Proofs for Some Conjectures of Rajaratnam and Takawira on the Peakedness of Handoff Traffic

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Abstract—**The purpose of this paper is to supplement a recent paper by Rajaratnam and Takawira, which deals with a model for the performance analysis of cellular mobile networks. The key performance measure is a second-order characteristic (peakedness) of the traffic stream that serves as a model for handoff traffic. We show that this quantity may be obtained by evaluating an explicit formula rather than by solving a set of equations. This result enables us to verify some conjectures formulated by Rajaratnam and Takawira on the basis of numerical experiments. We also show the uniqueness of the solution to a system of nonlinear equations, required in the performance analysis, as conjectured by Rajaratnam and Takawira.**

Index Terms—**Carried traffic, cellular mobile network, handoff traffic, output process, output traffic, peakedness factor, smooth traffic, tandem system.**

I. INTRODUCTION

IN a series of papers [1]–[5], Rajaratnam and Takawira have proposed a method for analyzing the performance of cellular mobile networks. At the heart of their method, and more specifically at the heart of their cell-traffic model, lies the simple tandem service system depicted in Fig. 1. It consists of a first cell containing N servers and a second cell with infinitely many servers. Customers (calls) arrive at the first cell according to a Poisson process with intensity λ. If, upon arrival of a call, at least one of the servers in the first cell is free, the call seizes an arbitrary free server and keeps it occupied during the call's service time; if all servers in the first cell are busy, the call is lost. After being served in the first cell, a call leaves the system with probability $1-p$ and is transferred with probability p to the second cell, where it is served for a second time. Service times are mutually independent random variables and also independent of the arrival process; the service times in cell i are identically distributed with mean μ_i^{-1}, $i = 1, 2$.

The stream of calls entering the second cell models the *handoff traffic* from a cell in a cellular network and is characterized by the mean $M_h(p)$ and variance $V_h(p)$ of the stationary number of occupied servers in the second cell. (It will be convenient to explicitly indicate dependence on p.) The question is how these quantities depend on the parameters of the model and, more concretely, whether explicit expressions for $M_h(p)$ and $V_h(p)$ can be given, perhaps in specific settings. Rajaratnam and Takawira have obtained partial solutions to these problems. Also, they have formulated a number of conjectures on the basis of numerical experiments. We will now briefly describe their findings and the supplementary results we intend to establish in this paper.

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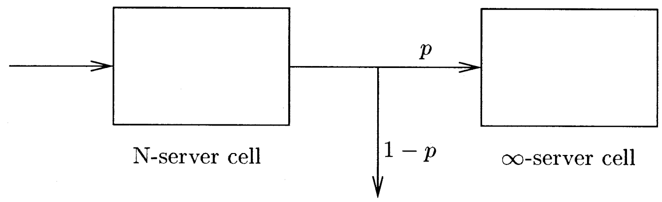

Fig. 1. A tandem service system.

For future reference, we note that the mean $M_c \equiv M_c(N, a)$ and variance $V_c \equiv V_c(N, a)$ of the stationary number of occupied servers in the first cell (known as the mean and variance of the *carried traffic*) are given by

$$M_c = a(1 - B) \tag{1}$$
$$V_c = M_c - (a - M_c)(N - M_c) \tag{2}$$

where $a \equiv \lambda/\mu_1$ and $B \equiv B(N, a)$ denotes the Erlang loss function

$$B(N, a) \equiv \frac{\frac{a^N}{N!}}{1 + a + \frac{a^2}{2!} + \ldots + \frac{a^N}{N!}} \tag{3}$$

(see, for example, [6]).

It is shown in [4] and [5] that

$$M_h(p) = pM_h(1) \tag{4}$$
$$\frac{V_h(p)}{M_h(p)} = 1 - p\left(1 - \frac{V_h(1)}{M_h(1)}\right) \tag{5}$$

so that the problem of calculating $M_h(p)$ and $V_h(p)$ reduces to that of finding $M_{\text{out}} \equiv M_h(1)$ and $V_{\text{out}} \equiv V_h(1)$, the index "out" being mnemonic for *output traffic*. Evidently, by Little's formula

$$M_{\text{out}} = \frac{\lambda(1 - B)}{\mu_2} = \left(\frac{\mu_1}{\mu_2}\right) M_c \tag{6}$$

but a relation between V_{out} and V_c has been established only in the case that the service times in each cell are deterministic and equal, namely, $V_{\text{out}} = V_c$ (see [3]).

