LETTERS TO THE EDITOR

THE CORRECTNESS OF THE FORMULAE FOR THE RADIATION OF SOUND FROM A DUCT

There exists some confusion about the correctness of the formulae for the radiation of sound from a duct. Doak [1] gives formulae that differ from the results given by Zorumski [2]. The formulae in Morse and Feshbach [3] are in agreement with Zorumski's. In his paper Doak gives in formula (27) an expression for the pressure with a twofold summation. On the other hand one has a threefold summation when one substitutes in formula (13) of Zorumski's paper his formula (22). Numerical calculations show that the additional terms in the expression of Zorumski do not vanish. It can be shown, as follows, that the results of Doak are in general not correct. Doak assumes in his formula (23) that the following expression for a Green function is possible:

$$p_{\delta, \omega}(x_1, x_2, x_3; \omega; y_1, y_2, y_3) = \sum_{mn} \alpha_{mn}(x_3; \omega; y_3) \psi_{mn}(x_1, x_2) \psi_{mn}(y_1, y_2),$$

where the functions $\psi_{mn}(x_1, x_2)$ are the normalized orthogonal characteristic functions of the cross-section. These functions satisfy the equation

$$\partial^2 \psi_{mn} / \partial x_1^2 + \partial^2 \psi_{mn} / \partial x_2^2 + \kappa_{mn}^2 \psi_{mn} = 0$$

in the interior of the duct. Moreover they satisfy the boundary conditions at the wall of the duct. For a hard wall duct this means that the normal derivative is zero at the wall.

In his appendix Doak remarks that the characteristic functions are not a complete set for the terminating surfaces. Only functions which satisfy the same boundary conditions can be expanded in a series of these functions. For other functions one has to add functions ϕ_{mn} . This leads Doak to his formula (A4)

$$p_{\delta, \omega}(x_1, x_2, x_3; \omega; y_1, y_2, y_3) = \sum_{mn} \alpha_{mn}(x_3; \omega; y_3) \psi_{mn}(x_1, x_2) \psi_{mn}(y_1, y_2) + \beta_{mn}(x_3; \omega; y_3) \phi_{mn}(x_1, x_2) \phi_{mn}(y_1, y_2).$$

However, this formula is in general not correct. One can prove the existence of an expansion in this form only if three conditions are satisfied (see reference [3], §7, page 820).

- (i) For each function ψ_{mn} and each function ϕ_{mn} there must exist a number κ_{mn} in such a way that they are a solution of the above mentioned partial differential equation.
- (ii) The functions ψ_{mn} together with the functions ϕ_{mn} form an orthogonal system. This implies that each function ψ_{mn} is orthogonal to each function ϕ_{mn} .
- (iii) The set of functions ψ_{mn} and ϕ_{mn} is complete: that is to say that it is possible to expand the Green function $p_{\delta, \omega}$ in a series of these functions ψ_{mn} and ϕ_{mn} .

Doak makes the mistake that he supposes that it is always possible to satisfy these three conditions, but this is not true. He illustrates his paper with the formulae for a hard wall tube with rectangular cross-section and in this special case the three conditions can be satisfied because the characteristic functions $\cos(m\pi x/a)$ (which have zero normal derivative at $x = a$) and $\sin(m\pi x/a)$ (which are zero at $x = a$) are orthogonal. But for a duct with circular cross-section things become different. Here the characteristic functions are $e^{im\theta} J_m(\lambda_{mn} r)$, where J_m is the Bessel function of order m . For a hard wall tube the numbers λ_{mn} are such that $J'_m(\lambda_{mn} b) = 0$,

where b is the radius of the tube. These functions form an orthogonal set. One can complete this set with functions $e^{im\theta} J_m(\nu_{mn} r)$ which are zero at the wall: i.e., $J_m(\nu_{mn} b) = 0$. These functions are orthogonal. But the two sets together are not orthogonal because

$$\int_0^b r J_m(\lambda_{mn} r) J_m(\nu_{ml} r) dr \neq 0.$$

Here we remark that one can apply an orthogonalization procedure, but the result is a set of functions for which condition (i) is not satisfied. When the functions ψ_{mn} and ϕ_{mn} are not orthogonal one can not prove, even for the coefficients of the functions ψ_{mn} , the form as given in Doak's equation (A4). The conclusion is that the formulae of Doak are in general not correct, but for the special case of a hard-walled rectangular duct they are.

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REFERENCES

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2. W. E. ZORUMSKI 1973 *Journal of the Acoustical Society of America* **54**, 1667–1673. Generalized radiation impedances and reflection coefficients of circular and annular ducts.
3. P. M. MORSE and H. FESHBACH 1953 *Methods of Theoretical Physics*, Volumes I and II. New York: McGraw-Hill Book Company, Inc.

AUTHOR'S REPLY

I welcome Dr le Grand's letter on this subject (although I do not wholly agree that he has drawn the correct conclusions) because as I tried to explain in section 4 of my paper to which he refers (his reference [1]), I believe that in the past there has been much unnecessary confusion about the points at issue. It is only too clear that in spite of good intentions my remarks in the paper have not helped much.

The reason for any differences between my "aperture modal coefficients" (and their associated impedances, etc.) and the "generalized radiation impedances" of Zorumski, Morse and Feshbach *et al.* is that these different coefficients are indeed *defined* differently. Before again attempting to explain and justify the differences in these definitions, however, I would like to comment on what seem to me to be some misconceptions in Dr le Grand's critique.

First, the "hard-wall" Green function that I used is known to exist for many aperture-cum-baffle shapes. I have tacitly assumed only that it exists for the shape of interest in any particular problem. If it does, then on the aperture surface itself it is a well-defined, suitably continuous function of the surface co-ordinates of the aperture which *can* be expressed in terms of the complete, orthogonal set of functions based on the aperture surface (if in turn it exists) in the form I have given, and as quoted by Dr le Grand. This is not an assumption.