Exact quantum-mechanical propagator for an electron in a saddle-point potential and a magnetic field

Ruud van Damme

Department of Computer Science, Twente University of Technology, NL-7500 AE Enschede, The Netherlands

Frederik W. Wiegel

Center for Theoretical Physics, Twente University of Technology, NL-7500 AE Enschede, The Netherlands

Michael J. Harrison

Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan 48824-1116

(Received 12 May 1988; revised manuscript received 17 October 1988)

We study the time-dependent propagator for an electron which moves in a plane and which is subject to quadrupolar electric fields with certain reflection symmetries, and to a uniform magnetic field perpendicular to the plane. It is shown that the electric potential, which has a saddle point at the origin, can be produced by voltages applied to four symmetrically configured cylindrical electrodes whose axes are perpendicular to the plane. We use path-integral methods to derive a closed-form expression for the propagator, and comment in some detail on the structure of its normalization factor.

I. INTRODUCTION

Recent work¹ on heterojunctions and semiconductor superlattices has emphasized the properties of two-dimensional electron systems located in perpendicular magnetic fields. The thermal, magnetical, electrical, and optical characteristics of reduced dimensionality electron gases have been studied²⁻⁷ in multilayers, heterojunctions, and specifically in metal-oxide-semiconductor field-effect-transistor (MOSFET) devices. Experiments which study tunneling phenomena in ultrathin semiconductor heterostructures^{8,9} have helped elucidate the relative roles of coherence and incoherence in the tunneling escape process.

The experiments generally require the application of voltages to electrodes located at certain positions in the two-dimensional electron gas, which is itself surrounded by a distribution of ionized impurities associated with doping. When the electrodes are closely spaced even moderate voltages can result in large electric fields and electric field gradients which have components parallel to the plane in which the electronic motion occurs.

Four-terminal devices whose electrodes give rise to quadrupole fields have been studied both experimentally¹⁰⁻¹² and theoretically.¹³ Typical electrode separations l vary considerably in different kinds of fabricated multielectrode heterostructures.^{10,14} It has been possible to achieve Hall electrode separations of approximately 50 μm , and Hall electric fields up to 100 V/cm.¹⁵ Magnetic fields up to 8 T applied perpendicular to the surface of thin metal-oxide-semiconductor structures have revealed that electron tunneling processes which carry current through the device indeed show effects of the magnetic field on the two-dimensional electron gas formed by the inversion layer in the semiconductor.¹⁶

In this paper we consider tunneling processes for an electron in a two-dimensional electron gas, moving under the combined influence of a perpendicular magnetic field $\mathbf{B}$ in the z direction and a two-dimensional potential energy field $U(x,y) = (\gamma^2 m/2)(y^2 - x^2)$, where m is the electron mass and γ^2 measures the strength of the voltages applied to the electrodes. In Sec. III we show how the potential energy U arises naturally from a symmetric quadrupolar configuration of electrodes.

This model was recently studied by Fertig and Halperin,¹⁷ who derived an analytic expression for the transmission coefficient through the saddle point, as well as various approximate results. In view of the generality and simplicity of the model it seems worthwhile to derive an analytic expression for the time-dependent propagator. This is the subject of the next section.

II. PATH-INTEGRAL DERIVATION OF THE PROPAGATOR

We choose a symmetric gauge $\mathbf{A} = \frac{1}{2}\mathbf{B} \times \mathbf{r}$; for $\mathbf{B} = \nabla \times \mathbf{A} = (0, 0, B)$ we have the Hamiltonian

$$H = \frac{\left[p_x - \frac{e}{c} A_x \right]^2}{2m} + \frac{\left[p_y - \frac{e}{c} A_y \right]^2}{2m} + \frac{\gamma^2 m}{2} (y^2 - x^2), \quad (2.1)$$

where c is the speed of light, m the electron mass, $e = -|e|$ the electron charge, and $\mathbf{A} = (-By/2, Bx/2, 0)$. Hamilton's equations for $\dot{x}$, $\dot{p}_x$, $\dot{y}$, and $\dot{p}_y$ follow immediately from Eq. (2.1), and by eliminating $\dot{p}_x$ and $\dot{p}_y$ we arrive at the second-order equations of motion for the coordinates

$$\begin{aligned}\ddot{x} &= -\Omega\dot{y} + \gamma^2 x, \\ \ddot{y} &= \Omega\dot{x} - \gamma^2 y,\end{aligned}\quad (2.2)$$

where

$$\Omega = \frac{|e|B}{mc} \quad (2.3)$$

is the cyclotron frequency. The Lagrangian is found from (2.1) to have the form

$$L = \frac{m}{2} [(\dot{x}^2 + \dot{y}^2) + \gamma^2(x^2 - y^2) + \Omega(\dot{x}y - x\dot{y})]. \quad (2.4)$$

