A 25 kA, 2T, 78 kJ, 52 LITRE SUPERCONDUCTING TEST COIL. STRENGTH CALCULATIONS AND CONSTRUCTION.

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Abstract

Within the scope of our research program for a 25 kA superconducting rectifier, we have built a 25 kA s.c. coil being a single layer solenoid with a bore of 0.45 meter and a volume of 52 litre. The starting point for the design was to avoid any metallic structural material. This unique coil consists of 26 turns of a Rutherford cable in one layer covered at the outside with 6 millimeter glassfibre reinforced epoxy, in order to lower the azimuthal and axial stresses in the conductor to acceptable values. The coil has been vacuum impregnated with a glassfilling factor of 0.529. The paper describes the strength calculations and the construction details. A theoretical analysis of the mechanical behaviour of the glassfibre-epoxy-conductor lamination is given.

Introduction

It is a challenge to construct a high current superconducting coil which consists with exception of the conductor itself of nonmetallic parts being glassfibre reinforced epoxy. In this concept all possible objects of the resin and the glassfibres such as potting-, insulation-, and reinforcing material are integrated.

The coil has been built in order to test our 25 kA s.c. rectifier^{1,2}. For this purpose it was important to create a simple coil. Limits were set by the op-

rating current (25 kA), the length and critical current of the conductor (26 kA/3 T) and the available space in the cryostat.

The following points: the coil data and geometry, the Lorentz forces in the coil in axial, radial and azimuthal direction, the desirable thickness of the glassfibre-epoxy reinforcements, the determination of the constants of elasticity of the glassfibre epoxy, the vacuum impregnation process and the production of the coil, will successively come up for discussion.

Coil data and geometry

The solenoid is build up with 26 turns in one layer, has a bore of 0.45 meter, a length of 0.34 meter and a volume of 52 litre.

The inductance and energy-content are 0.25 mH and 78 kJ respectively. The central field is 1.45 T, the maximum field at the windings is 2.45 T at 25 kA. A 24 strand MCA Rutherford cable (26 kA/3 T) ($12.3 \times 2.05 \text{ mm}^2$) has to carry the current. The strand conductor is a 1.05 millimeter diameter MCA 367 fil. of $36.5 \text{ } \mu\text{m}$ diameter NbTi/Cu = 0.8 wire. The cable has not been soldered in the windings. The data about the forces and coil reinforcements are discussed below. The finished coil is shown in fig. 1.

Lorentz forces

In order to calculate the Lorentz forces in the windings the 26 turns are replaced by the windings cross-section with an average current density $\bar{J} = 944 \text{ A/mm}^2$. The current density in the strands is $J_S = 1200 \text{ A/mm}^2$.

Axial forces. The radial component $B_r(r, z)$ of the magnetic field on the volume element $2\pi r dr dz$ in the windings causes an axial force $dF_z = B_r(r, z) \cdot J \cdot 2\pi r dr dz$. In the case of thin solenoids the field component $B_r(r, z)$ at a certain position z is nearly independent of r ($r_1 \leq r \leq r_2$) (see fig. 2a). The axial pressure force increase steadily from zero at the end ($z = 0$) to $F_z^0(l/2)$ at the centre plane ($z = l/2$). We assume the average values

$$\bar{B}_r(z) = \frac{1}{\pi(r_2^2 - r_1^2)} \int_{r_1}^{r_2} B_r(r, z) 2\pi r dr$$

and

$$\bar{B}_r = \frac{2}{l} \int_0^{l/2} \bar{B}_r(z) dz = 0.43 \text{ T.}$$

The axial pressure force F_z^0 and stress σ_z^0 can be written as:

$$\begin{aligned} F_z^0(z) &= \int_0^z \int_{r_1}^{r_2} B_r(r, z) \bar{J} 2\pi r dr dz \\ &= \pi(r_2^2 - r_1^2) \int_0^z \bar{J} \bar{B}_r(z) dz \end{aligned}$$

Fig. 1. The finished 25 kA superconducting solenoid.

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Fig. 2. Longitudinal section of the coil with axial forces (a) and radial forces (b).

