Competition between geometrically induced and density-driven segregation mechanisms in vibrofluidized granular systems

C. R. K. Windows-Yule,¹ G. J. M. Douglas,² and D. J. Parker¹

¹*School of Physics and Astronomy, University of Birmingham, Edgbaston, Birmingham B15 2TT, United Kingdom*

²*School of Metallurgy and Materials, University of Birmingham, Edgbaston, Birmingham B15 2TT, United Kingdom*

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The behaviors of granular systems are sensitive to a wide variety of particle properties, including size, density, elasticity, and shape. Differences in any of these properties between particles in a granular mixture may lead to segregation, or “demixing,” a process of great industrial relevance. Despite the known influence of particle geometry in granular systems, a considerable fraction of research into these systems concerns only uniformly spherical particles. We address, for the case of vertically vibrated granular systems, the important question of whether the introduction of differing particle geometries entirely invalidates our existing knowledge based on purely spherical granulates, or whether current models may simply be adapted to account for the effects of particle shape. We demonstrate that while shape effects can indeed influence the dynamical and segregative behaviors of a granular system, the segregative mechanisms associated with particle geometry are decidedly secondary to those related to particle density. The relevant control parameters determining the extent of geometrically induced segregation are established. Finally, a manner in which shape effects may be accounted for in simulations utilizing purely spherical particles is proposed.

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I. INTRODUCTION

Granular materials play a vital role in a wide variety of both natural and industrial processes. An understanding of their behaviors is therefore highly desirable in numerous fields, such as the pharmaceutical, food, cosmetic, chemical, petroleum, polymer, and ceramic industries [1,2]; geophysics [3–5]; soil mechanics [6]; volcanology [7]; and even space research [8]. Due to their cross-disciplinary importance, granular materials have been the subject of much research over the past two centuries [9]; however, those familiar with the field will be aware that a significant proportion of this research concerns granulates composed entirely of spherical particles. Of course, in real-world systems this is seldom the case. This poses an important question: if the constraint of sphericity is relaxed, can existing laws simply be “tweaked” to accommodate the presence of geometrical differences between particles, or must our current understanding of granular physics based on uniformly spherical particles be fundamentally reassessed?

Although geometrical effects due to differences in particle size have been studied extensively [10,11], the specific influence of *shape* has received comparatively little attention, due in part to the limitations of current computational technology as well as the experimental difficulties of isolating shape differences from size differences [12]. Nonetheless, there exists strong evidence that particle shape alone can significantly affect the behaviors of granular systems [13]. Perhaps most notably, it has been shown that a granular mixture comprising particles with differing geometries may spontaneously separate into its individual constituents [14–18] when exposed to an energy source. This process, termed granular segregation, may also be driven by various other differences in particle properties, such as size [19,20], mass [21], or elasticity [22,23]. An open question of much significance to industry, where mixing and separation can be of great importance and particles often differ in shape, is whether models assuming spherical

particles are adequate to describe and predict the behaviors of these complex systems.

The aim of the present study is to directly investigate the validity of the sphere assumption, with particular emphasis on the highly industrially relevant phenomenon of granular segregation. We address various issues related to the influence of particle shape on dynamical and segregative processes in a granular system. First, we compare our experimental findings concerning purely shape-driven segregation with those of previous studies in order to verify existing hypotheses and give greater insight into the relevant mechanisms underlying these segregative processes. While previous studies have investigated the competition between size- and shape-driven segregation, systems of particles differing in both shape and *density* remain largely unstudied; we remedy this through the investigation of particles with equal volumes but differing geometries and masses. The study of combined shape and density segregation is of *direct relevance* to various applications, including the reprocessing of electronic waste [24], a contemporary issue of environmental and financial importance on a global scale [25]. Finally, we assess possible manners in which current sphere-based simulational models might be adapted in order to account for effects introduced by shape nonuniformity, allowing these simpler, more computationally efficient simulations to better imitate the behavior of more realistic granular systems.

II. EXPERIMENTAL DETAILS

Our experimental system consists of a cuboidal container affixed atop an electrodynamic shaker. A number, N , of particles is introduced to the container forming a granular bed, which is vibrated sinusoidally in the vertical direction. The use of vertical vibration as a method of energizing a granular system is chosen due to the industrial relevance of such a manner of excitation [26–29] and the relative simplicity of

FIG. 1. (Color online) Top: Scale diagrams showing the dimensions of the three differing particle geometries used in experiment: spherical (left), disklike (horizontal center), and cuboidal (right). For each particle type, plan (top) and side (bottom) views are shown (due to the symmetry of the particles used, the end view is, clearly, redundant). Bottom: Photographic images of the particles represented above. Closeup images of the particles' surfaces acquired using scanning electron microscopy may additionally be seen in Fig. 2.

