Design of a 30 GHz Bragg reflector for a Raman FEL

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A design of a Bragg reflector for a Raman FEL is described. It is shown that mode conversion occurs whenever the axial wavenumbers of the two modes fulfil the Bragg condition. With a constant ripple of the corrugation it is shown that the reflected radiation also contains higher order modes, assuming that the incident radiation consists only of a TE_{11} mode. The mode purity can be increased by increasing the length of the reflector at the expense of a smaller reflection bandwidth. A more flexible method is by applying a Hamming window to the corrugation of the reflector. Contributions of other modes to the reflected radiation can in that case be neglected. The reflector will be installed in a Raman laser to be able to compare the amplifier with the oscillator configuration. Therefore some preliminary results are also presented about the start-up of the Raman laser.

1. Introduction

The Raman type FEL situated at the University of Twente has so far been operated in an amplifier configuration with a maximum current I_b of the electron beam of about 900 A maximum [1,2]. In order to compare the behaviour of the laser in an amplifier and oscillator configuration the current of the electron beam has been reduced to about 250 A maximum [3]. With this current the laser can be operated with the two configurations.

Several choices for the mirrors for the oscillator are available. As the beam line contains no bends, the mirror situated at the upstream side of the undulator must contain a hole. The electron beam is dumped in the waveguide wall before it reaches the mirror at the downstream side of the undulator. Since the laser operates at wavelengths in the mm-region, one option for the downstream mirror is a mesh. Another option is to use a plate with a hole in it through which the electron beam passes. In the former case the electron beam is influenced by the mesh, but for the radiation field it is almost a perfect reflecting mirror, whereas in the latter case the radiation field is modified by the hole and the electron beam is hardly influenced. The effects of this hole can be reduced by increasing the area of the mirror, keeping the hole diameter constant. However, one then needs to taper the waveguide to the

outer diameter of the mirror to avoid impedance mismatch.

A drawback of these type of mirrors is that there is no mode selectivity and it is difficult to change the reflectivity. This can be overcome by using a Bragg type reflector consisting of a section of waveguide with a corrugated wall (see e.g. ref. [4] for an introduction). The electron beam will be undisturbed, and the properties of the mirror, such as reflected power, spectral width of reflection band and centre frequency of reflection band, can be controlled by changing parameters as length of the mirror, corrugation height and period of the corrugation. It will be shown that, by applying a spatial filter, the mode purity of the reflected radiation can be improved.

The space available inside the axial guide magnet forced us to choose a plate with a hole as the upstream mirror. For the downstream mirror a Bragg reflector is chosen because it gives the designer a flexibility not offered by the other type of mirrors.

2. Design of a Bragg reflector

The theory of Bragg reflectors is well known [4–6] and only the coupled mode equations will be given here. Consider a section of waveguide with a corrugated wall for which the radius is given by

$$r = r_0 + l_0 \cos(k_b z + \phi_0), \quad (1)$$

where r_0 is the mean radius of the waveguide, l_0 is the amplitude of the corrugation, $k_b = 2\pi/\lambda_b$, λ_b being

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the periodicity of the corrugation, and ϕ_0 is an arbitrary starting phase. The coupled mode equations for the cylindrical waveguide Bragg reflector are [5,6]

$$\frac{\partial f_i^+}{\partial z} = -j\Delta_{ii}f_i^+ + jG_{ii}f_i^- - j\sum_p H_{ip}f_{E,p}^- + j\sum_p G_{ip}f_{H,p}^-, \quad (2)$$

$$\frac{\partial f_i^-}{\partial z} = j\Delta_{ii}f_i^- - jG_{ii}f_i^+ + j\sum_p H_{ip}f_{E,p}^+ - j\sum_p G_{ip}f_{H,p}^+, \quad (3)$$

where

$$\Delta_{ii} = k_z - k_b/2, \quad (4)$$

k_z being the axial wavenumber of the incident mode. Since the corrugation is azimuthally symmetric, the Bragg reflector will not couple modes with different azimuthal indices [4]. The indices in the above equations refer to the radial indices of the modes. f_i^+ is the amplitude of the (incident) waveguide mode propagating in the forward direction and f_i^- is the amplitude of the backward propagating (reflected) wave. G_{ii} , G_{ip} , and H_{ip} are the coupling coefficient between the forward and backward components of the incident mode, the coupling coefficient between two TE or TM modes and the cross-coupling coefficient between a TE (H) and TM (E) mode respectively. Expressions for these coefficients can be found in refs. [5,6]. Solving the system of equations given by (2) and (3) from $z = 0$ to $z = L$, L being the length of the mirror, one finds the mode amplitudes from which the reflection R and

transmission T coefficients for each mode can be calculated

$$R = |f_i^-(z=0)|^2 / |f_i^+(z=0)|^2, \quad (5)$$

$$T = |f_i^+(z=L)|^2 / |f_i^+(z=0)|^2. \quad (6)$$

The phase shift of the reflected (ϕ_r) and transmitted (ϕ_t) components of a waveguide mode can also be obtained from the amplitudes f_i^+ and f_i^- ,

$$\phi_t = -\tan^{-1}(f_{im}^+(z=L)/f_{re}^+(z=L)), \quad (7)$$

$$\phi_r = -\tan^{-1}(f_{im}^-(z=0)/f_{re}^-(z=0)), \quad (8)$$

where the subscripts im and re stand for imaginary and real part of the complex amplitude.