The propagator $G(x_1, y_1, t_1 | x_0, y_0, t_0)$ from the initial position (x_0, y_0) of the electron at time t_0 to the final position (x_1, y_1) at time t_1 can now be calculated from its path-integral representation by following the procedures in any of the existing monographs on path integrals.¹⁸⁻²⁰ As the Lagrangian is quadratic this calculation is in principle straightforward, but it turns out to be remarkably tedious in practice. The result is most elegantly expressed in terms of

$$x_+ = x_1 + x_0, \quad (2.5a)$$

$$x_- = x_1 - x_0, \quad (2.5b)$$

$$y_+ = y_1 + y_0, \quad (2.5c)$$

$$y_- = y_1 - y_0, \quad (2.5d)$$

$$T = t_1 - t_0. \quad (2.5e)$$

Let ω and λ denote the positive square roots of

$$\omega^2 = \frac{1}{2}\Omega^2 + (\gamma^4 + \frac{1}{4}\Omega^4)^{1/2}, \quad (2.6a)$$

$$\lambda^2 = -\frac{1}{2}\Omega^2 + (\gamma^4 + \frac{1}{4}\Omega^4)^{1/2}. \quad (2.6b)$$

Both have dimension (time)⁻¹. In terms of ω and λ we define two functions $\tau_1(T)$ and $\tau_2(T)$ with dimension (time), by

$$\begin{aligned}\frac{1}{\tau_1(T)} &= \lambda \left[\cosh \left[\frac{\lambda T}{2} \right] \sin \left[\frac{\omega T}{2} \right] \right. \\ &\quad \left. + \sinh \left[\frac{\lambda T}{2} \right] \cos \left[\frac{\omega T}{2} \right] \right] \\ &\quad + \omega \left[\cosh \left[\frac{\lambda T}{2} \right] \sin \left[\frac{\omega T}{2} \right] \right. \\ &\quad \left. - \sinh \left[\frac{\lambda T}{2} \right] \cos \left[\frac{\omega T}{2} \right] \right], \quad (2.7a)\end{aligned}$$

$$\begin{aligned}\frac{1}{\tau_2(T)} &= \lambda \left[-\cosh \left[\frac{\lambda T}{2} \right] \sin \left[\frac{\omega T}{2} \right] \right. \\ &\quad \left. + \sinh \left[\frac{\lambda T}{2} \right] \cos \left[\frac{\omega T}{2} \right] \right] \\ &\quad + \omega \left[\cosh \left[\frac{\lambda T}{2} \right] \sin \left[\frac{\omega T}{2} \right] \right. \\ &\quad \left. + \sinh \left[\frac{\lambda T}{2} \right] \cos \left[\frac{\omega T}{2} \right] \right]. \quad (2.7b)\end{aligned}$$

The action for the classical path that passes through (x_0, y_0) at time t_0 and through (x_1, y_1) at time t_1 is found to be given by

$$\begin{aligned}S_c(x_1, y_1, t_1 | x_0, y_0, t_0) &= \frac{1}{4}m\tau_1\tau_2(\lambda^2 + \omega^2) \left[\sinh \left[\frac{\lambda T}{2} \right] \sin \left[\frac{\omega T}{2} \right] \left[\frac{x_+^2}{\tau_2} - \frac{y_+^2}{\tau_1} \right] + \cosh \left[\frac{\lambda T}{2} \right] \cos \left[\frac{\omega T}{2} \right] \left[\frac{x_-^2}{\tau_1} + \frac{y_-^2}{\tau_2} \right] \right] \\ &\quad - \frac{1}{4}m\tau_1\tau_2\Omega \left[\frac{x_+y_-}{\tau_2^2} - \frac{x_-y_+}{\tau_1^2} \right]. \quad (2.8)\end{aligned}$$

