Simulation of Multipolar Fiber Selective Neural Stimulation Using Intrafascicular Electrodes

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Abstract—A realistic, quantitative model is presented for the excitation of myelinated nerve fibers by intrafascicular electrodes. It predicts the stimulatory regions of any configuration of any number of electrodes, positioned anywhere inside the fascicle. The model has two parts. First, the nerve fiber is represented by a lumped electrical network and its response to an arbitrary extracellular potential field is calculated. Second, assuming a cylindrical geometry of the nerve bundle and its surroundings, an analytical expression for this field is derived. With realistic parameters, the model is applied to two cases: monopolar stimulation by a single cathode and stimulation by a specific tripolar configuration. It is shown that tripolar stimulation has the better spatial selectivity. Also tripolar stimulation is less sensitive to the conductivity of the medium surrounding the nerve and yields a more natural recruitment order.

I. INTRODUCTION

SURFACE electrodes on the skin, intramuscular electrodes, and cuff electrodes around a nerve bundle are the types of electrodes most commonly used in functional electrical stimulation. A disadvantage of the surface and the intramuscular electrodes is that movement of the extremities and contraction of the muscles alter the geometry of the tissues and thereby the potential field induced by the stimulus currents. As a result, the force achieved varies with body position. In addition, the largest motor units have the lowest thresholds and will be activated at every stimulatory pulse. Since these motor units generally have a low fatigue resistance, muscle fatigue can be a serious problem in using these electrodes [23].

Electrodes can also be applied onto the surface of the nerve bundle. One such electrode though does not offer the means for spatially selective stimulation of nerve fibers and the order in which the nerve fibers are recruited is largely depending upon their size. The larger diameter nerve fibers, which correspond with the larger, less fatigue-resistant motor units, have the lower excitation thresholds. Units of smaller size are activated only at an increased stimulus level. This unnatural (inverse) recruitment order results in a low fatigue resistance and an inadequate open-loop control of muscle force [21], [22], [27].

Manuscript received February 6, 1990; revised July 15, 1991.

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IEEE Log Number 9104908.

In order to reduce fatigue, Holle *et al.* [11] used a multipole electrode configuration at the surface of the phrenic nerve of dog for sequential stimulation of different parts of this nerve. Petrofsky [14] and Happak *et al.* [9] used multipolar cuff electrodes to achieve the same in the stimulation of the sciatic nerve of rat and the femoral nerve of sheep, respectively. All three reported an increased fatigue resistance. Recruitment order was not investigated.

Recruitment order was emphasized in studies by Baratta *et al.* [3] and Solomonow *et al.* [21], [22]. Recruitment of motor units according to their size has been established using a cuff arrangement having three poles along the nerve bundle, 3 mm apart. Contraction of muscles is blocked with a high frequency (600 Hz) supramaximal electrical stimulus on the most distal electrode (using the middle electrode as common). It is assumed that this high-frequency stimulus "exhausts" the endplates, thereby preventing muscle contraction [3]. Lowering the stimulus amplitude first "releases" the axons of small size (which have the highest excitation threshold), allowing them to conduct action potentials elicited by a low-frequency stimulus on the most proximal electrode. Compared to conventional nerve stimulation techniques a decreased susceptibility to fatigue is shown.

The nerve stimulation methods mentioned above have the disadvantage that they provide no muscle selectivity (if the nerve branch stimulated is connected to more than one muscle). Sweeney *et al.* [24] used a multipolar cuff electrode and multipolar stimulation techniques for selective stimulation of fascicles of the greater sciatic nerve of cat. Since the fascicles are more or less muscle specific, muscle selective stimulation is achieved to some degree. Muscle fatigue and recruitment order were not investigated.

Compared to the stimulation methods mentioned above a more ideal situation will be created when a method is developed that provides the means for independent stimulation of many individual or small groups of nerve fibers. Force then can be regulated by selecting the appropriate fibers and fatigue can be improved by nonsimultaneous, asynchronous stimulation of different fibers. Also a high degree of muscle selectivity can be achieved.

