Frequency Dependence of Thermal Excitation of Micromechanical Resonators

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Abstract

A theoretical and experimental study is presented on thermal excitation of micromechanical resonators. It is shown that there is a turnover frequency, ω_t , which is related to the square of the ratio of the penetration depth of the thermal wave into the structure and its thickness. For constant power dissipation at the surface of the structure, the bending moment is independent of the frequency ω at $\omega \ll \omega_t$, while at $\omega \gg \omega_t$, it is proportional to ω^{-1} . At high frequency we obtain a phase shift of $-\pi/2$. Vibration amplitudes and turnover frequencies measured on clamped beam structures agree well with the theory.

1. Introduction

Micromechanical resonators form an attractive field of research, since they are the basic part of a class of reliable and accurate sensors and actuators. One of the methods of driving a beam or membrane into resonance is via periodic heating of the surface in order to induce local strains which may result in driving moments if the strain varies at different levels in the beam or the membrane. Heating at one side of the structure will have the desired effect, as has already been demonstrated in practice [1-3].

In this paper we concentrate on the dynamic properties of electrothermal transduction. In particular, we are interested in how the mechanical moment, which forces the structure into vibration, is related to the power dissipated in a resistor on top of the

structure, the geometry of the structure (here the thickness of the beam) and the driving frequency. A good insight into this transduction is required for the careful design of electrothermally driven resonators, which are technologically simple to produce, be it at the expense of the power efficiency. This study is also a necessary step in the process that ultimately leads to the design of resonators which are thermally driven via an optical or otherwise radiative beam of energy impinging on the resonator.

2. Theory

The problem can be described as follows (see Fig. 1). By means of a thin-film resistor, we have a heat source on top of a resonator beam. The resistor is driven by a harmonic a.c. voltage $U_{AC} = U_{ac} \cos \omega t$ superimposed on a d.c. voltage U_{DC} . Assuming a temperature-independent resistance R , the electrothermally generated power is given by

$$P_i(t) = (U_{DC} + U_{AC})^2/R$$

or

$$P_i(t) = (U_{DC}^2 + 0.5U_{ac}^2)/R + 2U_{DC}U_{ac} \cos \omega t/R + 0.5U_{ac}^2 \cos 2\omega t/R \quad (1)$$

$$P_i(t) = P_{stat} + P_{dyn1}(t) + P_{dyn2}(t)$$

The design of the resistor film geometry is such that we may assume a homogeneously distributed dissipation of power. The heat flows into the silicon beam and the surrounding gas. Since the latter flow is only one percent of the total, we simply neglect it.

Fig. 1. Clamped beam with thin-film resistor.

The generated heat given in eqn. (1) has one static and two dynamic components. Underneath the resistor film the heat enters the beam perpendicular to the surface. The static component of the heat flow, together with the ambient conditions, will cause a static background temperature distribution, determined by the three-dimensional geometry of the beam and its suspension, which we take for granted [4]. The dynamic components in which we are interested will be modelled in only one dimension (the y -direction). This is a reasonable assumption as long as the beam thickness h is small compared to the length l , and width b , of the area covered by the resistor.

With this assumption we have a boundary-value problem governed by the equation [5]

$$\frac{\partial T}{\partial t} = \left(\frac{k}{\rho c} \right) \frac{\partial^2 T}{\partial y^2} \quad (2)$$

with k the thermal conductivity, ρ the specific mass and c the heat capacity of the silicon. The term $k/\rho c$ is referred to as the thermal diffusion constant α . At the surface ($y=0$) $P(t)$ is dissipated, causes a heat flow and consequently gives rise to a temperature distribution in the beam. Since eqn. (2) is linear in T , the solution $T(y, t)$ consists of three parts, attributed to the terms of $P(t)$ in eqn. (1):

$$T(y, t) = T_{\text{stat}}(y) + T_{\text{dyn1}}(y, t) + T_{\text{dyn2}}(y, t) \quad (3)$$