According to the general principles of path integration¹⁸⁻²⁰ the propagator is given *exactly* by the expression

$$G(x_1, y_1, t_1 | x_0, y_0, t_0) = F(T) \exp \left[\frac{i}{\hbar} S_c(x_1, y_1, t_1 | x_0, y_0, t_0) \right], \quad (2.9)$$

where the normalization factor F depends on the time difference $t_1 - t_0$ only. As the propagator has the property

$$\begin{aligned}G(x_1, y_1, t_1 | x_0, y_0, t_0) &= \int G(x_1, y_1, t_1 | x, y, t) \\ &\quad \times G(x, y, t | x_0, y_0, t_0) dx dy\end{aligned}\quad (2.10)$$

we find the following functional equation for F ,

$$F(T_1)F(T_2)\phi(T_1|T_2) = F(T_1 + T_2), \quad (2.11)$$

where the function ϕ is given explicitly by

$$\begin{aligned}\phi(T_1|T_2) &= \frac{\hbar}{i} \pi \{ [f_1(T_1) + f_1(T_2)] \\ &\quad \times [f_2(T_1) + f_2(T_2)] \\ &\quad - [f_3(T_1) - f_3(T_2)]^2 \}^{-1/2}.\end{aligned}\quad (2.12)$$

The functions f_1 , f_2 , and f_3 are given by the prefactors of x^2 , y^2 , and $2xy$ in the classical action

$$f_1(T) = \frac{1}{4}m(\lambda^2 + \omega^2) \left[\tau_1 \sinh \left[\frac{\lambda T}{2} \right] \sin \left[\frac{\omega T}{2} \right] + \tau_2 \cosh \left[\frac{\lambda T}{2} \right] \cos \left[\frac{\omega T}{2} \right] \right], \quad (2.13)$$

$$f_2(T) = \frac{1}{4}m(\lambda^2 + \omega^2) \left[-\tau_2 \sinh \left[\frac{\lambda T}{2} \right] \sin \left[\frac{\omega T}{2} \right] + \tau_1 \cosh \left[\frac{\lambda T}{2} \right] \cos \left[\frac{\omega T}{2} \right] \right], \quad (2.14)$$

$$f_3(T) = \frac{1}{8}m\Omega \left[\frac{\tau_1}{\tau_2} - \frac{\tau_2}{\tau_1} \right]. \quad (2.15)$$

When (2.11) is used for $T_1 = T + \epsilon$, $T_2 = T - \epsilon$, one finds

$$F(T + \epsilon)F(T - \epsilon)\phi(T + \epsilon|T - \epsilon) = F(2T) = \{F^2(T) + \epsilon^2[F''(T)F(T) - F'^2(T)]\}\phi(T|T)[1 + \epsilon^2\psi(T)] + O(\epsilon^3), \quad (2.16)$$

where primes denote differentiation with respect to T and where

$$\begin{aligned} \psi(T) &= \frac{1}{2\phi(T|T)} \left[\frac{\partial^2 \phi(T_1|T_2)}{\partial T_1^2} + \frac{\partial^2 \phi(T_1|T_2)}{\partial T_2^2} - 2 \frac{\partial^2 \phi(T_1|T_2)}{\partial T_1 \partial T_2} \right]_{T_1=T_2=T} \\ &= -\frac{1}{4} \left[\frac{f_1''(T)}{f_1(T)} + \frac{f_2''(T)}{f_2(T)} - 2 \frac{[f_3'(T)]^2}{f_1(T)f_2(T)} \right]. \end{aligned} \quad (2.17)$$

However, as (2.11) also gives

$$F(T)F(T)\phi(T|T) = F(2T), \quad (2.18)$$

Eq. (2.16) immediately yields

$$F''(T)F(T) - F'^2(T) + F^2(T)\psi(T) = 0. \quad (2.19)$$

Although this equation looks somewhat forbidding, the substitution

$$R(T) = \ln F(T) \quad (2.20)$$

leads to the inhomogeneous linear equation

$$R''(T) = -\psi(T). \quad (2.21)$$

We have to solve this equation subject to the initial condition that $R(T)$ should behave as

$$R(T) = -\ln \left[\frac{2\pi i \hbar T}{m} \right] \quad (2.22)$$

asymptotically for $T \rightarrow 0$.

