FIELD DEPENDENCE OF FISKE RESONANCES IN Nb-AlOx BASED JOSEPHSON JUNCTIONS

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Abstract--Fiske resonances have been measured in a rectangular Nb-Al₂O₃/Al-Nb Josephson junction as a function of the magnetic field applied parallel to a junction side. Due to the high quality factor of the junction, many resonant modes could be measured, using a special measuring technique. Each mode shows more lobes than has ever been reported before. The measured curves are in very good agreement with the high Q theory of Kulik and Q-values ranging from 40 to 450 have been obtained. Probably the surface resistance of the Al/Nb barrier dominates the microwave losses in the junction barrier at 4.2 K. Losses due to the quasi-particle tunneling current can be neglected.

Fiske resonances in a square junction at a field angle of $\pi/4$ rad were also measured. Two-dimensional resonant modes have been found. The field dependence of the (1,1) mixed mode agrees well with the theory of Nerenberg et al.

I. INTRODUCTION

When a magnetic field is applied to a tunnel junction, a steplike structure in the I-V characteristic can be observed. In the few years after the discovery of these so-called Fiske steps [1], it was quantitatively calculated how the stripline modes in a square junction interact with the ac Josephson current at a field angle of $\theta = 0$ [2,3]. These calculations are only valid under the assumption of a low quality factor Q. A theory for unlimited Q was developed shortly after by Kulik [4]. Experimental confirmation was given for example by measurements on high Q Nb-NbOx-Pb junctions [5].

Here measurements on Nb-Al₂O₃/Al-Nb junctions are presented and compared with theory. The resulting Q-factors will be discussed.

In addition measurements of Fiske resonances in a rectangular junction at a field angle of $\pi/4$ are shown. Two-dimensional Fiske resonances have been observed. The field dependence of the (1,1) mixed mode is also presented and compared with the theory of Nerenberg et al. [6].

II. THEORY

Fiske resonance steps are caused by the interaction between the ac Josephson current and the microwave cavity modes in the dielectric barrier of the junction [1-4].

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Consider a rectangular junction with length L, width W and barrier thickness t in a magnetic field that is applied in the xy-plane. Fig.1 shows the geometry. The one-dimensional case, where a magnetic field H is applied along the y-axis ($\theta = 0$) has first been described by Eck et al.[2] and Kulik [3]. The phase difference $\phi(x,t)$ in the Josephson current $j = j_1 \sin \phi(x,t)$ can be written as $\phi(x,t) = \phi_0 + \omega t - kx + \phi_1(x,t)$, where j_1 is the maximum critical current density, $\omega = 2eV/\hbar$, V is the applied dc voltage, $k = 2edH/\hbar c$, $d = 2\lambda_J + t$, and λ_J is the London penetration depth. Combining the Josephson equations with the Maxwell equations the following differential equation for $\phi_1(x,t)$ is obtained:

$$\frac{\partial^2 \phi_1}{\partial x^2} - \frac{1}{c^2} \left[\frac{\partial^2 \phi_1}{\partial t^2} + \gamma \frac{\partial \phi_1}{\partial t} \right] = \frac{1}{\lambda_J^2} \sin \phi, \quad (1)$$

where $\lambda_J = (\hbar c^2 / 16\pi e j_1)^{1/2}$ is the Josephson penetration length, $\bar{c} = c/(t/\epsilon_r d)^{1/2}$ is the velocity of the standing waves in the junction cavity, and ϵ_r is the relative dielectric constant. The reduction of the wave velocity in an SIS stripline is due to the kinetic inductance [7]. The term with the parameter $\gamma = 1/\tau = \omega/Q$ describes the damping due to losses in the system. The characteristic relaxation time τ is proportional to the quality factor Q. In the case of small coupling the term $\phi_1(x,t)$ can be regarded as a small perturbation and is neglected in the right hand term of eq. 1. The resulting expression for the dc current j_{dc} is a series of resonance peaks in the I-V characteristic at voltages $V_n = n\hbar c / 4eL$. The magnetic field dependence of the maximum amplitude of the n-th resonance is given by:

$$j_n(k) = j_1 \left[\frac{L}{2n\pi\lambda_J} \right]^2 \cdot Q_n \cdot F_n^2(k), \quad (2)$$

Fig.1: Rectangular tunnel junction in applied field H.

with

$$F_n(k) = \frac{4kL}{|k^2 L^2 - n^2 \pi^2|} \times \begin{cases} |\cos(kL/2)|, & n = 1, 3, 5, \dots \\ |\sin(kL/2)|, & n = 2, 4, 6, \dots \end{cases}$$

The formula is only valid if $j_n(k)$ is small compared to j_1 (small coupling), i.e. if $(L/2n\pi\lambda_j)^2 Q_n \ll 1$. Q_n is the quality factor of the n -th mode.

From eq. 2 we see that the resonances show an alternation of minima and maxima as a function of the field. The principal maximum of the n -th resonance occurs for a field obeying the relation $kL = n\pi$. The Josephson current, having the well-known Fraunhofer pattern, shares these characteristics. It can be regarded as the 0-th resonance.