vibrofluidized beds, which allows the fundamental dynamics of the system to be probed. Particles of three differing shapes—spherical, cuboidal, and disklike—are used; all particles have volumes equivalent to that of a sphere of diameter $d = 3$ mm to within a 10% margin of error. The specific spatial dimensions of the particles used are given in Fig. 1. Various materials, providing a range of differing densities, ρ , and hence masses, are used: chrome steel ($\rho = 7900 \text{ kg m}^{-3}$), glass ($\rho = 2500 \text{ kg m}^{-3}$), and nylon ($\rho = 1100 \text{ kg m}^{-3}$). A variety of binary combinations of particles are used with, in all cases, $N_A = N_B = N/2$, where N_X represents the number of particles of species X . The total particle number and the horizontal dimensions of the system ($L_{x,y}$) are varied in the ranges $N \in (1000, 3000)$ and $L_{x,y} \in (25, 100) \text{ mm}$, the system in each case being bounded in all horizontal directions by rigid, vertical perspex walls. The frequency f and amplitude A at which the system is driven are also varied in the ranges $f \in (10, 100) \text{ Hz}$ and $A \in (0.67, 6.7) \text{ mm}$. This relatively large parameter space allows us to investigate several important issues, such as the relevant parameter(s) determining the extent of shape segregation and the variation in the degree of segregation at differing system energies and densities, as well as allowing us to assess the generality of our observations. The large height, $L_z = 200 \text{ mm}$, of the containers minimizes the probability of particle collisions with the upper boundaries. The relatively large particle size means that air effects, which under certain circumstances can significantly effect segregation [30–32], may be safely neglected [33]. Effects due to static charge are also minimized through the use of a steel base [34].

As illustrated in Fig. 2, the micron-scale surface roughness possessed by particles can differ noticeably between species—

FIG. 2. (Color online) Scanning electron microscope (SEM) images showing the surface topography of (a) spherical, (b) cuboidal, and (c) disklike steel particles.

even species composed of the same material. However, these differences are unlikely to significantly affect the results presented here: while surface roughness can affect various factors such as adhesion [35], rolling and sliding friction [36], and the critical force required to initiate particle motion [37], for systems such as ours, which consist of a strongly driven assembly of large, heavy particles in a dilute, collisionally dominated [38] regime in which enduring contacts are limited, these parameters are unlikely to exert an important influence [39]. In fact, previous studies [40] have directly demonstrated that the two factors most likely to influence systems such as ours—the normal force between particles and the torsion friction coefficient—have little or no dependence on surface roughness even outside the dilute regime. A schematic diagram of our experimental system can be seen in Fig. 3.

FIG. 3. (Color online) Schematic diagram (not to scale) of the experimental system from which data are obtained. For simplicity, only a monodisperse bed of spherical particles is represented.

III. DATA ACQUISITION

A. Experimental data: Positron emission particle tracking

Experimental data are acquired using positron emission particle tracking (PEPT). PEPT is performed by radioactively labeling a single “tracer particle,” physically identical to other particles of its species, with a β^+ -emitting radioisotope. The positrons emitted due to this activation rapidly annihilate with electrons within the tracer, producing pairs of 511 keV γ -rays whose directions of motion are separated by 180° . By placing a granular system containing a radioactively labeled tracer particle between the detector heads of a dual-headed γ camera, multiple pairs of these back-to-back γ -rays may be used to triangulate the tracer’s position in three-dimensional space. A simple, visual representation of this process is provided in Fig. 3. For adequately active tracers, such triangulation events may be performed multiple times per second, allowing the particle’s motion to be tracked with a typical spatial resolution of the order of millimeters and a millisecond temporal resolution [41]. The highly penetrating nature of the 511 keV γ -rays used allows the particle’s dynamics to be tracked even deep within the interior of large, dense, and/or opaque systems. For systems in a nonequilibrium steady state, such as those discussed here, the single-particle motion recorded can, through the principle of ergodicity, provide a variety of information pertaining to the system as a whole, including density, velocity, and temperature fields; mean-squared displacements; diffusion coefficients; velocity autocorrelation functions; and convection rates. Of these quantities, the most pertinent to the current work are the packing density, η , and the granular temperature, T .

One-dimensional density profiles, such as those shown in Figs. 5 and 8, may be obtained by subdividing the computational volume into a series of thin “segments” in the vertical (z) direction, each of thickness Δz , and recording the fractional residence time, $F(z)$, of the tracer within each segment. For systems in a steady state, $F(z)$ is directly proportional to the packing density for each segment, allowing

$\eta(z)$ to be determined as

$$\eta(z) = \frac{2}{3} \frac{N F(z) d^3}{L_x L_y \Delta z}. \quad (1)$$

For the current experiments, a Δz of 0.25 mm is chosen. This value is suitably small to allow a detailed representation of the variation of packing density with z , while remaining large enough to ensure adequate statistics.

The acquisition of vertical T -profiles is achieved in a similar manner, with the system once again being subdivided into a number of cells. The magnitude of the mean particle velocity, $\bar{v}$, within a given cell is then determined by averaging the velocities corresponding to all data points falling within this cell. The *fluctuation velocity*, $\tilde{v} = |v - \bar{v}|$, is then determined for each data point, allowing the local granular temperature to be calculated as

$$T(z) = m \langle \tilde{v}^2 \rangle, \quad (2)$$

where $\langle \rangle$ represents an average over all data points within a segment. For systems containing two or more particle species, PEPT data may also be used to provide a quantitative measure of segregation. For bidisperse systems, two identical runs are conducted, each using a different tracer particle. The residence times—and hence the time-averaged local concentrations, ϕ —of each species in each region of the experimental volume can then be determined. With a knowledge of the fractional concentration of a given species in each of the system’s i cells, the *segregation intensity*, I_s [42], may be obtained using the equation

