Magnetoresistance of $\text{PrBa}_2\text{Cu}_3\text{O}_{7-\delta}$ thin films

G. K. van Ancum, M. A. J. Verhoeven, D. H. A. Blank, and H. Rogalla

Department of Applied Physics, University of Twente, P.O. Box 217, NL-7500 AE Enschede, The Netherlands

(Received 26 June 1995)

Transport of charge carriers in $\text{PrBa}_2\text{Cu}_3\text{O}_{7-\delta}$ (PBCO) is often described by variable-range hopping (VRH). Until now the VRH mechanism was confirmed merely on the basis of a temperature dependence of the resistivity following Mott's law. In this article we show a positive magnetoresistance in PBCO thin films, depending exponentially on the applied magnetic field. This provides substantial additional evidence for a variable-range hopping transport mechanism. Both a strong-field and a weak-field magnetoresistance can be identified. At temperatures above 30 K we observe weak-field magnetoresistance, at 4.2 K we detect a transition from weak-field to strong-field magnetoresistance at a magnetic field of approximately 4.5 T. In the weak-field regime the radius of the localized wave function is only affected marginally by the applied magnetic field. In the strong-field regime the radius of the localized wave function decreases with increasing magnetic field. From the measurements in the strong-field regime we obtain an estimate for the two-dimensional density of localized states in the PBCO thin film of approximately $2 \times 10^{13} \text{ 1/eV m}^2$.

I. INTRODUCTION

The temperature and electric-field dependence of the electronic transport properties of $\text{PrBa}_2\text{Cu}_3\text{O}_{7-\delta}$ (PBCO) are usually described in the framework of Mott's model for variable-range hopping (VRH).¹ In this description an electron hops between different localized states. The origin of localized states in PBCO is still under discussion.² PBCO was claimed to exhibit VRH as soon as the typical $\rho \propto \exp T^p$ dependence was found, with ρ being the PBCO resistivity, T is the temperature, and $p = -1/3$ for two-, or $-1/4$ for three-dimensional samples.³

In this article we provide additional evidence for a VRH transport mechanism by investigating the magnetoresistance of PBCO thin films. The magnetoresistance is defined as $\rho(H)/\rho(H=0)$, with $\rho(H)$ the resistivity in an applied magnetic field H . Measurement of magnetoresistance is a well-known tool in semiconductor physics. In classical transport processes, based on conduction in energy bands, the resistivity has a power-law dependence on the applied magnetic field.⁴ In a VRH transport mechanism the magnetic field squeezes the wave functions of the electrons trapped at the localized states in the plane perpendicular to the field direction. This leads to a decrease in the overlap integral between sites and, hence, to an exponential increase in the resistivity. An exponential positive magnetoresistance is an unambiguous evidence of the importance of the "tails" of the wave functions and can be regarded as strong evidence of a hopping conduction mechanism.⁵

II. THEORY

As discussed previously⁶ we describe the electronic transport process in PBCO thin films with a two-dimensional (2D) Mott model without taking electron-electron interactions into account.⁷ A more elaborate description of the theory of magnetoresistance in the case of localized wave functions in a three-dimensional volume can be found elsewhere.⁵ Here, we shall briefly summarize this theory for

the magnetoresistance of a 2D thin film.

The magnetic field is applied perpendicularly to the film surface, see Fig. 1. In the description of the influence of the magnetic field on the resistivity we have to distinguish a weak-field and a strong-field regime. The magnetic length λ separates the two regimes and is given by

$$\lambda = \sqrt{\hbar/eH}, \quad (1)$$

with e being the unit charge, $\hbar$ is Planck's constant, and H is the applied magnetic field. In the weak-field regime $\lambda > a_0$ and in the strong-field regime $\lambda \leq a_0$, with a_0 the radius of the localized wave function in zero magnetic field. In the weak-field regime the shape of the electron wave function is affected only little by the magnetic field and the radius of the localized wave function is assumed to be constant. In the strong-field regime the wave function of the electron is squeezed considerably in the xy plane of the film, thereby reducing the overlap integral between adjacent sites. This causes an increase of the resistance with an applied magnetic field. In the strong-field regime the in-plane radius a_H of the localized wave function as a function of the applied magnetic field is given by

$$a_H = a_0 / \ln \left(H \frac{k_B^2 \hbar^3}{m^2 e^3} \right) \quad \text{for } \lambda \leq a_0, \quad (2)$$

