Performance of Thermally Excited Resonators

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Abstract

A study of electrothermal excitation of micro-machined silicon beams is reported. The temperature distribution is calculated as a function of the position of the transducer, resulting in stress in the structure which reduces the resonance frequency. Test samples are realized and measurements of resonance frequency, vibration shape and vibration amplitude are carried out. There is a satisfactory agreement between theory and experiment at small thermal stresses. Near the buckling load we find distinct deviations from theory which are ascribed to mechanical imperfections of the beams.

1. Introduction

Recently, much interest has been shown in sensors based on integrated resonators with a measurand-dependent resonance frequency [1-7]. Such a resonator has to be excited and the resulting vibration detected in order to measure the resonance frequency. Electro- or optothermal excitation of the resonator has advantages from a technological point of view. Wilfinger *et al* [8] were the first to use electrothermal excitation for a resonator. Static heat elevation of the resonating structure is inherent in thermomechanical excitation, resulting in mechanical stresses which influence the resonance frequency and vibration amplitude. Therefore it is interesting to design a resonator with its excitor constructed so that there is a minimum temperature elevation at a maximum vibration amplitude. In this paper we focus our attention on a clamped-clamped beam resonator with a thin-film resistor as electrothermal excitor. We calculate the influence of the position of the thin-film resistor on the average temperature elevation and the resonance frequency shift as well as its influence on the vibration amplitude of the beam and compare the results with measurements on realized test samples.

2. Mechanical Model

The resonator (Fig 1(a)) is regarded as a homogeneous, prismatic clamped-clamped beam forced into vibration by a distributed driving moment $M_d(x)$. Without damping its vibration is described by the partial differential equation

$$EI \frac{\partial^4 W(x, t)}{\partial x^4} - \sigma A \frac{\partial^2 W(x, t)}{\partial x^2} + \rho A \frac{\partial^2 W(x, t)}{\partial t^2} = \frac{\partial^2 M_d(x, t)}{\partial x^2} \quad (1)$$

with $W(x, t)$ the displacement function, EI the bending stiffness and A the cross section of the beam, σ the axial stress and ρ the specific mass of the beam material. Solution of eqn (1) (with the help of Laplace transformation) for $\sigma = 0$ and a harmonic place-dependent driving moment $M_d(x)$ (see Fig 1(b)) results in an analytical expression

Fig 1 (a) Simplified geometry of a beam, (b) driving moment $M_d(x)$, (c) temperature function with uniform heat generation in resistor and finite conductivity of the substrate

for the frequency-dependent vibration shape $W(x, t)$. The model results in line symmetrical (even modes) or point symmetrical (odd modes) shape functions $W(x)$ for the beam resonance frequencies (see Fig. 4). It is seen from eqn. (1) that the vibration amplitude depends on the shape of the moment function $M_d(x)$, hence on x_o/l and δ/l in Fig. 1(b). The expected vibration amplitude at the first resonance frequency as a function of x_o/l (with $\delta/l = 0.2$ and $M_d = \text{constant}$) is given (see Fig. 8). The eigenvalues of eqn. (1) result in the stress-dependent resonance frequencies of the beam. The first-order resonance is approximated with

$$f_{\text{res}} \cong f_o (1 + \sigma/\sigma_{\text{cr}})^{1/2} \quad (2)$$

where f_o is the resonance frequency with no stress in the beam ($\sigma = 0$), $f_o = 1.028(E/\rho)^{1/2}h/l^2$, with E the effective Young's modulus, ρ the specific mass and h and l the thickness and the length of the beam, respectively. The critical Euler buckling load $\sigma_{\text{cr}} = 4\pi^2 EI/Al^2$ [9]

3 Heat Generation and Thermal Model

We now need to calculate the stress from the temperature distribution. The film resistor on top of the beam (Fig. 1(a)) is driven by a harmonic a.c. voltage $U_{\text{ac}} \cos \omega t$ superimposed on a d.c. voltage U_{dc} . The electrothermally generated heat is given by

$$P(t) = \frac{(U_{\text{dc}}^2 + 0.5 U_{\text{ac}}^2)}{R} + \frac{2 U_{\text{dc}} U_{\text{ac}} \cos \omega t}{R} + \frac{0.5 U_{\text{ac}}^2 \cos 2\omega t}{R} \quad (3)$$

with R the electrical resistance of the resistor. The generated heat consists of one static component $P_{\text{stat}} = (U_{\text{dc}}^2 + 0.5 U_{\text{ac}}^2)/R$ and two dynamic components of amplitude $P_{\text{dyn1}} = 2 U_{\text{dc}} U_{\text{ac}}/R$ and $P_{\text{dyn2}} = 0.5 U_{\text{ac}}^2/R$ respectively. The static component results in a static temperature distribution. With a uniform heat generation over the resistor and assuming that all the heat flows through the beam (by means of conduction) towards the clamped edges which are assumed to be ideal heat sinks, the temperature distribution is parabolic under the resistor and linear outside, as shown in Fig. 1(c). For the heat generation concentrated at the centre x_o of the resistor, the average temperature elevation ΔT_{av} integrated over the beam depends on x_o/l and is given by