Assuming that the service times are exponentially distributed, Rajaratnam and Takawira [4] show that V_{out} may be obtained by solving a set of $N+1$ simultaneous equations. On the basis of numerical experiments, they subsequently conjecture that the *peakedness (factor)* $Z_{\text{out}} \equiv V_{\text{out}}/M_{\text{out}}$ of the output traffic satisfies

$$Z_{\text{out}} < 1 \tag{7}$$
$$Z_{\text{out}} \to 1 \text{ as } a \downarrow 0 \text{ or } a \to \infty. \tag{8}$$

In Section II, we shall obtain an explicit expression for V_{out}, which will enable us in Section III to prove the conjectured properties of Z_{out}. It then follows immediately from (5) that the peakedness factor $Z_h(p) \equiv V_h(p)/M_h(p)$ of the handoff traffic has similar properties.

As part of their (approximative) performance analysis, Rajaratnam and Takawira require a solution (a, N) to the system of equations $m = M_c(N, a)$ and $v = V_c(N, a)$. Evidently, such a solution will not exist in general if one requires N to be an integer. However, it is conceptually and computationally easy to continue the Erlang loss function $B(N, a)$ to nonintegral values of the first argument by letting

$$B(x,a) \equiv \left\{ a \int_0^\infty e^{-at}(1+t)^x dt \right\}^{-1}, \quad x \geq 0 \tag{9}$$

(see, for example, [7] or [8]). With this interpretation of the Erlang loss function, Rajaratnam and Takawira [4] conjecture that there is a unique solution (a, x) to the system of equations

$$m = a - aB(x,a) \tag{10}$$

$$v = m - (a-m)(x-m) \tag{11}$$

for given values of m and v such that $0 < v < m$. In Section IV, we will verify this conjecture. (A similar problem involving *overflow traffic* was solved in [8].)

We conclude this introduction with the observation that the proof of (5) in [4] does not depend on the particular type of traffic involved. That is, the effect on peakedness of *thinning* an arbitrary traffic stream by accepting each call with probability p is expressed by

$$Z(p) = 1 - p(1-Z) \tag{12}$$

where Z and $Z(p)$ denote the peakedness factors of the original and the thinned stream, respectively. This relation was observed earlier in the context of exponential service times in [9].

II. Variance of the Output Traffic

Since service times are assumed to be exponentially distributed, the number of calls in the first cell is a birth–death process, and hence reversible (see, for example, Kelly [10]). It follows that, in equilibrium, the output process of the first cell is stochastically identical to the input process, that is, the arrival process of served calls. (The input process together with the *overflow process*—the arrival process of lost calls—constitute the arrival process to the first cell.) Calculating the variance V_{out} of the output traffic is therefore equivalent to computing the variance V_{fc} of the *freed carried traffic*, that is, the variance of the stationary number of occupied servers when a copy of the input process to the first cell is offered to a hypothetical infinite-size cell in which each call is served during an exponentially distributed service time (of mean μ_2) that is independent of its service time in the first cell.

Freed carried traffic has recently been studied in the more general setting of renewal arrival processes by Brandt and Brandt [11]. Specifying their results for the setting at hand, these authors present in [11, (32)] an explicit formula for V_{fc}, which, in our notation, reads as

$$V_{fc} = M_{fc} \left\{ 1 - \frac{B}{2(1-B)} \left(N + aB - M_c - \frac{a}{N+1-M_c} \right) \right\}. \tag{13}$$

Here, as before, $a = \lambda/\mu_1$, $B \equiv B(N,a)$ is the Erlang loss function (3), and M_c is the mean of the carried traffic given in (1). In addition, M_{fc} is the mean of the freed carried traffic (the mean number of occupied servers in the hypothetical cell). Clearly, by Little's law, $M_{fc} = \lambda(1-B)/\mu_2$, so that $M_{fc} = M_{\text{out}}$ in view of (6). Since $V_{\text{out}} = V_{fc}$, the solution to our problem of determining V_{out} can now be given as follows.

Theorem 1: The variance V_{out} of the output traffic can be represented as

$$V_{\text{out}} = M_{\text{out}} \left\{ 1 - \frac{B}{2(1-B)} \left(Y + aB - \frac{a}{Y+1} \right) \right\} \tag{14}$$

where $Y \equiv N - M_c$.