Fig. 3. The radial component of the magnetic field averaged over \$r\$ and with the force and stress in axial direction.

$$\sigma_z^0(z) = \bar{J} \int_0^z \bar{B}_r(z) dz, \quad (0 \leq z \leq \frac{l}{2}).$$

Their maxima are $F_z^0(l/2) = \pi R N I \bar{B}_r = -198 \text{ kN}$
 $\sigma_z^0 = l/2 \bar{J} \bar{B}_r = -70 \text{ MPa}$

Fig. 3 shows the \$z\$-dependence of \$B_r\$, \$F_z^0\$ and \$\sigma_z^0\$.

Radial force. As a consequence of the axial component \$B_z(r,z)\$ of the magnetic field the volume element \$2\pi r dr dz\$ experiences a radial force \$dF_r^0 = B_z(r,z) \bar{J} 2\pi r dr dz\$.

Volume elements at the inside of the windings experience a positive force and those at the outside a negative force (see fig. 2b). We again assume the average values

$$\bar{B}_z(z) = \frac{1}{\pi(r_2^2 - r_1^2)} \int_{r_1}^{r_2} B_z(r,z) 2\pi r dr$$

and

$$\bar{B}_z = \frac{2}{l} \int_0^{l/2} \bar{B}_z(z) dz.$$

The net radial force and pressure on the winding element \$2\pi r dr dz\$ are

$$dF_r^0 = \bar{J} dz \int_{r_1}^{r_2} B_z(r,z) 2\pi r dr$$

$$= \pi(r_2^2 - r_1^2) \bar{B}_z(z) dz \bar{J} \approx 2\pi R \bar{B}_z \bar{J} h dz$$

and

$$P_r^0(z) = \bar{J} \bar{B}_z(z) h$$

Fig. 4. The axial component of the magnetic field (a) and its average value. The peaks in (a) at both conductor sides are determined by the not uniform current density in the computer calculations and in the real conductor. These peaks do not influence the stress calculations.

Fig. 4 shows the \$r\$-dependence of \$B_z\$ at two values and the \$z\$-dependence of \$B_z\$.

Azimuthal force. The net positive radial force is compensated by the tensile force \$F_\theta^0\$ in the conductor in absence of any reinforcement. The azimuthal tensile force and stress are described by:

$$F_\theta^0 = \bar{J}_S \bar{B}_z R N A_C; \quad A_C = \text{area conductor}$$

$$\sigma_{\theta C}^0(z) = \bar{J}_S \bar{B}_z(z) R$$

The maximum tensile stress \$\sigma_{\theta C}^0(l/2) = 180 \text{ MPa}\$; the total force in the circumference of the windings is \$F_\theta^0 = 90 \text{ kN}\$.

Glassfibres-epoxy-conductor lamination

The stresses calculated above bring about a strain up to 0.2% which is unacceptable. To reduce this the windings have been reinforced with glass-fibre-epoxy. The mechanical behaviour of the glass-fibre-epoxy-conductor composite can be described by four equations:

$$d\sigma_{\theta}^{0C} = d\sigma_{\theta}^C + t\sigma_{\theta}^L; \quad d = N \cdot A_C / l = 1.62 \text{ mm} \quad (1)$$

$$h\sigma_z^{0C} = h\sigma_z^C + t\sigma_z^L; \quad h = 2.05 \text{ mm} \quad (2)$$

$$\frac{\sigma_{\theta}^L}{E_{\theta}^L} - \nu_{\theta Z} \frac{\sigma_z^L}{E_z^L} = \frac{\sigma_{\theta}^C}{E_{\theta}^C} - \nu_{\theta Z} \frac{\sigma_z^C}{E_z^C} = \epsilon_{\theta} \quad (3)$$

$$\frac{\sigma_z^L}{E_z^L} - \nu_{Z\theta} \frac{\sigma_{\theta}^L}{E_{\theta}^L} = \frac{\sigma_z^C}{E_z^C} - \nu_{Z\theta} \frac{\sigma_{\theta}^C}{E_{\theta}^C} = \epsilon_z \quad (4)$$

These equations divide the previous stresses in θ and z direction (σ_{θ}^{0C} , σ_z^{0C}) between the conductor (σ_{θ}^C , σ_z^C) and the glassfibre-epoxy part (σ_{θ}^L , σ_z^L) being conditioned by equal strain in both parts. This description is only valid in case of thin wall coils! In other cases the situation is more complex because of the strong r dependence of all stresses.