As the function $\psi(T)$, given by (2.17), can be written in the form

$$\begin{aligned} \psi(T) &= \frac{1}{2} \frac{d^2}{dT^2} \ln \{ \lambda \omega \sin(\omega T) \sinh(\lambda T) \\ &\quad - \frac{1}{2}(\lambda^2 - \omega^2)[1 - \cos(\omega T) \cosh(\lambda T)] \} \\ &= -\frac{1}{2} \frac{d^2}{dT^2} \ln(\tau_1 \tau_2) \end{aligned} \quad (2.23)$$

the general solution of (2.21) is of the form

$$R(T) = \alpha T + \beta + \frac{1}{2} \ln(\tau_1 \tau_2), \quad (2.24)$$

where α and β are constants. When this expression is matched asymptotically to (2.22) for $T \rightarrow 0$, one finds

$$\alpha = 0, \quad (2.25a)$$

$$\beta = \ln \left[(\lambda^2 + \omega^2) \frac{m}{4\pi i \hbar} \right]. \quad (2.25b)$$

Combining (2.20) with the last three equations one finds

$$F(T) = \frac{\lambda^2 + \omega^2}{4\pi i \hbar} m(\tau_1 \tau_2)^{1/2}. \quad (2.26)$$

There are, of course, several alternative ways to write F and S_c , but our use of the variables λ , ω , τ_1 , and τ_2 seems a natural choice. Equation (2.9) with S_c given by (2.8) and F by the last equation forms the main analytic result of this paper.

It is of interest to consider various limiting cases of the propagator. The first case is the limit $\gamma \rightarrow 0$ in which the electric saddle-point potential vanishes. As the voltage on the electrodes goes to zero, so that $\gamma^2 \rightarrow 0$, we have $\omega \rightarrow \Omega$, which is the frequency measured in cyclotron resonance experiments on a two-dimensional electron system subjected to a perpendicular magnetic field²¹ (for $\gamma^2 \neq 0$, ω can be shown to represent the angular frequency of the only stable and periodic solution to the classical equation of motion, namely an electron orbit whose average position is centered at the origin of coordinates, where the potential energy vanishes). In this limit $\lambda \rightarrow 0$ and the exact solution reduces to

$$F(T) = \frac{\Omega m}{4\pi i \hbar \sin(\Omega T/2)}, \quad (2.27a)$$

$$\begin{aligned} S_c(x_1, y_1, T | x_0, y_0, 0) \\ = \frac{1}{4} m \Omega^2 [(x_1 - x_0)^2 + (y_1 - y_0)^2] \cot(\Omega T/2) \\ + \frac{1}{2} m \Omega (x_1 y_0 - x_0 y_1). \end{aligned} \quad (2.27b)$$

This result checks with the formula quoted by Feynman and Hibbs,¹⁸ provided an obvious misprint is corrected there.

Similarly, as the magnetic field goes to zero, so that $\Omega \rightarrow 0$, we obtain $\lambda \rightarrow \gamma$, which we show in Sec. III is proportional to the square root of the positive voltage applied to the electrodes maintained at nonzero potential. It is straightforward to show that our exact result now reduces to

$$F(T) = \left[\frac{\gamma m}{2\pi i \hbar \sin(\gamma T)} \right]^{1/2} \left[\frac{\gamma m}{2\pi i \hbar \sinh(\gamma T)} \right]^{1/2}, \quad (2.28a)$$

$$\begin{aligned} S_c(x_1, y_1, T | x_0, y_0, 0) \\ = \frac{1}{2} m \gamma \left[\frac{(x_1^2 + x_0^2) \cosh(\gamma T) - 2x_1 x_0}{\sinh(\gamma T)} \right. \\ \left. + \frac{(y_1^2 + y_0^2) \cos(\gamma T) - 2y_1 y_0}{\sin(\gamma T)} \right]. \end{aligned} \quad (2.28b)$$

This can be trivially deduced from a corresponding formula in Ref. 18.

To close this section we would like to remark that the two periods, $\tau_1(T)$ and $\tau_2(T)$, become infinite as $T = (t_1 - t_0) \rightarrow 0$ in the propagator G . The physical significance of τ_1 and τ_2 is reflected in their appearance in the final expression for the normalization factor F , in (2.26). We observe that as $t_1 \rightarrow t_0$, the absolute magnitude of F , and consequently that of the propagator G , becomes large for any finite value of $\lambda^2 + \omega^2 = 2(\gamma^4 + \Omega^4/4)^{1/2}$. But a large value for $|G|$ implies a large probability amplitude for propagation of the particle from (x_0, y_0) to (x_1, y_1) during the time interval $T = (t_1 - t_0)$. If we now regard T as the uncertainty in time measurement for the moving particle, then as $T \rightarrow 0$ the uncertainty principle between time and energy requires that the uncertainty in particle energy be quite large. Provided the particle energy depends on position, as it does in the present case of crossed electric and magnetic fields, the large uncertainty in energy therefore implies a large probability amplitude $|G|$ for particle propagation from location (x_0, y_0) to location (x_1, y_1) . Thus the physical significance of the large values of τ_1 and τ_2 as $T \rightarrow 0$ resides in an expression of the uncertainty principle through the large value for $|G|$.

