Applications of a model for silicon resonant sensors

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Abstract

A recently developed model for excitation and detection mechanisms of silicon resonant sensors is experimentally verified by applying it to practical devices. Various system aspects of resonators with piezoelectric and electrostatic excitation/detection come into focus. The experimental results agree reasonably well with simulation results obtained by the model.

1. Introduction

Silicon resonant sensors [1, 2] are suitable for measurement of mechanical quantities (force, pressure, acceleration, etc.) with high precision. The operating principle of silicon resonant sensors is that the resonance frequency of a micromachined silicon structure (for example, a prismatic beam) shifts as the measurand changes. The resonance frequency is the sensor output signal. Recently, we developed a systematic unifying model to describe the excitation and detection mechanisms of this sensor family [3]. The model is based on a system approach: the resonator, including the transducers for excitation/detection is regarded as a full electromechanical system. In ref. 3, the model is derived, and a comparative review of excitation and detection mechanisms for micromechanical resonators is given. In this paper, we demonstrate the practical applicability of the model.

2. The sensor model

In order to activate and sense the vibration, one or more electromechanical transducers for excitation and detection have to be incorporated in the sensor system. For this purpose, various mechanisms are suitable. Here, we restrict ourselves to mechanisms of the reversible type. Energy can be exchanged with the system through the transducers. The energy flow through a single transducer is characterized by an intensive variable (e.g., a voltage u) and a corresponding extensive variable (e.g., a charge q). The power flow equals the product of the intensive variable and the time derivative of the extensive variable, which is called extensity flow variable, or just flow. Every transducer serves as a power port

of the system. The number of electrical ports of the system equals the number of electromechanical transducers.

The system has also mechanical ports. Since the mass, stiffness and damping of the resonator are distributed, an infinite number of mechanical ports is required. An elegant discretization is obtained when the mechanics of the resonator is described in terms of its eigenstates, also called vibration modes. In that case, the modal displacements y_i are the extensive variables, and the modal forces P_i are the intensive variables. The modal approach is similar to a Fourier series: the shape of the vibrating resonator is expanded in the mode shapes. The number of mechanical ports of the system equals the number of modes taken into account. Since resonant sensors are operated near resonance, good approximations can be obtained with only a few mechanical ports.

Thus, the total system has $n+m$ energy ports, where m stands for the number of electromechanical transducers and n for the number of modes taken into account. A bond-graph representation of the sensor system is shown in Fig. 1. Small harmonic variations around an equilibrium state can be described by the matrix equation

$$\begin{bmatrix} \dot{\bar{u}} \\ \bar{P} \end{bmatrix} = \begin{bmatrix} \mathbf{E} & \mathbf{C} \\ \mathbf{C}^T & \mathbf{M} \end{bmatrix} \begin{bmatrix} \bar{q} \\ \dot{\bar{y}} \end{bmatrix} \quad (1)$$

where $\mathbf{E}$, $\mathbf{M}$ and $\mathbf{C}$ are an electrical, a mechanical and an electromechanical coupling matrix, respectively, $\bar{u}$ and $\bar{q}$ are vectors of length m , and $\bar{P}$ and $\dot{\bar{y}}$ are vectors of length n . The dot indicates differentiation with respect to time and the overbar indicates amplitude. Equation (1) is called the constituent equation of the system.

Fig 1 Bond-graph representation of a resonant sensor with m electromechanical transducers for excitation/detection purposes and n vibration modes taken into account

Fig 2 Cross section (a) and electrical scheme (b) of the piezoelectrically operated resonant force sensor

3. Electrical transfer function of a piezoelectric resonator

At the MESA Institute, research is performed on a piezoelectrically operated resonant force sensor [4]. The sensor has two electromechanical transducers for excitation and detection. The transducers consist of a thin piezoelectric ZnO film sandwiched between a tungsten and an aluminium electrode. This three-layer structure is placed on top of the resonator (see Fig 2). Only the first two modes of vibration are taken into account. Hence, the system model has two electrical and two mechanical ports.

The mechanical submatrix $\mathbf{M}$ of the constituent eqn (1) merely consists of the mechanical spring effect of the modes. Off-diagonal elements of this matrix, as well as the electrical submatrix $\mathbf{E}$, are zero. Diagonal elements of $\mathbf{E}$ represent the impedance of the capacities of the ZnO layers. Finally, the coupling matrix $\mathbf{C}$ reflects the electromechanical transducing capabilities of the excitation/detection elements. Important parameters in the matrix elements of $\mathbf{C}$ are the piezoelectric coefficient, d_{31} of the ZnO, the intrinsic coupling factor of the ZnO, the thickness of the piezoelectric film and a geometrical factor which is a measure of the efficiency of a transducer to couple energy to or from a specific mode. For the exact details concerning the matrix elements we refer to refs 3 and 5.