$$I_s = \left[\frac{\sum_{i=1}^{i=N_c} (\phi_i - \phi_m)^2}{N_c} \right]^{\frac{1}{2}}, \quad (3)$$

where N_c is the total number of cells within the computational domain and ϕ_m is the mean concentration for the system as a whole (for an equally weighted, bidisperse system, $\phi_m = 0.5$). Although the cells into which the experimental volume is divided may be of any dimensionality and any size, the segregation observed in our current setup occurs almost exclusively in the vertical direction. Thus, in order to ensure optimal statistics, we take our cells as the same 0.25 mm segments used to create η profiles as described above. To ensure that such a cell size is appropriate, I_s values were additionally calculated using cells of thickness 0.5 and 0.125 mm; in all cases, the segregation intensities produced were found to be effectively invariant with Δz . The obtained I_s is then normalized by the maximum value achievable such that an I_s of 1 corresponds to a completely segregated system, while $I_s = 0$ indicates perfect mixing. For the binary systems investigated here, in which $N_A = N_B$, the normalization constant is equal to 0.5, as can be easily determined from Eq. (3).

PEPT data are acquired over a period $t \in (3600, 7200)$ s, with data from larger and less strongly excited systems being acquired over a longer duration due to the increased time required for the tracer to fully explore the system. To ensure a steady state, the acquisition of each data set is preceded by a period of 1000 s during which the system is excited, but no data are recorded. For the systems investigated here, this period should be more than adequate to allow a steady

state to be achieved [43,44]. The steady state of all systems is confirmed by subdividing each data set into a series of overlapping 200 s segments and ensuring consistency in $\eta(z)$ between each segment. For additional information regarding PEPT and its applicability to monodisperse and bidisperse systems, please refer to Refs. [41,45–49].

B. Discrete particle simulations: MERCURYDPM

In addition to experimental results acquired using PEPT, additional data were obtained from simulations produced using the University of Twente’s MERCURYDPM software package [50–53]. The MERCURY code is an open-source package for performing simulations using the discrete particle method (DPM), also known as the discrete element method (DEM). For the current work, we implement a frictional spring-dashpot model with a linear elastic and a linear dissipative contribution for both the normal and tangential forces acting between particles in contact. To reproduce as closely as possible the experimental system described above, system parameters such as the driving frequency (f), the amplitude (A), the particle number (N), the system dimensions (L_x, L_y, L_z), and the material densities of the various particles used (ρ) are all implemented as their exact experimental values. The determination and implementation of the systems’ dissipative and frictional coefficients are discussed in detail in Sec. IV C.

A more detailed description of the contact law used can be found in Ref. [54] or, alternatively, in the supplemental material of our previous paper [43]. For a detailed discussion of contact laws in general, we refer the reader to the review by Luding [55].

IV. RESULTS AND DISCUSSION

A. Geometrically induced segregation in equal-density systems

We begin by discussing purely shape-driven segregation, i.e., the situation for which particles are identical in terms of their total volume and material properties, differing only in their geometries. Previous studies posit that there are two main mechanisms underlying shape segregation: differences in grains’ “effective size” [15], and differences in their maximal packing fractions [16]. Since the presence of two competing segregation mechanisms will clearly present complications in any analysis, we choose to deliberately suppress the latter of these two mechanisms; this is achieved in two ways: First, we use strong driving, which will frustrate the packing-related mechanism by rapidly breaking up the closely packed regions prerequisite to this manner of segregation [16,56]. Second, we focus on relatively dilute systems, wherein excluded volume effects are minimal. Particles with similar average random loose packings [15] were also deliberately chosen. The reasoning behind the choice to suppress the packing-based segregation mechanism and focus instead on that arising from differences in effective size is simple: while the former is only applicable for a specific range of system densities and driving strengths, and can be fairly easily eliminated if desired, the latter is thought to apply for a much wider parameter space, and is considerably harder to fully suppress. From a practical standpoint, this means that while one segregative mechanism can relatively easily be “avoided” if desired (e.g., in industrial

FIG. 4. Two-dimensional velocity vector fields for (a) a system of steel squares and spheres driven with a frequency $f = 40$ Hz and amplitude $A = 1.67$ mm, showing the randomized flow field typical of all systems explored, and (b) a similar system of horizontal extent $L_{x,y} = 100$ mm whose sidewalls are composed of (highly dissipative) lead.

applications), a fuller understanding of the other is necessary if the resultant effects are to be accounted for.

To further isolate the segregation mechanism in which our interest lies, and indeed generally ensure that the system under investigation is as simple as possible—thus better allowing the study of the system’s behavior at a fundamental level—care was taken to ensure the absence of any significant sidewall influence on the granulates’ flow profiles. Toward that end, checks were performed on the depth-averaged velocity fields for the x - z , y - z , and x - y planes to confirm the lack of any major variation in the flow fields near the container’s walls. An example of such a flow profile can be seen in Fig. 4; this image compares a typical velocity field produced using the experimental setup explored within this manuscript to that of a similarly composed yet wider system possessing highly dissipative (lead) sidewalls, which may be expected to strongly influence the system’s flow behavior [57,58]. While the latter system displays considerable coordinated motion, induced by the presence of the dissipative side boundaries, the former demonstrates seemingly randomized motion throughout its spatial extent, as illustrated by the short lengths (indicative of a small magnitude of particles’ net velocity in a given direction) and random orientation of the time-averaged velocity vectors, strongly suggesting an absence of significant wall influence on flow within the systems discussed here. Since, in particular, the presence of sidewall-induced convection rolls within a bed is known to affect segregation [10], additional measures were taken to ensure the absence of such convective motion. Specifically, fast Fourier transforms of the tracer’s vertical trajectory were taken and analyzed to check for the presence of distinct “peaks,” which might suggest a periodic circulatory motion associated with convection. Any data sets showing such peaks were discarded.