Further it is assumed that in the conductor part the constants of elasticity are equal for both directions ($E_{\theta}^C = E_z^C$, $\nu_{\theta z}^C = \nu_{z\theta}^C$). Generally this is not true if some complicated cable structure is used as a conductor. The constants of elasticity of the glassfibre-epoxy part will be calculated below. Considering the value of σ_{θ}^{0C} with regard to σ_z^{0C} and $\sigma_{\theta z}^C$ the Poisson ratio $\nu_{\theta z}$ can be neglected in equation (3), not in (4). Equations (1) and (3) give then a simple relation which illustrates the reduction of the azimuthal tensile stress in the conductor as a result of the glassfibre-epoxy reinforcement with a thickness t :

$$\frac{\sigma_{\theta}^{0C}}{\sigma_{\theta}^C} = 1 + \frac{t E_{\theta}^L}{d E_{\theta}^C} \quad (5)$$

A large value of E_{θ}^L , the azimuthal modulus of elasticity of the glassfibre-epoxy part, has been achieved by winding the glassfibre tape, i.e., the long warp fibres in the azimuthal direction. In conclusion the maximum stresses ($z = l/2$) in the conductor decreases by means of the reinforcement ($t = 6$ mm) as follows:

$$\left. \begin{array}{l} \sigma_{\theta}^{0C} = 180 \text{ MPa} \\ \sigma_z^{0C} = -70 \text{ MPa} \end{array} \right\} \xrightarrow{t = 6 \text{ mm}} \left. \begin{array}{l} \sigma_{\theta}^C = 78.7 \text{ MPa} \\ \sigma_{\theta}^L = 27.3 \text{ MPa} \\ \epsilon_{\theta} = 0.07\% \\ \sigma_z^C = -31.4 \text{ MPa} \\ \sigma_z^L = -12.8 \text{ MPa} \\ \epsilon_z = -0.045\% \end{array} \right.$$

Several hypotheses³ are known which predict when mini cracks between the fibres and the epoxy appear depending on the constants of elasticity, the angles of direction of fibres and stresses. Fig. 5 shows the predictions for this coil. The current in the coil may increase until 30 kA before mini cracks appear. The safety factor according to this criterion is about 1.5.

Before the fibres crack the forces may increase until:

$$F_{\theta} \text{ (fibre crack)} = 860 \text{ kN} \gg 90 \text{ kN}$$

$$F_z \text{ (fibre crack)} = 1.6 \text{ MN} \gg 198 \text{ kN}$$

So the safety factor with this criterion is greater

Fig. 5. The maximum stresses in the glassfibre-epoxy part before mini cracks appear.

than 8. The buckling resistance of the fibre-epoxy part has been checked. The buckling stress⁴ σ_b is a factor 20 larger than the normal axial stress $\sigma_z^L = -12.8$ MPa

$$\sigma_b \geq \frac{1}{4} (E_{\theta} \cdot E_z)^{\frac{1}{2}} \cdot \frac{t}{R} = 260 \text{ MPa}$$

Fibre reinforced plastics

The mechanical properties of a homogeneous material can be described by the E-modulus and the Poisson ratio ν . It is a task for the designer to find these quantities at cryogenic temperatures in order to secure safe operation of the product. The shell-shaped construction (lamination) of the fibre reinforced epoxy resins consists of multilayers of resin (R) and fibers (F) with relative thicknesses t_i . Each layer of glassfibre-epoxy can have a different angle of direction β_i . If a layer is deformed at first the strain of resin (ϵ_R) and fibres (ϵ_F) will be the same. For the parallel (to the fibres) direction holds $\epsilon_R = \epsilon_F = \epsilon_{//}$ and:

$$E_{//} = \frac{\sigma_{//}}{\epsilon_{//}} = \phi E^F + (1 - \phi) E^R$$

The $E_{//}$ increases linearly with the fibre filling factor ϕ .

