=1}^n\sum_{k=1}^{\infty}\frac{1}{\lambda_k}\left(\psi_k,\frac{\partial\xi}{\partial x_m}\right)^2 \quad (\text{from Green's formula}) \\
 &= \sum_{m=1}^n\left(\frac{\partial\xi}{\partial x_m},(-\Delta)^{-1}\frac{\partial\xi}{\partial x_m}\right) \quad (\text{from (3.2b)}) \\
 &= \sum_{m=1}^n\left(\frac{\partial\xi}{\partial x_m},\frac{\partial\xi}{\partial x_m}\right)_{H^{-1}} \quad (\text{from Lions, 1969, p. 192}) \\
 &= \sum_{m=1}^n\left|\frac{\partial\xi}{\partial x_m}\right|_{H^{-1}}^2 \\
 &\leq C|\xi|^2 \quad (\text{from Lions and Magenes, 1972, p. 85, } \partial/\partial x_i \in \mathcal{L}(H; H^{-1})) \quad (4.10)
 \end{aligned}$$

and from the fact that ϕ_i is an orthonormal basis in H ,

$$|\phi_i|^2 = 1, \quad (4.11)$$

we have

$$E\{X_{ij}(t)X_{ij}(s)\} \leq \text{Const.} |\Lambda(t, s)|_{L_2(V'; V)}^2. \quad (4.12)$$

Now, we get

$$\begin{aligned} E\left\{\left|f_{ij}^{(t_f)} - E\{f_{ij}^{(t_f)}\}\right|^2\right\} &\leq \frac{\text{Const.}}{t_f^2} \int_0^{t_f} \int_0^{t_f} |\Lambda(t, s)|_{L_2(V'; V)}^2 dt ds \\ &\leq \frac{\text{Const.}}{t_f} \left(\int_0^{t_f} |\Lambda(\tau)|_{L_2(V'; V)}^2 d\tau + \int_{-t_f}^0 |\Lambda(\tau)|_{L_2(V'; V)}^2 d\tau \right) \\ &\leq \frac{\text{Const.}}{t_f} \left(\int_{-\infty}^{\infty} |\Lambda(\tau)|_{L_2(V'; V)}^2 d\tau \right) \\ &\leq \text{Const.}/t_f. \quad \square \end{aligned} \quad (4.13)$$

Lemma 2.

$$\begin{aligned} \lim_{t_f \rightarrow \infty} f_{ij}^{(t_f)} &= \sum_{p,m=1}^n \text{tr} \left\{ \phi_i \frac{\partial}{\partial x_p} \left(\frac{\partial}{\partial x_p} \left(\frac{\partial}{\partial x_m} (-\Delta)^{-1} \right)^* \phi_j \frac{\partial}{\partial x_m} \Lambda(0) \right)^* \right\} \\ &= \sum_{p,m=1}^n \left[\frac{\partial}{\partial x_p} \left(\frac{\partial}{\partial x_m} (-\Delta)^{-1} \right)^*, \phi_i \frac{\partial}{\partial x_p} \left(\phi_j \frac{\partial}{\partial x_m} \Lambda(0) \right)^* \right] \quad a.s. \quad \square \end{aligned} \quad (4.14)$$

The proof of this lemma can be easily derived from Theorem 3 and Gaussian property of u . For more details, refer to Aihara and Sunahara (1989) and Wong (1971, p. 69).

Lemma 3. $\exists C_3 > 0$ (independent of j and t_f),

$$E\left\{\left|h_j^{(t_f)}\right|^2\right\} \leq C_3(1/t_f + |I_j(k)|^2) \quad (4.15)$$

where $I_j(k)$ is given by (4.17) below and

$$\sum_{j=1}^{\infty} |I_j(k)|^2 \rightarrow 0, \quad \text{as } k \rightarrow \infty. \quad (4.16)$$

Proof. We set

$$I_j(k) = \frac{1}{t_f} E \left\{ \int_0^{t_f} \sum_{p,m=1}^n \left(\phi_j \frac{\partial u(t)}{\partial x_p}, \frac{\partial}{\partial x_p} \left(\frac{\partial}{\partial x_m} (-\Delta)^{-1} \right)^* \left(a^0(x) - \sum_{q=1}^k a_q \phi_q(x) \right) \frac{\partial u(t)}{\partial x_m} \right) dt \right\} \quad (4.17)$$