From our results, it is first notable that, for particles of equal volume and mass but differing geometry, the species possessing the higher average *radius of gyration* will segregate upward (see Fig. 5). This was found to be the case for all combinations of particle shape, particle number, container size, and for all driving parameters. The radius of gyration is defined as $r_g = \sqrt{\frac{I}{m}}$ (with I the moment of inertia) and, when

FIG. 5. (Color online) One-dimensional vertical packing profiles for various bidisperse-by-shape systems: (a) spheres (solid line) and cuboids (dashed line) driven with a frequency and amplitude of 40 Hz and 1.67 mm, respectively; (b) cuboids (solid line) and disklike particles (dashed line) driven with the same f and A ; (c) spheres (solid line) and cuboids (dashed line) driven with $f = 100$ Hz and $A = 0.67$ mm; (d) spheres (solid line) and cuboids (dashed line) driven with $f = 80$ Hz and $A = 0.42$ mm, producing an equal Γ value to case (a), but with a reduced V . In all cases, $N = 1800$ and $L_{x,y} = 80$ mm. Although, for clarity, error bars are not included in the images presented, it is worth noting that the calculated η values for any given region of space carry errors of less than 5%, with the exception of the most dilute regions ($\eta < 0.01$), where the low particle density will clearly result in greater statistical noise. Results are shown for both experiment [orange (light gray)] and simulation (black). Details regarding the manner in which the simulations shown approximate the effects of the particles' nonspherical geometries may be found in Sec. IV C.

averaged over a particle's principal axes of rotation, it provides a measure of an object's "effective size" [15,59,60]. Our findings agree qualitatively with those of Roskilly *et al.* [15], with both our experimental systems and their simulations demonstrating a partially segregated steady state for similar species combinations.

The effect of varying driving parameters on segregation was also investigated. Our results show that while the degree of segregation of a bidisperse-by-geometry granular system is strongly dependent on driving, for frequencies of 20 Hz and above, the specific combination of f and A used is seemingly irrelevant, as is the dimensionless acceleration, $\Gamma = \frac{\omega^2 A}{g}$.

This somewhat surprising Γ independence is evidenced by that fact that, for a system of spheres and cuboids, when the system's vibrational frequency and amplitude are altered such that Γ varies between 5.4 and 26.9 while the base velocity $V = \omega A$ is held at a constant 0.42 ms^{-1} , the segregation intensity, I_s , maintains an approximately constant value of 0.36 ± 0.03 [see, as an example, Figs. 5(a) and 5(c)]; conversely, for a constant $\Gamma = 17$ and V varied in the range $0.19 \leq V \leq 0.66$, I_s is found to vary considerably, from $I_s = 0.07 \pm 0.2$ to 0.39 ± 0.3 . Qualitatively similar results are also observed for other shape combinations and system sizes, as illustrated in Fig. 7. Thus we conclude that the key control parameter for the system's driving is the peak base velocity, V . This result makes physical sense, as two of the main parameters affecting segregation in dilute systems (packing density and temperature) are both known to scale with V [61–64].

The breakdown of the observed Γ independence for $f \lesssim 20$ is likely related to the fact that at these low frequencies, the bed's behaviors depend sensitively on the specific form of the driving used—i.e., one can no longer expect particle motion to be decoupled from that of the base [65]. Thus, in this situation, the bed's state will begin to vary depending on the precise frequency and amplitude of vibrations (as opposed to simply the combination thereof, $V = \omega A$), hence introducing an additional dependence of the segregative behaviors on f , A , and Γ alongside the established V dependence. For the high- f case, however, the vibrating base can be treated simply as a source of energy [65], meaning that alterations in driving will only manifest themselves as alterations in $V \propto \sqrt{E}$. Since in the current study we are interested in analyzing the fundamental behaviors of the system, our discussion will focus mainly on the high-frequency case, thus avoiding the complexities introduced by the additional dependencies at low f .