We note that Y can be interpreted as the mean number of free servers in the first cell, so that $Y > 0$.

For some specific parameter values, we have compared the results of evaluating (14) with the results that Rajaratnam and Takawira [4, Fig. 7] obtained by solving a system of linear equations, and found complete agreement.

III. Proofs for the Peakedness Conjectures

Theorem 1 tells us that the peakedness factor $Z_{\text{out}} \equiv V_{\text{out}}/M_{\text{out}}$ of the output traffic is given by

$$Z_{\text{out}} = 1 - \frac{B}{2(1-B)} \left(Y + aB - \frac{a}{Y+1} \right). \tag{15}$$

We can now prove the following theorem, verifying conjecture (8).

Theorem 2: Let N be a fixed positive integer; then $Z_{\text{out}} \to 1$ as $a \downarrow 0$ or $a \to \infty$.

Proof: It is obvious that if $a \downarrow 0$, then also $B \equiv B(N,a) \to 0$, while $Y \equiv N - M_c = N - a(1-B) \to N$. Hence the first part follows immediately from (15).

To prove the second part, we note from (3) that, as $a \to \infty$, we have

$$\frac{1}{B} = 1 + \frac{N}{a} + \frac{N(N-1)}{a^2} + \frac{N(N-1)(N-2)}{a^3} + \mathcal{O}\left(\frac{1}{a^4}\right). \tag{16}$$

As a consequence

$$\begin{aligned} \frac{Y}{B} + a &= \frac{N-a}{B} + 2a \\ &= a + \frac{N}{a} + \frac{2N(N-1)}{a^2} + \mathcal{O}\left(\frac{1}{a^3}\right) \\ \frac{Y+1}{B} &= \frac{N-a+1}{B} + a \\ &= 1 + \frac{2N}{a} + \frac{3N(N-1)}{a^2} + \mathcal{O}\left(\frac{1}{a^3}\right) \\ \left(\frac{a}{B}\right)^2 &= a^2 + 2aN + 3N^2 - 2N + \mathcal{O}\left(\frac{1}{a}\right) \end{aligned}$$

so that

$$a\left(\frac{Y}{B}+a\right)\left(\frac{Y+1}{B}\right)-\left(\frac{a}{B}\right)^2=\mathcal{O}\left(\frac{1}{a}\right). \qquad (17)$$

Evidently, if $a \to \infty$, then $B \to 1$ and $Y \to 0$, while, in view of (16), $a(1-B)/B \to N$. Since we can rewrite (15) as

$$Z_{\text{out}}=1-\frac{B^2}{2(Y+1)}\frac{B}{a(1-B)} \times\left(a\left(\frac{Y}{B}+a\right)\left(\frac{Y+1}{B}\right)-\left(\frac{a}{B}\right)^2\right)$$

these facts together with (17) imply that $Z_{\text{out}} \to 1$ as $a \to \infty$, as required. □

It requires more effort to prove the next theorem, which verifies conjecture (7).

Theorem 3: For any positive integer N and real $a > 0$, we have $Z_{\text{out}} < 1$.

Proof: We define

$$H \equiv \sum_{i=0}^{N}\frac{a^i}{i!}$$

(so that $HB = a^N/N!$) and let

$$C \equiv H^2\left\{(Y+aB)(Y+1)-a\right\}. \qquad (18)$$

Substitution of $Y = N - a(1-B)$ shows us that

$$C = H^2\left\{2a^2B^2-\left(3a^2-a(3N+2)\right)B +a^2-2a(N+1)+N(N+1)\right\}. \qquad (19)$$

Clearly, C is a polynomial in a of maximum degree $2N+2$, so that we can write

$$C \equiv C(a)=\sum_{i=0}^{2N+2}c_i a^i. \qquad (20)$$

We see from (15) and (18) that

$$Z_{\text{out}}<1 \iff (Y+aB)(Y+1)>a \iff C>0 \qquad (21)$$

and we will prove that $C > 0$ by showing that each coefficient c_i in (20) is nonnegative, while $c_0 > 0$.