Currently, the possibilities are being explored of producing a selective multichannel stimulus device that can be implanted inside a nerve fascicle with minimal damage [19], [20]. In principle, electrodes positioned in closer

contact to the fibers will show better selectivity. An important step in the development of such a device is the construction of models that predict the selectivity of any given electrode configuration. This paper describes a model that simulates the response of myelinated nerve fibers in a cylindrical fiber bundle to electrodes positioned arbitrarily inside this bundle. The selectivity and recruitment order of two electrode configurations, a monopole and a specific tripole, are compared by application of this model.

II. METHODS

The stimulation model is developed for a nerve fiber in a cylindrical, anisotropic conductive medium and consists of two main parts. In the first part, the response of a nerve fiber to an electrical field will be calculated. This part of the model uses a modified version of the excitation model as developed by McNeal and Rattay [12], [16], [17]. The second part of the model calculates the potential fields generated by electrode configurations placed inside the nerve bundle.

A. Electrical Excitation of a Myelinated Nerve Fiber

Myelinated nerve fibers are represented by an equivalent electric network model (Fig. 1). The membrane at a node of Ranvier is approximated by a voltage source V_r , a capacitor C_m , and a resistor R_m . The axonal plasma between two nodes is modeled by an ohmic resistance R_i and the myelin-sheath is regarded to be a perfect insulator. I_n denotes the current through the membrane at node n . $V_{e,n}$ and $V_{i,n}$ are the external and internal voltages at this node. V_r is the membrane restpotential and V_n the deviation from restpotential at node n . Using $V_n = V_{i,n} - V_{e,n} - V_r$, the following equation can be derived:

$$(V_{n-1} + V_{n+1} - 2V_n) + (V_{e,n-1} + V_{e,n+1} - 2V_{e,n}) = R_i C_m \frac{d(V_n)}{dt} + \frac{R_i}{R_m} V_n. \quad (1)$$

At rest $V_n = V_{e,n} = 0$ for all n . An analysis of the electrical network shows that stimulation with a short rectangular current pulse of duration T , imposing an external potential $V_{e,n,T}$, raises the membrane potential at node n by (see Appendix):

$$\Delta V_n \cong \frac{T}{R_i C_m} (V_{e,n-1,T} + V_{e,n+1,T} - 2V_{e,n,T}) \quad (2)$$

$$T < R_i C_m$$

$$T < R_m C_m.$$

The node, and therefore the corresponding nerve fiber, is excited when this variable surpasses a threshold value V_{thresh} ($\cong 20$ mV, see "Discussion").

$R_i C_m$ is roughly constant for all fibers [12]. Thus, for a given stimulus time, reaching threshold is completely determined by $(V_{e,n-1,T} + V_{e,n+1,T} - 2V_{e,n,T})$. This is the so called activating function of one fiber [16], [17].

Fig. 1. The electric network equivalent of a myelinated nerve fiber.

TABLE I
VARIABLES AND CONSTANTS

V_r	membrane resting potential
C_m	nodal capacitance
R_m	nodal resistance
R_i	axial internodal resistance
I_n	membrane current at node n
$V_{e,n}$	external potential at node n
$V_{i,n}$	internal potential at node n
V_n	deviation from V_r of the membrane potential
V_{thresh}	threshold potential for activation of the nodal membrane
T	duration of the stimulus pulses
ϕ	potential field in the extracellular medium
λ	internode length
f	activating function
σ_p, σ_z	conductivity of the nerve bundle in radial and longitudinal direction
σ_s	conductivity of the perineural sheath
σ_e	conductivity of the surroundings of the fiber bundle
I_{cath}	cathodal current
I_{an}	anodal current
a	radius of the fiber bundle
X_j, Y_j, Z_j	position of the electrode j
r	distance between the cylinder axis and the electrode
N_o	nodal density
K	$N_o / (6\pi^2 \sigma_p^2 \sigma_z V_{\text{thresh}}^3 (R_i C_m)^3)$