$$\Delta T_{\text{av}} = P_{\text{stat}} \frac{l[x_o/l - (x_o/l)^2]}{2bh\lambda} \quad (4)$$

with b , h and l the width, the thickness and the

length of the beam, respectively and λ the heat conductivity of the beam material (assumed here to be independent of the temperature elevation). With $\lambda = 157 \text{ W/(m}^\circ\text{C)}$ for silicon, beam dimensions $b = 1 \text{ mm}$, $h = 30 \text{ }\mu\text{m}$, $l = 10 \text{ mm}$ and position $x_o/l = 0.5$, we find that $\Delta T_{\text{av}}/P_{\text{stat}} = 265^\circ\text{C/W}$. With a distributed heat generation over the resistor and $\delta/l = 0.2$, we find that $\Delta T_{\text{av}}/P_{\text{stat}} = 261^\circ\text{C/W}$ (see Fig. 6). Two-dimensional finite-element numerical analysis with the heat sink at the substrate bottom shows a temperature distribution which deviates from the analytical model at the resistor location and the clamped edges of the beam. At both locations a temperature difference between the top and the bottom surfaces of the beam results in a local static mechanical moment. Due to the finite thermal conductivity of the substrate, the numerical analysis predicts a slightly higher (2%) value for average temperature elevation ΔT_{av} . If we assume an extra thermal impedance R_s between the beam ends and the surroundings, the average temperature ΔT_{av} relative to the surroundings significantly increases (see Fig. 6, where the extra impedance is $0.2 R_b = 0.2 l/bh\lambda$).

The average temperature elevation ΔT_{av} of the beam induces a compressive axial stress $\sigma = -E\alpha \Delta T_{\text{av}}$, with E the Young's modulus and α the coefficient of thermal expansion. The relation between the first-order resonance frequency f_{res} and P_{stat} (see Fig. 5) can be found from the relation between the stress and the average temperature elevation ΔT_{av} , the critical Euler load σ_{cr} and eqn. (2). The critical static power generation at which the beam will buckle is given by

$$P_{\text{stacbr}} = \frac{2\pi^2}{3} \frac{\lambda b}{\alpha[x_o/l - (x_o/l)^2]} \left(\frac{h}{l}\right)^3 \quad (5)$$

with x_o/l the relative transducer position. For a beam with $b = 1 \text{ mm}$, $h = 30 \text{ }\mu\text{m}$, $l = 10 \text{ mm}$, $x_o/l = 0.5$, $\alpha = 2.5 \times 10^{-6}/^\circ\text{C}$ and $\lambda = 157 \text{ W/(m}^\circ\text{C)}$ the beam will buckle at $P_{\text{stacbr}} = 44.6 \text{ mW}$ with $\Delta T_{\text{avcr}} = 11.8^\circ\text{C}$.

The beam vibration is excited with the additional harmonic temperature distribution induced by the dynamic power with amplitude P_{dyn1} . The thermal gradients (and so a thermal moment) in the thickness direction of the beam result in a proportional mechanical moment $M_d(x, t)$, which excites the beam into vibration [10].

4. Realization of Test Samples

Figure 2 shows a test sample with a beam width of 1 mm and a length of 10 mm. The thin-film resistor covers the width of the beam by 2 mm. The centre positions x_o are at 0, 1, 2, 3, 4

Fig 2 Lay-out of the clamped-clamped beams with a Ni-Fe film as electrothermal transducer and aluminium metalization

or 5 mm from the beam end with a final beam thickness of 30 μm

The starting material is a 2 in (100) phosphorus-doped n-type silicon wafer polished on one side. First alignment markings on both sides of the wafer are realized. Then a SiO_2 layer of 1 μm (3 h, 1150 $^\circ\text{C}$, wet N_2) is grown and removed from the top side of the wafer. The resistor is realized from an evaporated Al film (0.3 μm) and a Ni-Fe layer (20–80%, RF sputtered, 250 W, 6 min, 50 nm). Both layers are patterned by wet chemical etching. The beams are dry-etched from a membrane (anisotropic etch KOH in H_2O , 50 g/100 ml, 75 $^\circ\text{C}$) in a SF_6 plasma (1.2 $\mu\text{m}/\text{min}$) with a positive resist etch mask. The back SiO_2 layer is removed to reduce the axial stress in the beams.