Kulik removed the restriction of low Q and calculated j_{dc} in the limit of $\lambda_j \rightarrow \infty$ for unlimited Q [4]. He arrived at

$$j_n(k) = j_1 J_0(a/2) J_1(a/2) F_n(k) \quad (3)$$

for the magnetic field dependence of the maximum amplitude of the n -th resonance (J_0 and J_1 are Besselfunctions). The value a is the solution of the equation $J_0'(a/2) = a/(Z_n F_n(k))$, where $Z_n = (L/n\pi\lambda_j)^2 Q_n$. In the case of a low quality factor, i.e. for $a \rightarrow 0$, $J_0(a/2)$ and $J_1(a/2)$ will become equal to 1 resp. $a/4$, and it follows that $j_n(k) = (j_1/4) Z_n F_n(k)$. This is exactly the low Q expression for $j_n(k)$ in eq. 2.

The temperature dependent quasiparticle tunneling current j_{qp} should be added as an additional term to eqs. 2 and 3. If losses due to this quasiparticle current dominate, the quality factor Q_n can be written as $Q_n = \omega_n/\gamma = \omega_n RC$, where $R = R(V, T)$ is the voltage and temperature dependent quasiparticle tunneling resistance $V/j_{qp}(V, T)$ at voltage V , and C is the capacitance per unit area.

In the more general case, where the field is applied under an arbitrary angle θ with the y -axis, not only one-dimensional cavity modes interact with the ac Josephson current, but the full set of two-dimensional modes (n, m) arises. These resonances occur at voltages

$$V_{nm} = \frac{\hbar c}{4eL} [n^2 + m^2 (L/W)^2]^{1/2} \quad (4)$$

An expression for the current in this case was calculated by Nerenberg et al. [6]. Their result, however, is only valid in the low Q approximation. These two-dimensional modes have indeed been observed [8-10].

III. FABRICATION

The samples are Nb-Al, AlOx, Al-Nb junctions (SNINS-type). A multilayer of 300 nm Nb, 3 nm Al, about 1 nm Al-oxide, 3 nm Al, and 30 nm Nb was deposited by DC-magnetron sputtering [11] and structured with lift-off as a square of $70 \times 70 \mu m^2$. The junction shapes were defined by anodization (SNAP process), for the measured junctions respectively a square of $50 \times 50 \mu m^2$ and a square of $40 \times 40 \mu m^2$ rotated 45° with respect to the base multilayer square. Finally, a 165 nm thick Nb counter electrode was deposited of size $60 \times 60 \mu m^2$.

IV. MEASUREMENT METHOD

The measurements were performed in a magnetically shielded environment at 4.2 K and 1.6 K. To assure

mode n	V_n (mV)	Q_{meas} 4.2 K	Q_{qp} 4.2 K	Q_{meas} 1.6 K	Q_{qp} 1.6 K
F1	0.19	43	210	--	3500
F2	0.36	87	680	--	7500
F3	0.55	110	1360	450	10000
F4	0.72	100	2180	340	13100
F5	0.88	110	2900	270	16000

Table 1: Quality factors of Fiske resonances, as derived from measurements (Q_{meas}) and calculated from the quasi-particle tunneling current (Q_{qp}).

minimal distortion of the magnetic field, the field was applied perpendicular to the current leads ($5 \mu m$ wide), that are connected to the sides of the junction.

We used a saw-tooth current source with adjustable dc-offset so that jumping to the Josephson current or a larger resonance at a lower voltage is prevented. In this way each resonance can be measured independently. This method was also used by Paternò and Nordman [5]. For our junctions with very low and nearly constant sub-gap current we had to adapt the measuring scheme by combining it with a bias circuit, consisting of a shunt resistor placed parallel to the series combination of the junction and a current measuring resistor.

V. RESULTS

The amplitudes of the Fiske resonances of a $50 \times 50 \mu m^2$ square junction were measured as a function of the magnetic field, that was applied along a junction side ($\theta = 0$). The junction has a critical current of 3.2 mA ($j_c = 128 A/cm^2$). From the good fit of the $I_c(H)$ curve with the expected Fraunhofer pattern we conclude that the current density and the field distribution is homogeneous over the junction area and a London penetration depth λ_L of about 50 nm can be calculated. Taking this value for λ_L we obtain a Josephson penetration length λ_J of $45 \mu m$, thus $2\lambda_J$ is large compared to L and self-field effects can be neglected. The spacing $\Delta V = 0.18$ mV between the Fiske resonances (see table 1) gives a $\bar{c}$ of 0.029c.

The field dependence of the resonant modes $n = 1$ up to $n = 5$ was measured at 4.2 K and 1.6 K. The modes $n = 1, 2$ and 3 at 4.2 K, and the modes $n = 3$ and 4 at 1.6 K are shown in fig. 2 and fig. 3, respectively. The theoretical curves according to the high Q theory of Kulik are also given.