More generally, in a nerve bundle along the z direction in a potential field $\phi(x, y, z)$, the activating function $f(x, y, z, \lambda)$ of a fiber with a node at position (x, y, z) and with internodal distance λ is given by

$$f(x, y, z, \lambda) = \phi(x, y, z - \lambda) + \phi(x, y, z + \lambda) - 2\phi(x, y, z). \quad (3)$$

When the potential field during stimulation is known, $f(x, y, z, \lambda)$ can be calculated with λ as parameter. The threshold condition for stimulation ($\Delta V_n > V_{\text{thresh}}$) now becomes

$$f(x, y, z, \lambda) > V_{\text{thresh}} R_i C_m / T. \quad (4)$$

B. The Potential Field in a Cylindrical, Anisotropic Medium

The fiber bundle (fascicle) is idealized as a homogeneous, infinitely extending and anisotropic conductive cylinder with radius a , oriented along the z -axis (Fig. 2). It has radial conductivity σ_p and longitudinal conductivity σ_z . The cylinder is surrounded by a layer that represents the thin perineurium around the fascicle. It has a conductivity σ_s . The fiber bundle is immersed in a homogeneous, infinite, and isotropic medium, having conductivity σ_e . This medium models the environment of the nerve. The influence of the epineurium can be incorporated in the

Fig. 2. The cylindrical model of the fiber bundle and its surroundings.

perineural layer or as part of the external medium. Stimulation electrodes are idealized as point current sources inside the cylinder. The mathematics needed to calculate the potential field are comparable to that developed by other investigators, (e.g., [1], [10], [18], [29]), and is described in the Appendix. A new aspect is the positioning of a point current source arbitrarily inside the cylinder instead of centrally or outside [1], [27].

C. Presentation of the Results: Activating Contours

For each specific value of the internodal distance λ , the activating function as defined by (3) is a continuous function of x, y, z and can be mapped in isoactivating contour plots. Closed (three-dimensional) surfaces can be identified inside which the activating function surpasses a threshold value such that any node (on any fibre with a internodal distance λ) inside these regions will become active whereas nodes outside will not (irrespective of the actual node distribution within the fibre bundle).

To introduce such contour plots more clearly, a simple example is shown in which the potential field and the activating function are mapped for a cathodal monopole in an infinite, isotropically conductive ($\sigma_\rho = \sigma_z = \sigma$) fiber bundle. The fibers are aligned in the z direction and each single fiber has a constant internodal distance λ . The cathode is placed at the origin and conducts a stimulus current I_{cath} . Now the induced potential at position (x, y, z) is given by

$$\phi(x, y, z) = \frac{-|I_{\text{cath}}|}{4\pi\sigma\sqrt{x^2 + y^2 + z^2}}. \quad (5)$$

For a node of Ranvier at position (x, y, z) that belongs to a fiber having a internodal distance λ , the corresponding activating function has the value:

$$f(x, y, z, \lambda) = \frac{|I_{\text{cath}}|}{4\pi\sigma} \left\{ \frac{2}{\sqrt{x^2 + y^2 + z^2}} - \frac{1}{\sqrt{x^2 + y^2 + (z + \lambda)^2}} - \frac{1}{\sqrt{x^2 + y^2 + (z - \lambda)^2}} \right\}. \quad (6)$$

Fig. 3. The potential field for the stimulation of nerve fibers in an infinite, homogeneous medium. Isovalues are in volts. Conductivity $\sigma = 0.1 \Omega^{-1} \text{m}^{-1}$. Stimulus current $I_{\text{cath}} = 20 \mu\text{A}$. Internodal distance $\lambda = 1000 \mu\text{m}$.

Fig. 4. The activating function for the situation in Fig. 3.

The maps of the potential field and the corresponding activating function are shown in Figs. 3 and 4, respectively, for a cathodal current of $20 \mu\text{A}$, a conductivity $\sigma = 0.1 \text{ m}^{-1} \Omega^{-1}$ and an internodal distance of $1000 \mu\text{m}$.