In what follows, we concentrate on the dynamic part with the same frequency (ω) as the input voltage. Solving eqn. (2) by separation of variables in complex form with

$$T_{\text{dyn1}}(y, t) = \text{Re}\{\theta(y, t)\} \quad (4)$$

we obtain the general solution

$$\theta(y, t) = [B_1 \exp(\sigma y) + B_2 \exp(-\sigma y)] \exp(i\omega t) \quad (5)$$

with $\sigma = (1+i)/\delta$ and $\delta = (2\alpha/\omega)^{1/2}$. Note that this solution consists of two (damped) wavelike parts in the positive (B_2) and the negative (B_1) y -directions. The complex constants B_1 and B_2 are determined by the boundary conditions at $y=0$, where the heat flows in, giving rise to the condition $\partial\theta(y, t)/\partial y = -P_0/k \exp(i\omega t)$ (P_0 is short for the complex amplitude of $P_{\text{dyn1}}(t)/\text{m}^2$) and at $y=h$, where we assume $\partial\theta(y, t)/\partial y = 0$. The latter assumption is justified because the thermal conductivity of the gas is small compared to that of silicon. This leads after some calculation to the final result

$$\theta(y, t) = \left[\frac{\exp(-\sigma h)}{2 \sinh(\sigma h)} \exp(\sigma y) + \frac{\exp(\sigma h)}{2 \sinh(\sigma h)} \times \exp(-\sigma y) \right] \frac{P_0}{\sigma k} \exp(i\omega t) \quad (6)$$

with $\sigma = (1+i)/\delta$ and $\delta = (2\alpha/\omega)^{1/2}$.

We distinguish two limiting cases, $\delta \ll h$ and $\delta \gg h$. For $\delta \ll h$ the ratio $B_1/B_2 = \exp[-2(1+i)h/\delta]$ approaches zero, so B_1 can be neglected. In this case, the temperature distribution is given by

$$T_{\text{dyn1}}(y, t) \cong \theta_0 \exp(-y/\delta) \times \cos(\omega t - y/\delta - \pi/4) \quad (7)$$

with $\theta_0 = P_0\delta/2k$, the amplitude of the temperature variation at the surface $y=0$. This solution is depicted in Fig. 2. We see a

Fig. 2. Time- and position-dependent temperature distribution; note the phase difference of $\pi/4$ at $y=0$ and the phase lag within the beam.

damped wave structure, with δ the penetration depth of this temperature distribution. In this limit the beam can be considered as 'thermally thick', i.e., the fluctuating distribution does not reach the underside of the beam.

Since $\delta = (2\alpha/\omega)^{1/2}$, every beam has a critical frequency ω_c , below which the beam can no longer be seen as 'thermally thick', the constant B_1 can no longer be neglected and we get a temperature distribution which is the superposition of two wavelike parts. In the end we have $\delta \ll h$, a 'thermally thin' film. Figure 3 shows the modulus of the (complex) temperature distribution $\theta(y, t)$ as a function of the ratio δ/h (note that the actual temperature $T_{\text{dyn1}}(t)$ is the real part of $\theta(y, t)$).

This information about the temperature distribution must be translated into information about the mechanical moments which ultimately drive the beam. Therefore we define the thermal moment M_t as the first moment of the temperature distribution over the beam thickness:

$$M_t(t) = (1/h) \int_0^h \theta(y, t)(h/2 - y) dy \quad (8)$$

It can be proved that this thermal moment is directly proportional to the mechanical moment M_b ($M_b(t) = Ebh\beta M_t(t)$, with β the

Fig. 3. Modulus of (complex) temperature distribution as a function of the position y/h and as a function of the ratio of thermal diffusion length and beam thickness.

Fig. 4. Mechanical moment $M_b(t)$. Modulus and phase of the moment as a function of the frequency; beam width $b = 1$ mm, beam thickness $h = 270 \mu\text{m}$, $E = 1.69 \times 10^{11} \text{ N/m}^2$, $\nu = 0.06$, $\beta = 2.3 \times 10^{-6} \text{ K}^{-1}$, $k = 100 \text{ W/mK}$, $\alpha = 6.2 \times 10^{-5} \text{ m}^2/\text{s}$, $P_{\text{dyn1}} = 10^4 \text{ W/m}^2$, $f_t = 1620 \text{ Hz}$.

thermal expansion coefficient [6]). After some calculations we finally arrive at

$$M_b(t) = P_0 \beta EI (12/k) \times \frac{1}{\gamma^2} \left\{ \frac{1}{2} + \frac{1}{\gamma} - \frac{2}{\gamma[1 + \exp(-\gamma)]} \right\} \times \exp(i\omega t) \quad (9)$$

with EI the bending stiffness of the beam and $\gamma = h\sigma = (1 + i)h/\delta$.