The system is described by eight variables: four intensive and four flow variables. To calculate the electrical transfer function, i.e., the ratio $\bar{u}_2/\bar{u}_1$, seven equations are required. Four equations are obtained from the constituent equation. The other three are

phenomenological equations. They describe how the power ports are loaded or controlled by the outside world. The two mechanical ports are loaded by viscous friction forces (which can be obtained from modal quality factors) and inertia (the modal masses). The electrical port used for the detection is loaded by a resistance R_2 . The excitation port is controlled by a voltage source.

Simulation results of the transfer function $\bar{u}_2/\bar{u}_1$ are given in Fig 3. Simulations are performed for three values of R_2 , namely, $R_2 = 0.1R_{\text{char}}$, $R_2 = R_{\text{char}}$ and $R_2 = 10R_{\text{char}}$. R_{char} is the modulus of the impedance of the detection transducer at the first resonance frequency $R_{\text{char}} = 1/\omega_1 C_{\text{ZnO}}$, where ω_1 is the angular resonance frequency of the first mode, and C_{ZnO} is the capacity of the ZnO transducer.

The load impedance R_2 forms a high pass filter in combination with the capacity of the transducer C_{ZnO} . The turnover frequency of this inherent filter is given by $f_{\text{to}} = 1/(2\pi C_{\text{ZnO}} R_2)$. The filter characteristic is clearly visible in Fig 3. If $R_2 \ll R_{\text{char}}$, the resonance peaks are on the 20 dB/decade flank of the filter and the loop of the second mode is larger than that of the first mode. For $R_2 \gg R_{\text{char}}$, both resonance frequencies are higher than the turnover frequency and the loop of the first mode is the larger one. Another observation is that the phase at resonance shifts from 0° to -90° as the load impedance varies from $R_2 \ll R_{\text{char}}$ to $R_2 \gg R_{\text{char}}$. Considerations like this are of the utmost importance when designing feedback electronics for the oscillator.

The above characteristics have been experimentally verified. Figure 4 shows the measured polar plots for frequencies around the first resonance frequency. The rotational orientation and the sizes of the resonance loop are determined by position of the turnover frequency of the filter with respect to the first resonance frequency. The aforementioned high-pass frequency characteristic manifests itself by a semicycle in the first quadrant of the polar plot. This is shown in Fig 5.

Fig 3 Bode diagram of the piezoelectric force sensor for three values of R_2

Fig 4 Measured polar plots of the transfer function of the piezoelectric force sensor for three values of R_2 . The plots are for frequencies near ω_1

Fig 5 Measured polar plot of the first two resonances of the piezoelectric force sensor. The Figure also shows the last part of a hemicycle, which is a consequence of the inherent high pass filter

Fig 6 Cross section (a) and electrical scheme (b) of the test structure for determination of piezoelectric properties

4. Characterization of piezoelectric materials

The sensor model can be exploited quantitatively to determine piezoelectric properties of ZnO films using the resonance method. For this purpose, a cantilever resonator with one ZnO transducer (see Fig 6) was used. The device can be modelled as a system with one electrical and one mechanical port (one mode is taken into account).

The impedance of this device, measured at the electrical port, can be written as

$$Z(\omega) = \frac{\bar{u}}{q} = \frac{1}{j\omega C_1} + \frac{1}{j\omega C_2} \frac{1}{1 + j \frac{\omega}{\omega_1} \frac{1}{Q_1} - \frac{\omega^2}{\omega_1^2}} \quad (2)$$

where the constants C_1 and C_2 are composed of piezoelectric, electric and mechanical properties of ZnO and the dimensions/shape of the transducer and the resonator. C_1 and C_2 can be obtained from the matrix elements of the constituent equation. For the exact expressions of C_1 and C_2 we refer to refs 3 and 5.

The first term of eqn (2) results in a straight line extending along the negative imaginary axis of the polar impedance plot (see Fig 7). The second term only contributes near resonance and results in a loop. Properties of the ZnO can be obtained from the diameter of this loop, provided that the dimensions of the structure and its Q factor are known. $\phi_{\text{loop}} \propto (d_{31}/\epsilon_{33}s_{11})^2$.