III. PHYSICAL REALIZATION OF THE SADDLE-POINT POTENTIAL

The electron motion which we considered in Sec. II occurs in a uniform perpendicular magnetic field and in a particular quadrupolar two-dimensional electrostatic potential. In this section we show that such a potential may be produced by four pairwise symmetrically shaped and configured electrodes which are placed appropriately in pairs astride the x and y axes, respectively. The four electrodes are imagined to be cylinders of arbitrary, but symmetric, cross section. They are mirror images of each other in pairs with respect to reflections in the two coordinate planes containing the z axis. Each electrode is taken to have bilateral symmetry about the coordinate plane that passes through it. Let the pair of electrodes cut by the y - z plane be maintained at zero potential, and the pair of electrodes cut by the x - z plane have a potential $V > 0$ applied to them both. We fix the distance from the z axis of the two equally distant electrodes at zero potential, but permit the total separation l of the two equally distant electrodes at potential V to vary.

Under the conditions stipulated above the electrostatic potential $\phi(x, y)$ has reflection symmetry through both the x, z and y, z coordinate planes: $\phi(x, y) = \phi(x, -y) = \phi(-x, y)$. We recall that ϕ satisfies Laplace's equation in two dimensions.

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0. \quad (3.1)$$

Now define $G(x) = \phi(x, 0)$ and introduce the notation $G_n(x) = d^n G / dx^n = \partial^n \phi / \partial x^n |_{y=0}$. Since $\phi(x, y) = \phi(x, -y)$, ϕ is an even function of y when x is held fixed. It follows that all odd partial derivatives of ϕ with respect to y are odd functions of y which must vanish at $y = 0$. Therefore the Taylor-series expansion of $\phi(x, y)$ in powers of y , for fixed x , can contain only even powers:

$$\phi(x, y) = \sum_{n=0}^{\infty} \frac{\partial^{2n} \phi}{\partial y^{2n}} \bigg|_{y=0} \frac{y^{2n}}{(2n)!}. \quad (3.2)$$

Successive even differentiations of (3.1) lead to the result

$$\frac{\partial^{2n} \phi}{\partial y^{2n}} = (-1)^n \frac{\partial^{2n} \phi}{\partial x^{2n}}, \quad (3.3)$$

so that from the definition of $G_n(x)$ we have

$$\frac{\partial^{2n} \phi}{\partial y^{2n}} \bigg|_{y=0} = (-1)^n G_{2n}(x). \quad (3.4)$$

The power series for ϕ then becomes

$$\phi(x, y) = \sum_{n=0}^{\infty} (-1)^n G_{2n}(x) \frac{y^{2n}}{(2n)!} \quad (3.5)$$

Since $\phi(x, y) = \phi(-x, y)$, $G(x)$ is an even function of x , and all its odd derivatives vanish at $x = 0$, with its even derivatives being even functions of x . Therefore $G_{2n}(x)$ are even functions, and

$$G_{2n}(x) = \sum_{m=0}^{\infty} \frac{d^{2m} G_{2n}}{dx^{2m}} \bigg|_{x=0} \frac{x^{2m}}{(2m)!}. \quad (3.6)$$

Let $A_{mn} = d^{2m} G_{2n} / dx^{2m} |_{x=0}$. Then $A_{mn} = A_{nm}$ and we may write the electrostatic potential as a power series in x and y :

$$\phi(x, y) = \sum_{n, m=0}^{\infty} (-1)^n \frac{A_{mn}}{(2n)!(2m)!} x^{2m} y^{2n}, \quad (3.7a)$$

where

$$A_{mn} = \frac{d^{2(m+n)} \phi(x, 0)}{dx^{2(m+n)}} \Big|_{x=0}. \quad (3.7b)$$

From the linearity of the dependence of $\phi(x, y)$ on the boundary value V we may write $\phi(x, 0) = \phi(x, 0; V, l) = Vg(x, 0; l)$, where V is the voltage applied to both electrodes astride the x axis, which are separated by the variable distance l . As $g = \phi(x, 0)/V$ is independent of V and depends parametrically on l alone, one has

$$A_{mn} = Vg_{m+n}^{(l)}, \quad (3.8a)$$

where

$$g_{m+n}^{(l)} = \frac{d^{2(m+n)} g(x, 0; l)}{dx^{2(m+n)}} \Big|_{x=0} \quad (3.8b)$$

depends only on l and on the specific geometry of the electrodes.