5. Experimental

We used a Michelson interferometer and a Mach-Zehnder heterodyne interferometer for measuring the beam vibration. The Michelson interferometer is based on a 5 mW He-Ne gas laser with a beam width of 0.6 mm. For constant sensitivity the interferometer is kept in quadrature with a low frequency feedback loop and a piezo-controlled reference mirror. A gain/phase analyser (HP 4194A) is used to determine the resonance frequencies and their quality factors. Figure 3 shows a typical result obtained with this set-up. The Mach-Zehnder heterodyne interferometer is based on a 10 mW He-Ne gas laser and has a measuring spot of 40 μm diameter in combination with a computer-controlled device translation table. Resonance frequency shifts of 30% are observed due to mounting the wafer on the device

Fig 3 Vibration amplitude of a beam as a function of the frequency measured with a Michelson interferometer. The dip at the first-order resonance $f = 800$ Hz is due to the strong non-linearity of the interferometer. The second resonance at $f = 2$ kHz does not belong to the collection of transversal resonance modes.

holder and to forces induced by the probe pens used for contacting the resistor.

6. Results and Discussion

Figure 4 shows both the measured and the predicted beam vibration shapes for the first- and second-order resonances. The measured shapes are qualitatively similar to the predicted ones. The differences may be due to the temperature distribution, or inhomogeneous mechanical properties and are the subject of current investigation.

The resonance frequencies of the beams as a function of the static power P_{stat} are given in Fig 5. The frequency values are normalized at the resonance frequency for the stress-free beam. With a thickness of $h = 30$ μm we expect $f_0 = 2.633$ kHz. The measured f_0 values for the different beams vary from $f_0 = 2.15$ kHz to

Fig 4 Calculated and measured shapes of the beam vibration $W(x)$ at the first- and second-order resonance frequencies. The transducer is at $x_0/l = 0.1$ with $\delta/l = 0.2$.

$f_0 = 2.5$ kHz The measured relation of the first two resonance frequencies $f_1/f_0 = 2.76$ indicates that the beams are stress-free ($\sigma \rightarrow 0$) for $P_{\text{stat}} \rightarrow 0$. With increasing power P_{stat} we see that the measured resonance frequencies diverge from the expected ones when power P_{stat} approaches the critical power P_{statcr} . This is probably caused by the clearly visible static deflection of the beam before the actual buckling load is reached. This indicates that the measured resonance frequency behaviour is influenced by (initial) beam imperfections. A similar effect is found for resonating membranes [3] and thin-film microbridges [2].

The average temperature elevation $\Delta T_{\text{av}}(x_0/l)/P_{\text{stat}}$ is recalculated from the relative frequency change df/dP_{stat} for $P_{\text{stat}} \rightarrow 0$ (see Figs 5 and 6) and agrees well with the analytical thermal model, which predicts that the beams have an average temperature elevation of $\Delta T_{\text{av}} = 118^\circ\text{C}$ at the critical buckling load. The mechanical vibration amplitude W was shown to be proportional

Fig 5 Calculated (using eqn (5)) and measured normalized frequency shifts as a function of the heating power P_{stat} . The parameter is the relative position of the thin-film transducer x_0/l .

Fig 6 Calculated and measured average temperature elevations ΔT_{av} of the beam as a function of the relative transducer position x_0/l . The dotted line gives ΔT_{av} for an extra thermal impedance R_s between the beam ends and the surroundings.

Fig 7 The vibration amplitudes relative to P_{dyn1} as a function of the static power P_{stat} . The lines show the expected behaviour assuming constant vibration energy. The parameter is the relative transducer position x_0/l .

to the dynamic power P_{dyn1} only if the static power P_{stat} was kept constant. On the contrary, when the static power P_{stat} is varied, the expected vibration amplitude increases according to $W \sim 1/(1 - P_{\text{stat}}/P_{\text{statcr}})^{1/2}$, assuming a decreasing frequency according to eqn (2) and a constant vibration energy. The measured stronger increase for W as illustrated in Fig 7, may be caused by the frequency dependence of the driving moment M_d or by the static deflection also induced by P_{stat} . The vibration amplitude W as a function of the relative transducer position x_0/l shows a poor resemblance to the theory, as shown in Fig 8. Possible causes for the differences are (i) the different quality factors of the various beams, (ii) excitation of the beam due to the dynamic elongation under the resistor (another excitation mechanism), and (iii) the vibration amplitude depends on a static beam deflection (for $P_{\text{stat}} = 0$ due to the realization process).

Fig 8 Calculated and measured vibration amplitudes as a function of the relative transducer position x_0/l (measured for $P_{\text{stat}} \rightarrow 0$, see Fig 7).

7. Conclusions

Thermal excitation inherently causes a rise in temperature of the resonator. This temperature rise induces an axial stress which significantly reduces the resonance frequency. This influence is much larger than the influence of the temperature dependency of the material parameters ρ and E . The influence of the transducer position x_0/l on the average temperature elevation ΔT_{av} and the critical buckling power P_{statcr} is quantitatively described and shows a good agreement with experimental results. The extreme sensitivity to stresses indicates that the thermal as well as the mechanical packaging of resonators may have a significant influence on the performance. The beam mechanics in the vicinity of the buckling load and the effectiveness of the excitation mechanism as a function of the axial stress σ need investigating further.

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