The relative permittivity ϵ_{33} can be determined by a capacitance measurement. The compliance s_{11} and d_{31} cannot be separated without performing a pure mechanical measurement. The ratio $d_{31}/\epsilon_{33}s_{11}$ can be interpreted as a figure of merit for piezoelectrically operated resonators.

Two samples were measured. A typical impedance measurement is shown in Fig 7. It is noteworthy that the measured diameter of the loop was corrected for the series and parallel capacitances, which are the result of an SiO_2 layer and the extension of the film on the substrate, respectively (see Fig 6). Table 1 summarizes the results. Literature values of bulk ZnO are also

Fig 7 Polar impedance plot of the test structure

TABLE 1 Material properties of sputtered ZnO film

Sample	ϵ_{33}/ϵ_0	d_{31}/s_{11} (C/m ²)	d_{31} (pC/N)
I	9	0.19	-1.5*
II	9	0.25	-2.0*
bulk	11.3	0.64	-5.1

*Value determined using the s_{11} value of bulk ZnO

given The ZnO was sputtered with a substrate temperature of 425 °C, an RF power of 1800 W for 1 min, and 1000 W for the remainder of the time (2 h) The oxygen pressure was 0.5 Pa The thickness of the film was 3.68 µm It was assumed that the whole film shows a homogeneous piezoelectric activity

5. Loaded Q s

Since high Q resonant sensors generally have a high resolution, a high Q is usually pursued The Q of the resonance, being defined as the ratio of the stored vibration energy and the periodically dissipated energy, is often determined by mechanical considerations However, energy can be dissipated not only at mechanical ports, but also at electrical ports by resistively loading the transducers In this manner, it is possible that a resonator exhibits a Q which is considerably lower than the Q based on mechanical considerations

The Q , based on dissipation in the electrical domain, of a resonator system with one vibration mode and one transducer is given by [3]

$$Q_{el} = \frac{1}{k^2} \frac{|1 + E/Z_{load}|}{|E| \operatorname{Re}(1/Z_{load})} \quad (3)$$

where the electromechanical coupling factor, defined as $k^2 = C^2/ME$, was supposed small The minimum Q_{el} is obtained for $Z_{load} = |E|$ and equals $Q_{el, min} = 2/k^2$ At this load impedance, maximum signal-to-noise ratio is obtained In most cases, the coupling factor is rather small and Q_{el} is overruled by the Q based on mechanical considerations For example, the piezoelectric force sensor (Section 3) has a coupling factor $k = 0.01$, which makes $Q_{el, min} = 2 \times 10^4$ However, it is well known that the coupling factor of a resonator driven by an electrostatic air-gap mechanism [3] can be varied from 0 to 1 by increasing the bias voltage from 0 to the pull-in voltage This is shown in Fig 8

Fig 8 Q_{el} as a function of the ratio $U_{bias}/U_{pull-in}$ for various values of the impedance mismatch $|E|/Z_{load}$ The mechanical Q was assumed infinite and Z_{load} was assumed real (system loaded resistively)

Fig 9 Experimental set-up (top) and measurement results (bottom) showing the effect of an electric load on the system's quality factor

Figure 9 shows experimental results of an electrostatically operated resonator, the load impedance of which was varied Clearly, the measured Q is limited by dissipation in the electric domain (the mechanically based Q was approximately 500) The bias voltage was set at 90% of the pull-in voltage, resulting in a coupling factor $k = 0.7$ This would imply a minimum $Q_{el} = 2/k^2 = 5$, however, due to parasitic capacitances the load is not purely resistive A complex load results in a higher $Q_{el, min}$ [5]

6. Conclusions

Excitation and detection mechanisms should be regarded as integral parts of silicon resonant sensor systems A recently developed model has been verified by various experiments, emphasizing system aspects The model appears to be a powerful tool to describe the functioning of silicon resonant sensors and their interaction with the environment

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References

- 1 G Stemme, Resonant silicon sensors, *J Micromech Microeng*, 1 (1991) 113–125

- 2 H A C Tilmans, M Elwenspoek and J H J Fluitman, Micro resonant force gauges, *Sensors and Actuators A*, 30 (1992) 35-53
- 3 A Prak, T S J Lammerink and J H J Fluitman, Review of excitation and detection mechanisms for micromechanical resonators, *Sensors Mater*, accepted for publication
- 4 C J van Mullem, F R Blom, J H J Fluitman and M Elwenspoek, Piezoelectrically driven silicon beam force sensor, *Sensors and Actuators A*, 25-27 (1991) 379-383
- 5 A Prak, Silicon resonant sensors operation and response, *Ph D Thesis*, University of Twente, 1993