At 4.2 K all modes could be measured very well. Only the $n = 1$ and $n = 2$ modes show an unexplained suppression of the principal lobe. The $n = 2$ and $n = 5$ mode show a so-called zero-field step. Such zero-field steps have been reported before [5,12]. They can be described theoretically in terms of vortex motion inside the junction [13].

At 1.6 K modes $n = 1$ and 2 could not be measured well. The junction switched to the Josephson current or to zero field resonances for field values, where the resonance was comparatively small. The modes $n = 3, 4$, and 5 could be measured well.

We fitted the measured curves with the theory of Kulik, using Q as the only free parameter. In all cases we used the same values for λ_L , λ_J , and $\bar{c}$. The Q -value that gives the best fit with the measured curve was used. In all cases a reasonable good fit is obtained. In

Fig.2: Fiske resonance amplitude as a function of the magnetic field at 4.2 K. Modes a) $n = 1$, b) $n = 2$, and c) $n = 3$. Theoretical curves are given by solid lines.

table 1 the resonance voltages V_n , the fitted Q -values, and the Q -values, that can be calculated if the quasiparticle tunneling resistance is the dominating source of losses, are shown. The measured quality factors are about an order of magnitude larger than the values Paternò and Nordman found for high Q Nb-NbOx-Pb junctions [5].

Possible loss mechanisms in the junction barrier cavity determining the Q -factor are the quasi-particle tunneling current, the surface resistance of the electrodes bounding the junction cavity, geometrical non-uniformities (e.g. surface roughness), and radiation from the junction. The measured Q -values are a factor of about 5 to 60 smaller than the values Q_{qp} , calculated from the quasiparticle tunneling resistance. Further the

Fig.3: Fiske resonance amplitude as a function of the magnetic field at 1.6 K. Mode a) $n = 3$ and b) $n = 4$. Theoretical curves are given by solid lines.

measured Q values do not increase strongly with increasing mode number as in the case of the calculated Q_{qp} . These are indications that Q is not dominated by the quasi-particle tunneling resistance.

A temperature dependence of the measured quality factors is observed. Since losses due to geometrical non-uniformities or to radiation are not expected to depend on temperature, we assume that the losses at 4.2 K are dominated by the surface resistance of the Nb/Al bilayers on both sides of the Al_2O_3 cavity.

A quantitative expression for Q due to surface resistance is given in ref.14, where Q is proportional to σ_2/σ_1 , σ_2 and σ_1 being respectively the inductive and resistive component of the surface conductance. At low temperature ($T < T_c/2$) the quality factor mainly depends on temperature through σ_1 , which falls exponentially to zero with decreasing temperature. Thus Q increases with decreasing T as is observed experimentally. At 1.6 K, however, temperature independent losses may have become important limiting the increase of Q .

Fiske resonances in a rectangular junction of $40 \times 40 \mu m^2$ have been measured at 1.6 K. The field was applied at an angle $\theta = \pi/4$. The junction has a critical current of 2.1 mA. The same values for j_c , λ_L , and λ_J apply as for the $50 \times 50 \mu m^2$ junction.

Both single and mixed modes have been observed. The lower index modes are listed in table 2. Especially the mixed modes are well pronounced. The voltages at which the modes occur differ slightly from the theoretically expected values. Some modes, e.g. the (2,0) mode, could

mode n m	V_{nm} meas. (mV)	V_{nm} calc. (mV)
1 0	0.24	0.24
1 1	0.39	0.34
2 1	0.56	0.53
2 2	0.70	0.68
3 0	0.73	0.72
3 1	0.80	0.76
3 2	0.86	0.86
3 3	1.02	1.01

Table 2: Resonant modes observed in a rectangular junction of $40.40 \mu\text{m}^2$ in a field at an angle of $\theta = \pi/4$.

not be observed.

The (1,1) mode, measured at 0.39 mV, is plotted as a function of the field in fig. 4, with a curve according to the low Q model of Nerenberg et al. for a realistic value $Q = 200$. The places of the extrema agree very well with this model. Also the small lobe is correctly predicted. The measured resonance amplitude of the main lobe shows a large deviation from theory, probably because the condition that the resonance current should be much smaller than the Josephson current is not fulfilled for this high Q junction.

VI. CONCLUSIONS

We measured one and two-dimensional Fiske resonances in Nb/Al,Al-oxide,Al/Nb Josephson junctions up to the 5th mode. This is possible because of the measuring technique used and the high Q-values in these devices. The measurements on one-dimensional modes are very well described by the theory developed by Kulik. High quality factors, ranging from 40 to 450, have been found. They depend strongly on temperature, indicating that microwave losses are dominated by a temperature dependent loss mechanism, probably the surface resistance of the Al/Nb layer adjacent to the dielectric barrier.

The field dependence of the (1,1) mixed mode is well described by the theory of Nerenberg et al. The measured resonance voltages of the mixed modes are well predicted by theory.

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