We now attempt to provide a cogent explanation for the observed segregative behaviors. Figure 6 shows the data from Figs. 5(c) and 5(d) replotted alongside the corresponding temperature profiles for each species. The temperature profiles follow, in all cases, the general qualitative form expected of a dilute, vibrofluidized bed [66], initially decreasing, before passing through a minimum and increasing with large z . However, as illustrated in Fig. 6(a), the magnitude of T can vary significantly between differing particle species within the same bed. Specifically, we find that where such energy nonequipartition is observed, the species endowed with the larger r_g invariably possesses the higher energy. This can be understood through analogy with the findings of Wildman *et al.* [48], who show that large particles—analogue to our higher- r_g particles—will indeed, through modified collision and energy dissipation rates, carry higher temperatures. This disparity between temperatures for differing species leads to the existence of additional forces within the system [67], which influence the segregation of unlike particles. Brey *et al.* [68]

FIG. 6. (Color online) Packing profiles [blue (dark gray)] from Figs. 5(c) and 5(d) replotted alongside their respective vertical temperature distributions [orange (light gray)]. In all instances, data corresponding to the lighter species are represented by a dotted line, and the heavier species are shown by a solid line.

propose a control parameter $\chi = \frac{m_\alpha T_\beta}{m_\beta T_\alpha}$ whose value determines whether a given species may be expected to rise or sink within the system—for $\chi > 1$, species β will tend to segregate upward, and vice versa. For the case in which both species are equally massive, therefore, one would anticipate that the more energetic particles would tend to segregate toward the top of the system, as is indeed observed. It is worth noting additionally that, as exemplified in Fig. 6, for the case $\chi \rightarrow 1$, $I_s \rightarrow 0$, providing some measure of support for our hypothesis.

Despite the system's clear V dependence, no clear monotonic relationship between V and I_s is observable, as is evident from Fig. 7; this is perhaps not surprising, as V affects a number of system parameters, such as temperature, density,

FIG. 7. (Color online) Experimental data showing the variation of segregation intensity, I_s , with differing driving parameters. Data are shown for systems comprising cuboidal and spherical particles (circles) as well as for systems composed of disks and spheres (triangles). In each case, the blue dashed lines denote data sets in which V is varied at a fixed acceleration $\Gamma = 12$ and the red dotted lines represent cases in which V is held constant at $V = 0.19$ while Γ is adjusted. For all data shown, $N = 2400$ and $L_{x,y} = 60$.

and hence particle collision rate and void fraction, all of which will individually affect the degree of segregation exhibited by a system. For similar reasons, a simple relation between the value of χ and I_s is not observable from the available data, as greater disparities between T (which would act to increase I_s) generally occur in more dilute systems (where I_s is typically reduced). Although the establishment of a concrete relationship between I_s and V is a subject worthy of future research, it is not of primary concern with regard to the aims of this current work. In fact, the simple determination of the correct control parameter alone may potentially prove highly useful in future research, e.g., through the knowledge that a greater variation in segregative behaviors may be achieved by altering V as opposed to Γ .

B. Competition between geometrically induced and density-driven segregation

We consider next the case in which particles differ in both shape and mass, while still maintaining equal volumes. It is first notable that the geometric mechanism of segregation is decidedly secondary to the density-related mechanism. This observation is found to hold across the entire range of parameter space investigated, and is exemplified in Fig. 8(b); here, we see a system of (heavy) steel cuboids and (light) nylon spheres. Whereas in the equal density case (see Fig. 5) the nonspherical component would be expected to segregate upward, in this instance we see that the lighter nylon spheres instead migrate to the upper region; it is also notable that the *extent* of segregation is also much greater for the case of differing ρ . Similar behavior is observed for glass/steel systems [see Fig. 8(c)], nylon/glass systems, and for all shape combinations. Comparison with an equivalent system of purely spherical particles [Fig. 8(a)] shows that, on a qualitative level, the system comprising nylon spheres and steel cuboids resembles much more closely the bidisperse-by-density case than the bidisperse-by-shape. This is a pleasing result, as it suggests that the introduction of differing particle shapes does not inherently invalidate existing research into segregation based on uniformly spherical granulates. Nonetheless, there remain certain significant discrepancies between the pure-sphere (PS) and geometrically nonuniform (GNU) cases; for the remainder of this paper, we will attempt to not only explain the possible origins of these discrepancies, but also to propose methods by which they can be accounted for and incorporated into current models.

It is notable from Fig. 8 that, somewhat counterintuitively, the GNU system in (b) exhibits *stronger segregation* ($I_s = 0.67$) than its PS counterpart ($I_s = 0.42$). Indeed, this is typical of all systems studied. The increased I_s for the GNU case can be explained by the clearly observable increase in packing density for these systems, resulting in a reduction in void space between particles, thus allowing for more complete density-induced segregation [43]. This increased η can in turn be explained by an increase in the number of degrees of freedom for the GNU case—the greater conversion of translational to rotational kinetic energy will result in an effective reduction in the system's temperature, and hence a denser packing [69]. The particularly strong segregation in image (d) ($I_s = 0.86$) may be explained by the fact that, in this

FIG. 8. (Color online) Vertical packing profiles for systems comprising particles of equal volume, but differing in both geometry and material density. Data are shown for (a) steel and nylon spheres ($I_s = 0.42$), (b) steel cuboids and nylon spheres ($I_s = 0.67$), (c) steel cuboids and glass spheres ($I_s = 0.41$), and (d) steel cuboids and nylon disklike particles ($I_s = 0.86$). Systems (a)–(c) are driven at a frequency of 65 Hz and an amplitude of 1.03 mm, and they consist of $N = 1800$ particles. System (d) is driven at $f = 40$ Hz, $A = 3.57$ mm. For each image, the heavier component is represented by a solid line, and the lighter component is represented by a dashed line; orange (light gray) lines denote experimental data, while simulated results are shown in black.

case, both segregative mechanisms are working in unison, as opposed to in opposition.