FIG. 1. Schematic drawing of the measurement setup.

with k_B being Boltzmann's constant and m is the electron mass. The magnetoresistance is derived using percolation theory, in close analogy to Ref. 5. The weak-field magnetoresistance for a 2D system can be found to be

$$\frac{\rho_{\text{weak}}(H)}{\rho(0)} \propto \exp\left(\frac{H^2}{k_B T}\right). \quad (3)$$

The two-dimensional magnetoresistance in the strong-field regime is given by

$$\frac{\rho_{\text{strong}}(H)}{\rho(0)} \propto \exp(\sqrt{H/k_B T}). \quad (4)$$

The transition from weak to strong magnetoresistance takes place at a critical field H_c , that can be found by equating the exponents of Eqs. (3) and (4):

$$H_c \propto T^{1/3}. \quad (5)$$

III. EXPERIMENT

In the experiments we used various thin PBCO films of about 50 nm thickness, grown by off-axis sputter deposition on SrTiO_3 substrates. X-ray measurements indicate that the films are c -axis oriented, and are optimally oxygen loaded. More details on the deposition and film quality can be found elsewhere.⁸ On the PBCO thin film we deposited silver contacts with a thickness of approximately 60 nm.

Measuring the very high resistance of PBCO thin films at low temperatures requires special attention. The contact resistance between the silver electrodes and the PBCO may obscure the PBCO resistance in the measurements. To differentiate between contact resistance and PBCO resistance we employ the transmission line model.⁹ For the IV measurements we apply a Keithley Source Measure Unit (SMU), combining a voltage source with 100 μV accuracy, and a current meter with 5 fA resolution. Triaxial wiring is used from the SMU to the cryostat. A driven guard surrounds the sensitive wire down to the sample holder to prevent leakage currents. The whole insert, including the sample holder, is surrounded by a grounded shield to reduce noise. The measurement technique is discussed more extensively in Ref. 6.

In preliminary experiments we found values for the magnetoresistance of the order of a few percent. Consequently, discriminating the effect of the applied magnetic field poses strict demands on the temperature stability during experiments.

To measure the magnetoresistance of thin PBCO films we employed two different setups. The first series of measurements were done in a standard flow cryostat, enabling us to change the temperature of the sample, and to apply magnetic fields up to 5 T. From Mott's equation for the temperature dependence of variable-range hopping¹ we find

$$\frac{\Delta\rho(T)}{\rho(T)} = -\frac{T_0^{1/3}}{4T^{4/3}} \Delta T, \quad (6)$$

with T_0 of the order of 10^5 K for PBCO.¹⁰ In our flow cryostat the temperature stability is of the order of 0.1 K. Allowing 1% resistivity change $\Delta\rho/\rho$ caused by a temperature

FIG. 2. Magnetoresistance at 100, 60, and 30 K (sample No. 1). The drawn lines represent weak-field dependence.

fluctuation ΔT , Eq. (6) implies that the operating temperature in the flow cryostat is limited to temperatures above approximately $T=30$ K.

To be able to measure accurately at a lower temperature, we did a second set of measurements in a liquid helium bath. Here the temperature was fixed at 4.2 K, and the magnetic field was swept from 0 to 12 T. The temperature stability at the position of the sample was monitored to be better than 1 mK, enabling us to measure the magnetoresistance within 0.5% error at 4.2 K.

IV. MEASUREMENTS

Figure 2 gives the magnetoresistance of a thin PBCO film for temperatures of 100, 60, and 30 K and applied magnetic fields ranging from 0 to 5 T. To test for weak or strong-field behavior, we plotted $\ln^2[\rho(H)/\rho(0)]$ versus $\ln H$. The slope of the drawn lines represent a weak-field $\rho(H)/\rho(0) \propto \exp H^2$ dependence, which clearly fits the data. These measurements show that the magnetoresistance of PBCO in the temperature and field range of the experiments is positive and depends exponentially on the square of the applied magnetic field. The magnetoresistance increases for lower temperatures, which is consistent with the theory.

In Fig. 3 the zero electric-field magnetoresistance of a different PBCO film at 4.2 K in magnetic fields ranging from 0 to 12 T is plotted as $\ln^2[\rho(H)/\rho(0)]$ versus $\ln H$. The drawn lines represent $\rho(H)/\rho(0) \propto \exp H^2$, and $\rho(H)/\rho(0) \propto \exp H^{1/2}$

FIG. 3. Magnetoresistance at 4.2 K (sample No. 2). The drawn lines represent weak- and strong-field dependences, as indicated.