If $\hat{\epsilon}_R$ and $\hat{\epsilon}_F$ are the maximum strains without fracture of the resin and fibre part respectively mini cracks in the resin will occur at the stress

$$\hat{\sigma}_{//} = \hat{\epsilon}_R \cdot E_{//} \quad (\hat{\epsilon}_R < \hat{\epsilon}_F).$$

The fracture stress of the total composite will be $\sigma_{//} \text{ (fracture)} \approx \hat{\epsilon}_F \cdot \phi E^F$ (see figure 6).

Fig. 6. The relation between $E_{//}$, E_1 and the fibre filling factor $\phi(a)$ and the stress-strain curve for a composite.

For the transverse direction the experimentally determined relation³ holds:

$$\frac{1}{E_{\perp}} = \frac{\phi}{E^F} + \frac{(1 - \nu^F)^2 (1 - \phi)^{1.45}}{E^R} \quad (0.3 < \phi < 0.7)$$

The deformation of each layer i is now described by:

$$\begin{aligned} \epsilon_{//} &= \frac{\sigma_{//}}{E_{//}} - \nu_{//\perp} \frac{\sigma_{\perp}}{E_{\perp}} & \epsilon_{\perp} &= \frac{\sigma_{\perp}}{E_{\perp}} - \nu_{\perp//} \frac{\sigma_{//}}{E_{//}} \\ \nu_{\perp//} &= \phi \nu^F + (1 - \phi) \nu^R & \nu_{//\perp} &= \frac{E_{\perp}}{E_{//}} \nu_{\perp//} \end{aligned}$$

For each layer i the stresses in the relevant direction θ and z of the coil can be added:

$$\begin{aligned} \sigma_{\theta} &= \sum_i t_i \sigma_{//i} \cos^2 \beta_i \\ \sigma_z &= \sum_i t_i \sigma_{//i} \sin^2 \beta_i \\ \tau_{\theta z} &= \sum_i t_i \sigma_{//i} \sin \beta_i \cos \beta_i = \tau_{z\theta} \end{aligned}$$

wherein β_i is the angle between the fibres (//) and the relevant θ direction.

The constants of elasticity for our lamination. In our case the lamination has been build up with glass-fibre tape. The long warp fibres make an angle $\beta = 1.4^\circ$ with the relevant direction. So in this case ($\cos^2 1.4^\circ = 0.999$) the model of the glassfibre-epoxy composite can be simplified to a two part model representing the warp fibres-epoxy part in θ direction (β) and the weft fibres-epoxy part in the z direction ($90^\circ + \beta$). The relative thicknesses of these parts is $t_{//} = 0.535$ and $t_{\perp} = 0.465$ respectively. With the formulae above, Table I and the substitution schemes of the θ and z parts (fig. 7), the constants of elasticity in both directions can be determined:

$$\begin{aligned} E_{//} &= 0.42 \text{ MPa} \\ E_{\perp} &= 36.1 \text{ MPa} \\ \nu_{//} &= 0.270 \\ \nu_{\perp} &= 0.232 \end{aligned} \rightarrow \begin{aligned} E_{\theta}^L &= t_{//} E_{//} + t_{\perp} E_{\perp} = 39.3 \text{ MPa} \\ E_z^L &= t_{//} E_{//} + t_{\perp} E_{\perp} = 38.8 \text{ MPa} \\ \nu_{z\theta}^L &= t_{//} \nu_{//} + t_{\perp} \nu_{\perp} = 0.253 \\ \nu_{\theta z}^L &= t_{//} \nu_{//} + t_{\perp} \nu_{\perp} = 0.250 \end{aligned}$$

Fig. 7. Substitution schemes of the glassfibre-epoxy lamination in θ direction (a) and in z direction (b).