We first note from (19) that $c_0 = N(N+1) > 0$, as required. Let us next assume that $0 < i \le N$. It then follows from (19) that

$$\begin{aligned}
c_i &= \sum_{k+l=i-2}\frac{1}{k!l!}-\sum_{k+l=i-1}\frac{2(N+1)}{k!l!}+\sum_{k+l=i}\frac{N(N+1)}{k!l!}\\
&\ge \sum_{k=0}^{i}\frac{N(N+1)}{k!(i-k)!}-\sum_{k=0}^{i-1}\frac{2(N+1)}{k!(i-k-1)!}\\
&= \sum_{k=0}^{i}\frac{(N+1)(N-2i+2k)}{k!(i-k)!}\\
&= \sum_{k=0}^{i}\frac{(N+1)(N-i)}{k!(i-k)!}-\sum_{k=0}^{i-1}\frac{N+1}{k!(i-k-1)!}\\
&\quad +\sum_{k=1}^{i}\frac{N+1}{(k-1)!(i-k)!}\\
&= \sum_{k=0}^{i}\frac{(N+1)(N-i)}{k!(i-k)!}\ge 0.
\end{aligned}$$

The following step is to observe from (19) that

$$\begin{aligned}
c_{N+1} &= \frac{3N+2}{N!}+\sum_{k=0}^{N-1}\frac{1}{k!(N-k-1)!}-\sum_{k=0}^{N}\frac{2(N+1)}{k!(N-k)!}\\
&\quad +\sum_{k=1}^{N}\frac{N(N+1)}{k!(N-k+1)!}\\
&= \frac{N}{N!}+\sum_{k=0}^{N-1}\left(\frac{1}{k!(N-k-1)!}-\frac{2(N+1)}{k!(N-k)!}\right.\\
&\qquad\left.+\frac{N(N+1)}{(k+1)!(N-k)!}\right)\\
&= \frac{N}{N!}+\sum_{k=0}^{N-1}\frac{N(N-k)-(k+1)(k+2)}{(k+1)!(N-k)!}\\
&= \frac{N}{N!}+\sum_{k=1}^{N}\frac{N}{k!(N-k)!}-\sum_{k=0}^{N-1}\frac{k+2}{k!(N-k)!}\\
&= \frac{N}{N!}+\sum_{k=1}^{N}\frac{N}{k!(N-k)!}-\sum_{k=1}^{N}\frac{N-k+2}{k!(N-k)!}\\
&= \sum_{k=2}^{N}\frac{k-2}{k!(N-k)!}\ge 0.
\end{aligned}$$

We subsequently obtain from (19) by straightforward calculations that

$$c_{2N+2}=c_{2N+1}=c_{2N}=0$$

so it remains to check whether $c_i \ge 0$ for $N+1 < i < 2N$. To this end, we first derive a representation for c_{N+j}, $1 < j \le N$. Namely, from (19), we have

$$\begin{aligned}
c_{N+j}-\frac{N-j+1}{N!(j-1)!} &= -\frac{3}{N!(j-2)!}+\frac{3N+2}{N!(j-1)!}\\
&\quad +\sum_{k=j-2}^{N}\frac{1}{k!(N+j-k-2)!}\\
&\quad -\sum_{k=j-1}^{N}\frac{2(N+1)}{k!(N+j-k-1)!}\\
&\quad +\sum_{k=j}^{N}\frac{N(N+1)}{k!(N+j-k)!}-\frac{N-j+1}{N!(j-1)!}\\
&= \sum_{k=j-1}^{N-1}\frac{1}{k!(N+j-k-2)!}\\
&\quad -\sum_{k=j-1}^{N-1}\frac{2(N+1)}{k!(N+j-k-1)!}\\
&\quad +\sum_{k=j-1}^{N-1}\frac{N(N+1)}{(k+1)!(N+j-k-1)!}\\
&= \sum_{k=j-1}^{N-1}\frac{1}{k!(N+j-k-2)!}\\
&\quad -\sum_{k=j-1}^{N-1}\frac{N+1}{k!(N+j-k-1)!}
\end{aligned}$$

$$
\begin{aligned}
&+\sum_{k=j-1}^{N-1}\frac{(N+1)(N+j-k-1)}{(k+1)!(N+j-k-1)!}\\
&-\sum_{k=j-1}^{N-1}\frac{j(N+1)}{(k+1)!(N+j-k-1)!}\\
=&\sum_{k=j-1}^{N-1}\frac{1}{k!(N+j-k-2)!}\\
&-\sum_{k=j-1}^{N-1}\frac{j(N+1)}{(k+1)!(N+j-k-1)!}\\
=&\sum_{k=j-1}^{N-1}\frac{(k-j+1)(N-k-1)-j}{(k+1)!(N+j-k-1)!}
\end{aligned}
$$

so that

$$c_{N+j}=\frac{N-j+1}{N!(j-1)!}+\sum_{l=0}^{N-j}\frac{l(N-j-l)-j}{(j+l)!(N-l)!},\quad 1<j\le N.$$