The apparent role of rotational kinetic energy, E° , in the observed segregative processes within our system raises an interesting question—might E° alone, or a gradient thereof, lead to segregation? In a recent work [70], we have shown that the rotational modes of an elongated particle may act as a “reservoir” for translational kinetic energy, $\bar{E}$. Thus, it is possible that for the case of simple, buoyancy-driven segregation, particle rotational energies may affect species separation in various, contrasting manners. Consider, for instance, a system in which two particle species possess, on average, equal *translational* kinetic energies but differing *rotational* energies; since a particle of the species with higher E° is able, during a (dissipative) collision, to transfer

more energy from the rotational to the translational modes, such particles will generally maintain higher $\bar{E}$ values after multiple collisions, and they are therefore more likely to segregate upward through the system [67,68]. Conversely, for the situation in which two particle species possess the same *total* energy but differing ratios of $\bar{E}$ to E° , one would expect the inverse to be observed—the conversion of rotational to translational kinetic energy will, by definition, be an imperfectly efficient process due to its reliance on dissipative collisional and frictional interactions [71]. Moreover, a particle with an inherent tendency to possess a higher relative rotational energy may also be expected to gain a proportionally greater E° (and hence comparatively smaller $\bar{E}$) during an energizing particle-base interaction. In this case, one would predict the higher- E° particles to, on average, descend through the system. Of course, these proposed processes are highly speculative, and they can only be expected to be significant in the somewhat idealized situations presented—in the current work, the rotational energy of particles seems only to be relevant in its heat-sink-like effect as described in the previous paragraph. Nonetheless, this matter is certainly a topic worthy of future research.

C. Comparison with simulations

To support our hypotheses and, further, to demonstrate the possibility that simple, spherical models may potentially be adapted to predict the effects of shape segregation, we turn to a series of simulations performed using the MERCURYDPM software package. From our above discussion, it is clear that there exist two main geometrical effects that must be considered: the increase in effective size due to a particle’s radii of gyration, and the decrease in translational kinetic energy for particles in which a significant amount of energy is effectively “lost” to the rotational modes. We account for the first of these factors in our simulations by increasing the diameters of particles representing nonspheres by a factor equivalent to the ratio of the appropriate shape’s average r_g to that of an equivalent sphere. Thus, our cuboid particles may be represented by a sphere of $d = 3.8$ mm and our disklike particles by a 4.9 mm sphere. Since granular materials are known to violate the principle of energy equipartition [48,62,72], the introduction of energy loss to the rotational modes is not a trivial matter as with ideal molecular gases. We implement this energy loss through an *effective coefficient of restitution*, $\bar{\epsilon}$, incorporating energy loss due to both particle inelasticity and rotation, as well as other factors, such as friction. For the purposes of this paper, $\bar{\epsilon}$ is calculated in a manner similar to that described by Feitosa and Menon [72]. Specifically, for each particle type—i.e., each combination of material and shape used in experiment—data corresponding to a monodisperse experimental system of such particles are analyzed. Particle velocities immediately preceding and immediately following collisions are then measured and used to provide an estimate of the relevant restitution coefficient. By recording a large number of such collisions, we estimate $\bar{\epsilon}$ as the mean value of the obtained distribution. It is hoped that, with further research, it will be possible to reliably determine a relationship between $\bar{\epsilon}$ and particles’ average moments of inertia, thus allowing a *truly predictive* model. The frictional coefficient, μ , is set

to zero due to the inclusion of frictional energy losses in the implemented values of $\bar{\epsilon}$; such an assumption should not prove problematic, as specific frictional effects are typically small in dilute systems such as those described here [73]. Other than these adaptations, all parameters, including f , A , N , ρ , L_x , L_y , and L_z , are taken as their experimental values.

The simulations show strong agreement with experimental data in terms of the qualitative forms of the profiles and their variation with differing system compositions (as may be seen in Figs. 5 and 8) as well as in terms of the I_s values achieved. Indeed, for all cases in which, throughout the system, the typical spacing between particle centers remains greater than the maximal effective particle radius, simulational and experimental I_s values were found to agree to within 10%. For denser systems, however, where excluded volume effects significantly influence segregation, simulation and experimental results, as expected, begin to diverge. The agreement observed nonetheless provides encouraging support for the possibility that existing theoretical and simulational models may indeed be adapted to account for and predict effects due to the geometrical properties of particles.

V. CONCLUSIONS

Through a combination of experimental measurements and discrete particle simulations, we have investigated the influence of particle geometry on the segregative behaviors of vibrofluidized granular systems. Our results provide strong experimental evidence that particles' radii of gyration may

significantly affect the mixing and separation of granular mixtures, as hypothesized in previous, simulational works. We demonstrate that the degree of geometrically induced segregation shows a strong dependence on the velocity and hence mean energy input from the system's vibrating base, but little or no dependence on the specific frequency, amplitude, or acceleration with which the system is driven. We also propose, and provide support for, the possibility that geometric effects may be accounted for through relatively simple adaptations of existing simulational and theoretical models, thus providing an important indication that models developed based on uniform systems of spheres may indeed be extended to more realistic systems—in other words, our results strongly suggest that the sphere approximation is indeed viable for systems such as those described here. Moreover, we provide evidence that for systems in which particles differ both in density and shape, effects due to geometry are decidedly secondary to those arising from mass differences. Although the current findings concern specifically dilute, strongly driven systems, this work nonetheless provides an important step toward a generalized understanding of the role of particle geometry in granular dynamics.