In the Raman laser the FEL interaction basically takes place with the TE_{11} mode, though cyclotron instabilities can result in interaction with other, higher order, modes [2]. The Bragg reflector must therefore reflect the TE_{11} mode and preferably none of the other modes. The central frequency, chosen to be 30 GHz, is given by the Bragg condition,

$$k_z = k_b/2. \quad (9)$$

The period of the corrugation for reflection of the TE_{11} mode at 30 GHz thus becomes $\lambda_b = 5.37$ mm.

The system of equations (2) and (3) can be solved for any number of modes, but for practical reasons (i.e. for reasonable computing time) the number of modes simultaneously present is limited to two in the (modified) code [5] used. The incident mode at $z = 0$ is assumed to be a pure TE_{11} mode whereas the other mode is chosen to be the TM_{11} mode. The reflector is matched at $z = L$, i.e. no waves are reflected at the end of the mirror. The power reflection coefficient is shown in Fig. 1a for the case of $L = 120$ mm and $l_0 = 0.5$ mm. The transmission coefficient is shown in

Fig. 1. Power reflection coefficient (a) and transmission coefficient (b) for the TE_{11} mode (solid line) and TM_{11} mode (dashed line) for the Bragg reflector versus frequency. The incident mode is assumed to be a pure TE_{11} mode. The parameters of the Bragg reflector are $\lambda_b = 5.37$ mm, $l_0 = 0.5$ mm and $L = 120$ mm.

Fig. 1b. It can clearly be seen that the incident TE_{11} mode is primarily reflected as a TE_{11} mode at 30 GHz (FWHM = 1.7 GHz) and as a TM_{11} mode around 33.4 GHz (FWHM = 2.4 GHz). Note that the Bragg reflector acts as a mode converter at this frequency which is given by the Bragg condition

$$k_z^{TE} + k_z^{TM} = k_B. \quad (10)$$

At the design frequency of 30 GHz not all the power is reflected in the TE_{11} mode because of the side lobes of the TM_{11} reflection. Almost all the transmitted power is in the TE_{11} mode, while only a minor fraction is transmitted in the TM_{11} mode (see Fig. 1b) ^{#1}. The side lobes may be undesired in high gain FELs. One way to decrease the effects of side lobes is to make the reflection band around the centre frequency smaller, by increasing the length of the mirror, so that overlap between reflections will be less.

A different way to decrease or even remove the effects of side lobes and to obtain high mode purity is to apply a Hamming window to the corrugated part of the mirror. This is particularly interesting for overmoded waveguides where higher order modes are not damped. This type of waveguide is often used in FELs because propagation of the electron beam requires waveguide dimensions corresponding to overmoded operation for the radiation generated. A Hamming window can be applied to the Bragg reflector by modifying the corrugation height l according to

$$l = l_0 \left[0.54 - 0.46 \cos \left(\frac{2\pi z}{L} \right) \right], \quad (11)$$

where L is the length of the corrugated section which starts at $z = 0$.

The Raman FEL for which this mirror is designed has an electron beam duration of 100 ns. The radiation field can therefore only have about 6 round trips during which the electron beam is present. As the system starts from noise, and as we want to investigate the difference between an amplifier and oscillator configuration it was decided to use a power reflectivity around the 40% for the TE_{11} mode at the design frequency of 30 GHz. The single pass gain will be changed in the future by varying the length of the undulator. Again the system of coupled equations is solved with the Hamming window applied to the reflector with a corrugation amplitude of 0.2 mm. The power reflection coefficient is shown in Fig. 2. The side lobes have disappeared and the reflection bands for the two modes are completely separated. Even for the 0.5 mm ripple reflector one finds that the two modes are still

^{#1} This is a general property of the reflector, i.e. the mode of the transmitted radiation is basically the incident mode.

Fig. 2. Power reflection coefficient for the TE_{11} and TM_{11} mode. The parameters of the Bragg reflector are $\lambda_b = 5.37$ mm, $l_0 = 0.2$ mm and $L = 270$ mm (solid line) and $l_0 = 0.17$ mm (dashed line). The crosses indicate measured values of the reflection coefficient.

completely separated if the Hamming window is applied.

3. Measurement of reflection properties of the Bragg reflector

A Bragg reflector with $\lambda_b = 5.37$ mm, $l_0 = 0.2$ mm and $L = 270$ mm has been constructed with the Hamming window (7) applied to the ripple. The reflection properties are measured in order to validate the calculations made. The set-up is schematically shown in Fig. 3. A Hewlett Packard model HP8690B K_a-band sweep generator (26.5–40.0 GHz) and a model HP8755C amplitude analyser are used to generate the input power and analyse the forward and backward propagating power which are measured using one-directional couplers. The Raman FEL utilises a cylindrical waveguide.

Fig. 3. Schematic overview of the set-up used for measuring the power reflection of the Bragg reflector.