By choosing the separation l so that it satisfies the equation $g_2(l) = 0$ we may eliminate the quartic terms in the power series Eq. (3.7a), and after dropping the constant term in the potential we may rewrite the power series for ϕ in the form

$$\phi(x, y) = \frac{Vg_1(l)}{2} (x^2 - y^2) + Q(r^6). \quad (3.9)$$

With the definitions $\gamma^2 = |e|Vg_1(l)/m$ and $e = -|e|$ we find that sufficiently close to the origin the potential energy $U = e\phi$ can be written as in Eq. (2.1).

ACKNOWLEDGMENTS

F.W. and M.H. are indebted to Alan Perelson of the Los Alamos National Laboratory for pointing out the possibilities of the computer language REDUCE, and to George Bell for making their visit to that Laboratory possible. F.W. also wishes to acknowledge North Atlantic Treaty Organization (NATO) Research Grant No. RG-86/719, jointly received with Alan Perelson. R.v.D. acknowledges Bernard Geurts and Martin Streng for their interest in this work.

¹*Heterojunctions and Semiconductor Superlattices*, edited by G. Allan, G. Bastard, N. Boccarda, M. Lannoo, and M. Voos (Springer-Verlag, Berlin, 1986).

²H. L. Störmer, T. Haavasoja, V. Narayanamurti, A. C. Gosard, and W. Wiegmann, *J. Vac. Sci. Technol. B* **1**, 423 (1983).

³F. F. Fang and P. J. Stiles, *Phys. Rev. B* **28**, 6992 (1983).

⁴E. Gornik, R. Lassnig, G. Strasser, H. L. Störmer, A. C. Gosard, and W. Wiegmann, *Phys. Rev. Lett.* **54**, 1820 (1985).

⁵V. M. Pudalov, S. G. Semenchinskii, and V. S. Edel'man, *Zh. Eksp. Teor. Fiz.* **89**, 1870 (1985) [*Sov. Phys.—JETP* **62**, 1079 (1985)].

⁶S. B. Kaplan and A. C. Warren, *Phys. Rev. B* **34**, 1346 (1986).

⁷R. Meisels and F. Kuchar, *Z. Phys. B* **67**, 443 (1987).

⁸S. Tarucha, K. Ploog, and K. von Klitzing, *Phys. Rev. B* **36**, 4558 (1987).

⁹M. Tsuchiya, T. Matsusue, and H. Sakaki, *Phys. Rev. Lett.* **59**, 2356 (1987).

¹⁰G. Timp, A. M. Chang, P. Mankiewich, R. Behringer, J. E. Cunningham, T. Y. Chang, and R. E. Howard, *Phys. Rev. Lett.* **59**, 732 (1987).

¹¹K. von Klitzing, G. Dorda, and M. Pepper, *Phys. Rev. Lett.* **45**, 494 (1980).

¹²J. A. Simmons, D. C. Tsui, and G. Weimann, *Surf. Sci.* **196**, 81 (1988).

¹³F. M. Peeters, *Phys. Rev. Lett.* **61**, 589 (1988).

¹⁴B. E. Kane, D. C. Tsui, and G. Weimann, *Phys. Rev. Lett.* **61**, 1123 (1988).

¹⁵S. Komiyama, T. Takamasu, S. Hiyamizu, and S. Sasa, *Solid State Commun.* **54**, 479 (1985).

¹⁶A. Hartstein and R. H. Koch, *Phys. Rev. B* **35**, 6442 (1987).

¹⁷H. A. Fertig and B. I. Halperin, *Phys. Rev. B* **36**, 7969 (1987).

¹⁸R. P. Feynman and A. R. Hibbs, *Quantum Mechanics and Path Integrals* (McGraw-Hill, New York, 1965).

¹⁹L. S. Schulman, *Techniques and Applications of Path Integration* (Wiley, New York, 1981).

²⁰F. W. Wiegel, *Introduction to Path-Integral Methods in Physics and Polymer Science* (World Scientific, Singapore, 1986).

²¹B. A. Wilson, S. J. Allen, Jr., and D. C. Tsui, *Phys. Rev. B* **24**, 5887 (1981).