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- [1] J. Duran, *Sands, Powders, and Grains: An Introduction to the Physics of Granular Materials* (Springer-Verlag, New York, 2000).
 - [2] H. M. Jaeger, S. R. Nagel, and R. P. Behringer, *Rev. Mod. Phys.* **68**, 1259 (1996).
 - [3] C. H. Scholz, *Nature (London)* **391**, 37 (1998).
 - [4] R. M. Iverson, *Rev. Geophys.* **35**, 245 (1997).
 - [5] P. A. Johnson and X. Jia, *Nature (London)* **437**, 871 (2005).
 - [6] A. Khaldoun, E. Eiser, G. H. Wegdam, and D. Bonn, *Nature (London)* **437**, 635 (2005).
 - [7] I. M. P. Houghton, K. L. Aplin, and K. A. Nicoll, *Phys. Rev. Lett.* **111**, 118501 (2013).
 - [8] L. Sharma, J. T. Jenkins, and J. A. Burns, *Icarus* **183**, 312 (2006).
 - [9] M. H. Cooke, D. J. Stephens, and J. Bridgewater, *Powder Technol.* **15**, 1 (1976).
 - [10] A. D. Rosato, D. L. Blackmore, N. Zhang, and Y. Lan, *Chem. Eng. Sci.* **57**, 265 (2002).
 - [11] M. Schröter, S. Ulrich, J. Kreft, J. B. Swift, and H. L. Swinney, *Phys. Rev. E* **74**, 011307 (2006).
 - [12] K. Samiei, G. Berhe, and B. Peters, *Kona* **31**, 265 (2014).
 - [13] T. Börzsöenyi and R. Stannarius, *Soft Matter*, **9**, 7401 (2013).
 - [14] K. Ahmad and I. J. Smalley, *Powder Technol.* **8**, 69 (1973).
 - [15] S. J. Roskilly, E. A. Colbourn, O. Alli, D. Williams, K. A. Paul, E. H. Welfare, and P. A. Trusty, *Powder Technol.* **203**, 211 (2010).
 - [16] C. R. A. Abreu, F. W. Tavares, and M. Castier, *Powder Technol.* **134**, 167 (2003).
 - [17] R. Caulkin, X. Jia, M. Fairweather, and R. A. Williams, *Phys. Rev. E* **81**, 051302 (2010).
 - [18] K. R. LaMarche, M. J. Metzger, B. J. Glasser, and T. Shinbrot, *Phys. Rev. E* **81**, 052301 (2010).
 - [19] A. Rosato, K. J. Strandburg, F. Prinz, and R. H. Swendsen, *Phys. Rev. Lett.* **58**, 1038 (1987).
 - [20] J. B. Knight, H. M. Jaeger, and S. R. Nagel, *Phys. Rev. Lett.* **70**, 3728 (1993).
 - [21] S. C. Yang, *Powder Technol.* **164**, 65 (2006).
 - [22] R. Brito and R. Soto, *Eur. Phys. J. Spec. Top.* **179**, 207 (2009).
 - [23] D. Serero, I. Goldhirsch, S. H. Noskowitz, and M.-L. Tan, *J. Fluid Mech.* **554**, 237 (2006).
 - [24] N. Mohabuth, P. Hall, and N. Miles, *J. Miner. Eng.* **20**, 926 (2007).
 - [25] S. Sthiannopkao and M. H. Wong, *Sci. Total Environ.* **463**, 1147 (2013).
 - [26] N. Shishodia and C. R. Wassgren, *Phys. Rev. Lett.* **87**, 084302 (2001).
 - [27] D. U. Ringer and A. S. Mujumdar, *Drying Technol.* **2**, 449 (1983).
 - [28] S. Satija and I. L. Zucker, *Drying Technol.* **4**, 19 (1986).
 - [29] S. S. Hsiao, C. C. Liao, P. Y. Sheng, and S. C. Tai, *Exp. Fluids* **51**, 795 (2011).
 - [30] N. Burtally, P. J. King, and M. R. Swift, *Science* **295**, 1877 (2002).
 - [31] M. E. Möbius, X. Cheng, G. S. Karczmar, S. R. Nagel, and H. M. Jaeger, *Phys. Rev. Lett.* **93**, 198001 (2004).