Thermal contraction. The integral thermal contraction

$$\delta = \int_{T_1}^{T_2} \alpha dT = \Delta L/L$$

for the substitution schemes in fig. 7 are calculated to be $\delta_{\theta} = 3.12 \cdot 10^{-3}$ and $\delta_z = 3.45 \cdot 10^{-3}$. Comparison with the thermal contraction of copper ($3.24 \cdot 10^{-3}$) shows that as a consequence of the great fibre concentration the integral thermal contractions of all parts are nearly the same. The resulting mechanical stress between the conductor and the glassfibre-epoxy lamination due to the thermal contraction is given by

$$\frac{\sigma^L}{E^L} = \frac{(\delta^C - \delta^L)}{1 + \frac{tE^k}{hE^C}} \rightarrow \begin{aligned} \sigma_{\theta}^L &= 1.9 \text{ MPa} \\ \sigma_{\theta}^C &= -7.1 \text{ MPa} \\ \sigma_z^L &= -3.6 \text{ MPa} \\ \sigma_z^C &= 13.2 \text{ MPa} \end{aligned}$$

these stresses are quite low compared to the Lorentz forces and can be neglected. In absence of any glass-fibres ($\phi = 0$) these stresses would be $\sigma_{\theta}^C = \sigma_z^C = + 175 \text{ MPa}$!

Production of the coil

Glassfibre tape. Several tests were done with a fine woven and a coarse woven glassfibre tape. The best impregnation results were obtained for the coarse woven tape because of the greater fibre filling factor, the lower epoxy flow resistance and the more homogeneous weave (see Table II). Both tapes are made from the same VETROTEX⁵ E-glassfibres.

Table I Properties of relevant materials.

material	E MPa	ν	σ_{fracture} MPa	ϵ %	ρ kg/m ³	$\delta = \int_{T_1}^{T_2} \alpha dT$
CY221 HY979	6900	0.35	66	.95	1200	0.016
glassfibre	73000*	0.2	1500	2.14*	2550	0.006
Cu	124000	0.33	420	50.0	8960	0.0032
Cu/NbTi	113000*					
NbCuTi	92000*					

* 300 K

Table II Relevant data about two glassfibre tapes.

type	thickness μm	width mm	ϕ_{eff} kg/m ³	glassfilling ϕ calculated	ϕ measured	warp %	weft %
fine	80	30	1150	0.451	0.454	63.0	37.0
coarse	170	70	1354	0.531	0.529	53.5	46.5

Epoxy-resin. The epoxy acts as a binding agent, between fibres and conductor. For this case a well described⁶⁻⁹ epoxy CIBA CY221HY979 was chosen from the many epoxies available in order to be able to compare test results. Important for the working up of the epoxy are a few physical properties: working temperature (40 °C), potlife (1.2 hr), curing temperature (70 °C) and the starting viscosity η_0 (40°C) = 0.17 Pas.

Vacuum impregnation. The coil winding is surrounded by a vacuum seal which is stiff on the inside of the coil and flexible on the outside (Mylar foil). This last concept allows to reach the maximum possible glassfilling factor 0.53 because of the air pressure on the glassfibres via the flexible foil (see Table II). Fig. 8 shows the vacuum impregnation installation. The epoxy is supplied at the bottom and then rises through the windings up to a height $l(t)$.

Fig. 8. The vacuum impregnation installation.

The epoxy flow through the glassfibre-conductor packet can be described by:

$$-\frac{dp}{dx} = \frac{8\eta \bar{u}}{r_{\text{eff}}^2} \rightarrow \frac{8\eta}{r_{\text{eff}}^2} \frac{dl}{dt} = \frac{\Delta p}{l}$$

in which dl/dt is the average velocity of the epoxy, r_{eff} the effective radius of the pores, Δp the difference in pressure between coil and epoxy container, $\eta = \eta_0 e^{t/\tau}$ the time depending viscosity of the epoxy. The height $l(t)$ is given by:

$$l^2(t) = \frac{\Delta p r_{\text{eff}}^2}{4\eta_0} (1 - e^{-t/\tau}) \tau; \tau = \frac{t_{\text{potlife}}}{\ln 2} \quad (6)$$

If the viscosity is constant ($t < t_p$) then $l(t)$ is proportional to $\sqrt{t}$ (diffusion).