Using this result twice, we obtain for $2<j\le N$

$$
\begin{aligned}
c_{N+j-1}&=\frac{N-j+2}{N!(j-2)!}+\sum_{l=0}^{N-j+1}\frac{l(N-j-l+1)-j+1}{(j+l-1)!(N-l)!}\\
&=\frac{N-j+1}{N!(j-2)!}+\sum_{l=0}^{N-j}\frac{l(N-j-l+1)-j+1}{(j+l-1)!(N-l)!}\\
&=(j-1)\left(c_{N+j}-\sum_{l=0}^{N-j}\frac{l(N-j-l)-j}{(j+l)!(N-l)!}\right)\\
&\quad+\sum_{l=0}^{N-j}\frac{l(N-j-l+1)-j+1}{(j+l-1)!(N-l)!}\\
&=(j-1)c_{N+j}+\sum_{l=0}^{N-j}\frac{l(l+1)(N-j-l+1)}{(j+l)!(N-l)!}\\
&\ge(j-1)c_{N+j}.
\end{aligned}
$$

Since $c_{2N}=0$, it now follows by induction that we have $c_i\ge 0$ for $i=2N, 2N-1,\ldots,N+2$. Thus we have shown that all coefficients in (20) are nonnegative while $c_0>0$. As a result, we have $C>0$ and hence, by (21), $Z_{\rm out}<1$. □

We conclude this section with some remarks. First, the validity of Theorem 3 was claimed earlier by Kirstein [12, (4.11)], but, as explained in a correction to [12] in [13], his proof is in error. Then, recalling that the peakedness factor $Z_{\rm ov}$ of the overflow traffic satisfies

$$Z_{ov}=1-aB+\frac{a}{Y+1} \tag{22}$$

(see, for example, [6]), we note that (15) and Theorem 1 yield the interesting upper bound

$$Z_{\rm ov}<1+Y \tag{23}$$

supplementing the well-known lower bound $Z_{\rm ov}>1$ (see, for example, [8]). Conversely, the inequality $Z_{\rm ov}>1$ readily leads to the lower bound

$$Z_{\rm out}>1-\frac{BY}{2(1-B)}.$$

Since, by (1) and (2), the peakedness of the carried traffic is given by

$$Z_c\equiv\frac{V_c}{M_c}=1-\frac{BY}{1-B} \tag{24}$$

this bound can nicely be expressed as

$$Z_{\rm out}>\frac{1}{2}(1+Z_c). \tag{25}$$

IV. Uniqueness of a Solution

As announced, we will prove in this section the following theorem.

Theorem 4: For given values of m and v such that $0<v<m$, the system of (10) and (11) has a unique solution (a,x) such that $a>m$ and $x>0$.

Proof: Writing $\alpha\equiv(a-m)^{-1}$ and solving (11) for x, we find

$$x=m+(m-v)\alpha.$$

Hence, letting

$$f(x,a)\equiv\{aB(x,a)\}^{-1}=\int_0^\infty e^{-at}(1+t)^x dt$$

we can reformulate our problem as that of establishing that there is a unique positive solution to the single equation

$$g(\alpha)=\alpha$$

where

$$g(\alpha)\;\equiv f(m+(m-v)\alpha, m+\alpha^{-1}).$$

It is straightforward to verify that, under our assumption $0<v<m$, we have $g'(\alpha)<0$ for $\alpha>0$. Since $g(\alpha)\to\infty$ as $\alpha\downarrow 0$ and $g(\alpha)\to 0$ as $\alpha\to\infty$, the required result follows immediately. □

We finally note that uniqueness of the solution was claimed already by Katz in [14], but no proof was given. We must add that Katz uses a logarithmic interpolation formula rather than (9) to compute the Erlang loss function for a nonintegral number of servers.

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