- [32] X. Yan, Q. Shi, M. Hou, K. Lu, and C. K. Chan, *Phys. Rev. Lett.* **91**, 014302 (2003).
- [33] C. Zeilstra, M. A. van der Hoef, and J. A. M. Kuipers, *Phys. Rev. E* **77**, 031309 (2008).
- [34] Q. Shi, G. Sun, M. Hou, and K. Lu, *Phys. Rev. E* **75**, 061302 (2007).
- [35] D.-L. Liu, J. Martin, and N. A. Burnham, *Appl. Phys. Lett.* **91**, 043107 (2007).
- [36] M. H. Korayem and M. Zakeri, *Appl. Surf. Sci.* **257**, 6503 (2011).
- [37] S. Saito, H. T. Miyazaki, T. Sato, and K. Takahashi, *J. Appl. Phys.* **92**, 5140 (2002).
- [38] D. L. Blair and A. Kudrolli, *Phys. Rev. E* **67**, 041301 (2003).
- [39] S. Tatsumi, Y. Murayama, H. Hayakawa, and M. Sano, *J. Fluid Mech.* **641**, 521 (2009).
- [40] R. Fuchs, T. Weinhart, J. Meyer, H. Zhuang, T. Staedler, X. Jiang, and S. Luding, *Granular Matter* **16**, 281 (2014).
- [41] D. J. Parker, R. N. Forster, P. Fowles, and P. S. Takhar, *Nucl. Instrum. Methods Phys. Res., Sect. A* **477**, 540 (2002).
- [42] D. V. Khakhar, J. J. McCarthy, T. Shinbrot, and J. M. Ottino, *Phys. Fluids* **9**, 31 (1997).
- [43] C. R. K. Windows-Yule, T. Weinhart, D. J. Parker, and A. R. Thornton, *Phys. Rev. Lett.* **112**, 098001 (2014).
- [44] C. R. K. Windows-Yule and D. J. Parker, https://www.jstage.jst.go.jp/article/kona/advpub/0/advpub_2015004/_article.
- [45] R. D. Wildman, J. M. Huntley, J.-P. Hansen, D. J. Parker, and D. A. Allen, *Phys. Rev. E* **62**, 3826 (2000).
- [46] R. D. Wildman, J. M. Huntley, and D. J. Parker, *Phys. Rev. E* **63**, 061311 (2001).
- [47] R. D. Wildman, J. M. Huntley, and D. J. Parker, *Phys. Rev. Lett.* **86**, 3304 (2001).
- [48] R. D. Wildman and D. J. Parker, *Phys. Rev. Lett.* **88**, 064301 (2002).
- [49] C. R. K. Windows-Yule, T. Weinhart, D. J. Parker, and A. R. Thornton, *Phys. Rev. E* **89**, 022202 (2014).
- [50] A. R. Thornton, D. Krijgsman, A. te Voortwis, V. Ogarko, S. Luding, R. Fransen, S. Gonzalez, O. Bokhove, O. Imole, and T. Weinhart, in *DEM 6: Proceedings of the 6th International Conference on Discrete Element Methods and Related Techniques, 2013* (Colorado School of Mines, Colorado, 2013), p. 393.
- [51] A. R. Thornton, T. Weinhart, S. Luding, and O. Bokhove, *Int. J. Mod. Phys. C* **23**, 1240014 (2012).
- [52] A. R. Thornton, T. Weinhart, V. Ogarko, and S. Luding, *Comput. Methods Mater. Sci.* **13**, 197 (2013).
- [53] MercuryDPM.org
- [54] T. Weinhart, A. R. Thornton, S. Luding, and O. Bokhove, *Granular Matter* **14**, 531 (2012).
- [55] S. Luding, *Eur. J. Environ. Civil Eng.* **12**, 785 (2008).
- [56] A. D. Rosato and D. Yacoub, *Powder Technol.* **109**, 255 (2000).
- [57] J. Talbot and P. Viot, *Phys. Rev. Lett.* **89**, 064301 (2002).
- [58] C. R. K. Windows-Yule, N. Rivas, and D. J. Parker, *Phys. Rev. Lett.* **111**, 038001 (2013).
- [59] R. Verma, J. C. Crocker, T. C. Lubensky, and A. G. Yodh, *Phys. Rev. Lett.* **81**, 4004 (1998).
- [60] P. C. Hiemenz and R. Rajagopalan, *Principles of Colloid and Surface Chemistry* (Marcel Dekker, New York, 1997).
- [61] S. Luding, H. J. Herrmann, and A. Blumen, *Phys. Rev. E* **50**, 3100 (1994).
- [62] S. McNamara and S. Luding, *Phys. Rev. E* **58**, 2247 (1998).
- [63] V. Kumaran, *Phys. Rev. E* **57**, 5660 (1998).
- [64] J. A. Dijksman and M. van Hecke, *Europhys. Lett.* **88**, 44001 (2009).
- [65] P. Sunthar and V. Kumaran, *Phys. Rev. E* **64**, 041303 (2001).
- [66] R. Ramírez and R. Soto, *Physica A* **322**, 73 (2003).
- [67] J. E. Galvin, S. R. Dahl, and C. M. Hrenya, *J. Fluid Mech.* **528**, 207 (2005).
- [68] J. J. Brey, M. J. Ruiz-Montero, and F. Moreno, *Phys. Rev. Lett.* **95**, 098001 (2005).
- [69] I. Goldhirsch, *Powder Technol.* **182**, 130 (2008).
- [70] C. R. K. Windows-Yule, B. Maddox, and D. J. Parker, *Europhys. Lett.* **108**, 58006 (2014).
- [71] T. A. Knight and L. V. Woodcock, *J. Phys. A* **29**, 4365 (1996).
- [72] K. Feitosa and N. Menon, *Phys. Rev. Lett.* **88**, 198301 (2002).
- [73] C. C. Liao, S. S. Hsiao, and C. S. Wu, *Phys. Rev. E* **86**, 061316 (2012).