FIG. 4. The average hopping distance r_x in the direction of the applied electric field E for applied magnetic fields of 0, 4, 8, and 12 T at $T=4.2$ K.

dependences for weak and strong magnetic fields, respectively. We see an exponential positive magnetoresistance. For magnetic fields below approximately 4 T, a weak-field dependence can be observed. For $H > 5$ T the $\exp H^{1/2}$ dependence sets in, indicating strong-field magnetoresistance. A critical field $H_c \approx 4.5$ T at $T=4.2$ K can be identified. From this value, using Eq. (5), the critical field at 30 K is calculated to be approximately 9 T, explaining the weak-field dependence we observed earlier.

The impact of the applied magnetic field on the average hopping distance in the direction of the applied electric field can be evaluated using the same technique as described in Ref. 6. We find the average hopping distance to decrease with an increasing applied magnetic field, as well as with an increasing electric field, see Fig. 4. The influence of an applied electric field was discussed more extensively in Ref. 6: an increase in the applied electric field leads, at constant magnetic field, to a decrease in the average hopping distance in the direction of the applied electric field.

In a variable-range hopping transport process the hopping distance depends on the radius of the localized wave function. In the framework of Mott's theory, the average hopping distance r in the xy plane of the film can be found to be⁵

$$r(E=0, H) = \left(\frac{a_H}{2g(\mu)k_B T} \right)^{1/3}, \quad (7)$$

with $g(\mu)$ being the 2D density of localized states. The magnetic field squeezes the radius of the wave function of the localized state and thus reduces the overlap between adjacent sites. Therefore it becomes less attractive for an electron to hop to a distant site with a compatible energy to the initial site. By applying a magnetic field the electron is more likely to hop to a final site in its vicinity, thereby accepting a larger energy mismatch. A decrease in the radius of the wave function leads to a decrease in the average hopping distance and, thus, also to a decrease in the average hopping distance in the direction of the applied electric field. Combining Eqs. (7) and (2) we obtain in the strong-field regime

$$\frac{1}{r^3} = \frac{2g(\mu)k_B T}{a_0} \ln(H) + \text{const.} \quad (8)$$

The zero-field average hopping length can be approximated by the average hopping distance in the field direction for

FIG. 5. $1/r^3$ in zero electric field versus $\ln H$ at $T=4.2$ K. Weak- and strong-field regimes as indicated.

weak electric fields E . Then the average hopping distances in the x and y directions are equal and we can write

$$r(E=0) = \sqrt{r_x^2(E=0) + r_y^2(E=0)} \approx \lim_{E \rightarrow 0} r_x(E) \sqrt{2}. \quad (9)$$

In Fig. 5 we plotted $1/r^3$ versus $\ln H$. Clearly the two regimes can be identified: for $H < 3$ T (or $\ln H < 1$) the hopping distance is not dependent on the applied magnetic field. Here, the size of the wave function is virtually not affected by the magnetic field. For $H > 4.5$ T (or $\ln H > 1.5$), $1/r^3$ increases linearly with $\ln H$. We fitted a line through the strong-field data points. From the slope of that line, using $a_0 = 8.5$ nm,¹¹ we calculated the 2D density of localized states in the PBCO film as $g(\mu) = 2 \times 10^{13}$ 1/eV m².

V. CONCLUSIONS

PBCO thin films show an exponential positive magnetoresistance, evidence of the importance of the tails of the localized wave functions in the transport process. The measurements provide substantial additional evidence for a variable-range hopping mechanism.

At temperatures above 30 K we observed weak-field magnetoresistance, at 4.2 K we observed a transition from weak-field to strong-field behavior. The critical magnetic field was approximately 4.5 T at $T=4.2$ K which leads to a critical field of approximately 9 T at 30 K. In the experiments the temperature stability turned out to be a crucial factor.

The average hopping distance depends on the applied magnetic field, as well as on the applied electric field. In the weak-field regime the average hopping distance was not affected by the magnetic field. In this regime, the effect of the magnetic field on the radius of the localized wave function is marginal. In the strong-field regime the hopping distance decreases with increasing magnetic field, which can be explained by the squeezing of the wave function of the localized electrons in the plane of the film by the applied magnetic field. Analysis of the data yields an estimate for the density of localized states in the 2D PBCO thin film of $g(\mu) = 2 \times 10^{13}$ 1/eV m².

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