Figure 4 depicts M_b in terms of its phase and its amplitude. Again we have two asymptotic regions, separated by a characteristic turnover frequency

$$\omega_t = 12\alpha/h^2 = 6\omega\delta^2/h^2 \quad (10)$$

Note that $\delta = \delta(\omega)$ and that for $\omega = \omega_t$, $\delta = 0.408h$ and the phase of M_b is equal to $-\pi/4$. For $\omega \ll \omega_t$ eqn. (9) becomes

$$M_b(t) = P_0(\beta/2k)EI \exp(i\omega t) \quad (11)$$

The amplitude is frequency independent and because $M_b(t)$ is proportional to EI , the static displacement, which is proportional to M/EI , will be independent of the beam thickness. For $\omega \gg \omega_t$ we find

$$M_b(t) = -iP_0 \frac{Ebh\beta}{2\rho c\omega} \exp(i\omega t) \quad (12)$$

We see that the moment is inversely proportional to ω , while the phase has shifted over -90° . These results are of great relevance with respect to the design of thermally driven resonator structures.

3. Experiments

For the experiments on electrothermal excitation of micromechanical devices, we used two types of beams. Type A was produced by thinning and sawing beams from a (100)-oriented n-type silicon wafer, after a NiFe resistive film and Al contact film were deposited and patterned. The beams were fixed to a supporting frame using a cyano acrylic glue. Type B was produced from the same type of wafer, but by micromachining. The beam is connected to the support in one piece as shown in Fig. 1, using wet and dry etching after the deposition of the structures on top of the beam. Further details are given in ref. 6.

Transfer functions of the devices are measured with the help of a Michelson interferometer set-up and a network analyser. Accurate measurements of absolute vibration amplitudes were carried out with the help of a Mach-Zehnder heterodyne interferometer. For details, again see ref. 6.

The vibration amplitude of the beam turned out to be perfectly linear with P_{dyn} and the slope of the curves was in reasonable agreement with the predicted values as calculated from eqn. (11). We found a mean deviation of 10% for the samples tested.

In Fig. 5(a) we give an example of the experimentally derived curves for the amplitude and phase of the driving moment as a function of frequency. The agreement with the simulated results on the basis of the theory of Section 2, given in Fig. 5(b), is excellent. We have chosen this example because it shows the turnover frequency as well as the first resonance, while the turnover frequency is not in the region of influence of the resonance.

We have measured turnover frequencies for a number of beams with different thicknesses. According to eqn. (10), ω_t is inversely proportional to h^2 . Experimental results are shown in Fig. 6 together with a plot of ω_t (eqn. (10)), taking a silicon thermal conductivity of $k = 100 \text{ W/mK}$. The agreement is very good.

Fig. 5. (a) Measured and (b) expected beam tip displacement $W(l)$ as function of the frequency. Beam width $b = 2 \text{ mm}$, $h = 270 \mu\text{m}$, $l = 3.4 \text{ mm}$, $E = 1.69 \times 10^{11} \text{ N/m}^2$, $\nu = 0.06$, $\beta = 2.3 \times 10^{-6} \text{ K}^{-1}$, $k = 100 \text{ W/mK}$, $P_{\text{dyn}} = 3 \times 10^4 \text{ W/m}^2$. Measured turnover frequency $f_t = 1700 \text{ Hz}$, resonance frequency $f_0 = 35 \text{ kHz}$.

Fig. 6. Measured and expected turnover frequency $f_t (= 6\alpha/\pi h^2)$ as function of the beam thickness h . Thermal properties of the beam: $k = 100 \text{ W/mK}$, $\alpha = 6.2 \times 10^{-5} \text{ m}^2/\text{s}$.

4. Conclusions

The frequency-dependent temperature distribution in a single-layer beam driven with an electrothermal dissipator has been calculated on the basis of a one-dimensional heat flow model. The thermally induced mechanical moment is proportional to the first moment of the temperature distribution.

The significance of the dynamic behaviour of the thermal distribution is expressed in the

appearance of a characteristic turnover frequency ω_t , at which the phase between the applied heat and the induced mechanical moment shifts over 90° . The magnitude of the induced moment is constant for $\omega < \omega_t$ and proportional to ω^{-1} for $\omega > \omega_t$. This is of relevance, of course, for the design of any device based on electrothermal transduction. One important aspect is the choice of the beam dimensions to obtain the desired resonance frequencies relative to the characteristic turnover frequency.

Another important aspect is the design (in terms of the phase behaviour) of the electronic circuit in a closed-loop resonator.

The theory has been tested rather thoroughly with a number of experiments. The general outcome of these experiments gives a good confirmation of the theory as well as of the quality of both types of samples used.

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