The maximum height that can be reached is given by $h_{\infty}^2 = \Delta p r_{\text{eff}}^2 \tau / 4 \eta_0$. Equation (6) describes the process adequately.

An agreement has been found with the data of Hartwig⁸ concerning the η/η_0 in our case determined from (6) (see fig. 9). Because of the short potlife of the epoxy used, the coil has been filled in two stages. In stage one the lower half has been impregnated from the bottom. In stage two the upper half has been impregnated with fresh epoxy supplied through a ring at the middle of the coil. The filling process is shown in fig. 10.

Fig. 9. The time dependent relative viscosity of the epoxy. Comparison of measurements with data of Hartwig⁸.

Fig. 10. The measured double stage impregnation process.

Fig. 11. The cure cycle.

Curing. Cracks caused by chemical shrinkage have to be avoided by curing the coil with a temperature gradient. This gradient is achieved by a heater at the inside of the windings. The minimum curing time for this epoxy is 4 hr at 40°C and 8 hr at 70°C. The cure cycle is shown in fig. 11. After the cure cycle is completed all vacuum seals and coil former are removed.

Former winding. The next layers have been wound successively: an underlayer of coarse glassfibre tape (0.68 mm thick); 26 turns of the conductor (2.7 mm); 20 layers coarse tape (6.8 mm); and one layer fine tape (0.16 mm). The glassfibre tape in one layer has been wound with 200% overlap. The conductor has been wrapped around with one layer fine tape. After the fourteenth layer one connection terminal is led back to the other terminal so that both are at one side. Two heaters, two quench sensing coils and two voltage centre taps have also been placed.

Final remarks

The coil has been finished. Visual inspection of the cured glassfibre-epoxy lamination showed no faults. Tests with positive results were done with many samples concerning tensile strength, thermal cycling between 80 and 300 K, glass filling factor and the method of vacuum impregnation. Our method brings about an excellent volume glass filling factor of 0.53. A description of the vacuum impregnation process has been given. It showed good agreement with data known from references. The superconducting properties of the coil will be measured if the 25 kA s.c. rectifier is tested.

References

1. H.H.J. ten Kate, A.H.M. Holtslag, H.A. Steffens, J. Knoen and L.J.M. van de Klundert, "Design of a three phase thermally switched 25 kA, 1.5 kW superconducting rectifier-fluxpump," Proc. ICEC 9, Butterworth, Guildford UK, pp.753-756, 1982.
2. H.H.J. ten Kate, A.H.M. Holtslag, J. Knoen, H.A. Steffens and L.J.M. van de Klundert, "Status report of the three phase 25 kA, 1.5 kW thermally switched superconducting rectifier. Construction and some results," Proc. ASC Knoxville USA, 1982.
3. Internal report A.J. Ingen Housz.
4. G. Nonhoff, "Konstruieren mit Kunststoffen," Proc. Symp. der DECHEMA (dec. 1980), pp.217-243.
5. VETROTEX, "Eigenschaften von Vetrotex textiltglas," Product information paper E1 (1970), Gevetex Textiltglas-GmbH Düsseldorf BRD.
6. G. Hartwig, "Low-temperature properties of epoxy resins and composites", Adv. Cryogenic Eng., Vol. 24, pp.17-36, 1978.
7. G. Hartwig, "Low-temperature properties of potting and structural materials for s.c. magnets," IEEE MAG-11, pp.536-539, 1975.
8. G. Hartwig, B. Kneifel, "Viskosität von pulvergefüllten und reinen Epoxid harzen," Kunststoffe 67, pp.43-47, 1977.
9. P. Bauder, G. Hartwig, W. Weiss, "Die thermische ausdehnung von pulvergefüllten Epoxy harzen bis zu tiefen Temperaturen," Materials Science and Engineering, Vol. 22, pp.265-270 (1976).