Therefore a rectangular to cylindrical waveguide transition is needed together with a cylindrical taper to switch from standard K_a-band dimensions to the custom size of the Bragg reflector. The rectangular to cylindrical transition converts the rectangular TE₁₀ waveguide mode to a cylindrical TE₁₁ mode. Higher order modes present in the cylindrical section of the measurements will in principle not couple to the fundamental TE₁₀ rectangular waveguide mode as they are reflected at the transition. However the Bragg reflector is reversible, i.e. if at a frequency f_0 an incoming TE₁₁ mode is reflected into a TM₁₁ mode then at the same frequency an incoming TM₁₁ mode will be reflected in a TE₁₁ mode. Thus higher order modes can be measured with this setup, though the reflected power will depend on the double mode conversion that has taken place.

One side of the mirror is connected to the cylindrical taper while at the other side an outcouple horn (used in the Raman FEL) is mounted, i.e. the reflector is matched at that side and there are no reflections. Spurious reflections inside the measurement system limited the dynamic range to about 20 dB. The results are also shown in Fig. 2. The centre frequencies of the two reflection bands clearly coincide with the calculated ones. The total reflectivity measured is somewhat less than calculated. The amplitude ripple used in the calculation was 0.2 mm. The peak reflectivity at 30 GHz is reduced to 25% for a corrugation height of 0.15 mm whereas it is 30% for $l_0 = 0.17$ mm. The calculated reflection coefficient for the latter value is also shown in Fig. 2. For the TE₁₁ mode good agreement is found between the measured values and the calculation for $l_0 = 0.17$ mm. The double mode conversion, in the experiment, for the TM₁₁ mode should reduce the measured values to 87% of the theoretical ones. One can thus state that within the mechanical limits of making the corrugation, good agreement is found between the calculations and the measurements for the TE₁₁ and the TM₁₁ modes.

4. Start-up of the Raman FEL

In this section some preliminary results will be presented obtained with a numerical code. The code used is basically the one described in ref. [7]. The radius of the electron beam is such that the assumption of perfect orbits and that of neglecting the radial dependence of the undulator field is not valid. The model has been modified to include the full three dimensional electron orbits. The current version ^{#2}

^{#2} The model will be described in more details in a future publication.

Fig. 4. The logarithmic of the power (solid line) and the radiation phase (dashed line) as a function of the distance along the undulator. The initial power is assumed to be zero while the electrons are initialised with a uniform phase distribution with a small random disturbance (TDA-code). The main simulation parameters are $I = 200$ A, $B_u = 0.18$ T, $\lambda_u = 0.03$ m, $B_z = 1.0$ T, $\epsilon_n = 10^{-5} \pi$ mrad and $\gamma = 2$.

assumes an ideal helical undulator field. For now particular attention has been given to the start-up of the laser. Therefore the initial radiation amplitude is assumed to be zero in the calculations and the laser starts from noise in the electron beam. Two methods of electron beam initialisation have been used. One is the initialisation used by the FRED code and the other is used in the TDA code [8]. The TDA-initialisation has been modified to start with zero radiation field and a uniform phase distribution modified by a small random number [9]. The code has been run for the parameters of the Twente Raman FEL [2] with an undulator field of 0.18 T, a guide field of 1.0 T, a current of 200 A and a normalised emittance of $10^{-5} \pi$ mrad. Growth is found for a radiation frequency of 30 GHz. The growth and the phase of the radiation field is given in Figs. 4

Fig. 5. As Fig. 4 except that the electrons are initialised in the same way as in the FRED-code.

and 5 for the TDA- and FRED-initialisation respectively. The lethargy (i.e. the distance before exponential growth starts) is about the same for both initialisations. The FRED-initialisation gives however a lower field amplitude at the beginning of the exponential growth region resulting in a factor 5 lower output at the end of the undulator. The TDA-initialisation results in saturation at a level of about 12 MW whereas this is not the case for the FRED-initialisation. The phase of the radiation field changes more rapidly in the lethargy region for the FRED-initialisation than for the TDA-initialisation whereas during exponential growth the frequency shift is slightly less. By varying the length of the undulator it becomes possible to investigate the behaviour of an amplifier and oscillator configuration for different single pass gains.

5. Conclusions

A Bragg reflector has been designed with a power reflection coefficient of ≈ 0.4 at the design frequency of 30 GHz for an incoming and reflected wave in the TE_{11} mode. By applying a Hamming window to the corrugation it is possible to obtain high mode purity and still maintain a relative large FWHM of the reflection peak. It is found that the reflectivity is a sensitive function of the ripple amplitude for the parameter region investigated. A Bragg reflector has been constructed with $\lambda_b = 5.37$ mm, $l_0 = 0.2$ mm and $L = 270$ mm. Allowing for mechanical tolerances in making the corrugation, good agreement is found between the measured and calculated reflectivity.

Preliminary calculations show that the laser may just or just not saturate in a single pass depending on

the type of initialisation used for the electron beam in the simulation. Varying the interaction length makes it possible to investigate the different behaviour of an oscillator and amplifier configuration.

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