It is easy to show that from Lemma 2,

$$\begin{aligned} \sum_{j=1}^{\infty} |I_j(k)|^2 &\leq \left\{ \frac{\text{Const.}}{t_f} E \left\{ \int_0^{t_f} |u(t)|_v^2 dt \right\} \left| a^0 - \sum_{q=1}^k a_q^0 \phi_q \right| \right\}^2 \\ &\leq \text{Const.} \left| \Lambda(0) \right|_{L_1(V';V)}^2 \left| a^0 - \sum_{q=1}^k a_q^0 \phi_q \right|^2 \rightarrow 0, \quad \text{as } k \rightarrow \infty. \end{aligned} \quad (4.18)$$

From (4.1), we have

$$\begin{aligned} E \left\{ |h_j^{(t_f)}|^2 \right\} &\leq 3E \left\{ \frac{1}{t_f^2} \int_0^{t_f} \int_0^{t_f} \left\{ \sum_{p,m=1}^n \left(\phi_j \frac{\partial u(t)}{\partial x_p}, \frac{\partial}{\partial x_p} \left(\frac{\partial}{\partial x_m} (-\Delta)^{-1} \right)^* \sum_{q=k+1}^{\infty} a_q^0 \phi_q \frac{\partial u(t)}{\partial x_m} \right) - I_j(k) \right\} \right. \\ &\quad \times \left. \left\{ \sum_{p,m=1}^n \left(\phi_j \frac{\partial u(s)}{\partial x_p}, \frac{\partial}{\partial x_p} \left(\frac{\partial}{\partial x_m} (-\Delta)^{-1} \right)^* \sum_{q=k+1}^{\infty} a_q^0 \phi_q \frac{\partial u(s)}{\partial x_m} \right) - I_j(k) \right\} ds dt \right\} \\ &\quad + 3E \left\{ \frac{1}{t_f^2} \left| \int_0^{t_f} \sum_{p=1}^n \left(\phi_j \frac{\partial u(t)}{\partial x_p}, \frac{\partial}{\partial x_p} (-\Delta)^{-1} dw(t) \right) \right|^2 \right\} + 3 |I_j(k)|^2. \end{aligned} \quad (4.19)$$

By using the same procedure to derive the results of Lemma 1, we can show that there exists a $C > 0$ such that

$$\text{the 1st term of the right hand side of (4.19)} \leq C/t_f. \quad (4.20)$$

Furthermore, it follows that

$$\begin{aligned} E \left\{ \frac{1}{t_f^2} \left| \int_0^{t_f} \sum_{p=1}^n \left(\phi_j \frac{\partial u(t)}{\partial x_p}, \frac{\partial}{\partial x_p} (-\Delta)^{-1} dw(t) \right) \right|^2 \right\} \\ = \frac{1}{t_f^2} E \left\{ \int_0^{t_f} \sum_{m,p=1}^n \left(\phi_j \frac{\partial u(t)}{\partial x_p}, \frac{\partial}{\partial x_p} (-\Delta)^{-1} \left(\frac{\partial}{\partial x_m} (-\Delta)^{-1} \mathcal{Q} \right)^* \phi_j \frac{\partial u(t)}{\partial x_m} \right) dt \right\} \\ \leq (C/t_f) |\Lambda(0)|_{L_1(V';V)}. \quad \square \end{aligned} \quad (4.21)$$

Theorem 4. If $k \rightarrow \infty$, with $k/t_f \rightarrow 0$, as $t_f \rightarrow \infty$, then

$$\lim_{t_f \rightarrow \infty} \sum_{p,m=1}^n \left| \frac{\partial}{\partial x_p} \left(\frac{\partial}{\partial x_p} \left(\frac{\partial}{\partial x_m} (-\Delta)^{-1} \right)^* \left((\hat{a}_k^{t_f} - a^0) \frac{\partial \Lambda(0)}{\partial x_m} \right)^* \right) \right|_{L_2(H;H)} = 0 \quad (4.22)$$

in probability.

Proof. From Lemma 3, it follows that

$$\begin{aligned} E\left\{\left|F^{(t_f)}\left(\hat{a}_k^{t_f}-a^0(k)\right)\right|^2\right\} &= \sum_{j=1}^k E\left\{\left|h_j^{(t_f)}\right|^2\right\} \\ &\leq \text{Const.}\left(\frac{k}{t_f}+\sum_{j=1}^{\infty}\left|I_j(k)\right|^2\right) \rightarrow 0, \quad \text{as } k, t_f \rightarrow \infty. \end{aligned} \quad (4.23)$$

Using Lemma 2, we can derive (4.22) from (4.23). $\square$

5. Conclusion

We studied the identification problem for parabolic systems with spatially dependent parameter. Our state was assumed to be completely observed. The problem becomes more complicated for the case of partial observation. For the case of finite dimensional parameter vectors, this problem has been analyzed in Bagchi and Borkar (1984). The extension to infinite dimensional parameter vectors is not obvious at present.

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