In the plot of the activating function two regions can be distinguished. One region is seen around the cathode where the activating function has positive values. These positive values indicate that at a node of Ranvier (on a fiber with internodal distance λ) found in this region the potential of the internal membrane is raised. When a node is found within the closed region where $f(x, y, z, \lambda)$ exceeds a value of $V_{\text{thresh}} R_i C_m / T$ [see (4)] the threshold condition is fulfilled and this node, and thereby the corresponding nerve fiber, will be excited. As an example in Fig. 4 the area of excitation under the threshold condition $V_{\text{thresh}} R_i C_m / T = 0.2 \text{ V}$ (e.g., $V_{\text{thresh}} \cong 20 \text{ mV}$, $R_i C_m \cong 150 \mu\text{s}$, $T \cong 15 \mu\text{s}$, see "Discussion") is hatched. Outside this area the induced membrane potentials are too low to activate the nodes of Ranvier.

At one internodal distance to the left of the cathode a region is seen where the activating function has negative values. The internal membrane potential of a node in this region will be lowered and hence the membrane at this node will be hyperpolarized. A similar region is found to the right of the cathode (not shown).

Each isoactivating contour plot is only valid for the group of fibers which have the same internodal distance λ . To examine the response of the whole nerve fiber population (which is a mixture of different sized fibers having different λ 's) maps of activating functions corresponding to different values of λ have to be made.

III. RESULTS: APPLICATION OF THE MODEL

A. The Activating Function of a Monopole

Isovalue-lines of the activating function (volts) computed for monopolar cathodal stimulation of a nerve bundle are shown in Fig. 5. The bundle's axis is directed horizontally (z direction). A value of $500 \mu\text{m}$ was chosen as a realistic diameter of the fascicle (a) and a value of $1000 \mu\text{m}$ as a realistic internodal distance (λ). The activating function is shown in a plane through the cathode and the axis of the fiber bundle. The cathode is identified in the upper right corner of the plot by the $-$ symbol. Conductivities were chosen to be close to the bulk values of neural tissue (see "Discussion"). The current amplitude was $20 \mu\text{A}$ [20]. For the hatched area the same threshold condition was chosen as in Fig. 4.

The activating function for a monopole has mirror symmetry with respect to a plane through the current source, transverse to the fiber direction. As in the situation of Fig. 4 three regions are of interest, one region showing positive activating values and two regions showing negative values. Only two regions are shown. Positive values are found around the cathode. The activating function near the center of this region is dominated by the third term on the right-hand side of (3). Moreover, near the electrode, which is at position (X, Y, Z) , the primary field dominates the secondary field [see Appendix (24)]. Under these conditions the activating function can be approximated by

$$f(x, y, z, \lambda) \cong -2\phi(x, y, z) = \frac{2|I_{\text{cath}}|}{4\pi\sqrt{(\sigma_p\sigma_z)}\sqrt{(x-X)^2 + (y-Y)^2 + (z-Z)^2}\sigma_p/\sigma_z} \quad x, y, z \approx X, Y, Z. \quad (7)$$

For low stimulus strength the volume of the region, W , where $f(x, y, z, \lambda)$ fulfils the excitation condition $f(x, y, z, \lambda) > V_{\text{thresh}} R_i C_m / T$ is an ellipsoid having two axes of length $b = |I_{\text{cath}}| T / (\pi\sqrt{(\sigma_p\sigma_z)} R_i C_m V_{\text{thresh}})$ in the x - and the y -direction and one axis of length $c = |I_{\text{cath}}| T / (\pi\sigma_p R_i C_m V_{\text{thresh}})$ in the z direction. Thereby, the average number of fibers stimulated, N_{cath} , is given by

$$\begin{aligned} N_{\text{cath}} &\cong N_0 W = N_0 \frac{4}{3} \pi \frac{b^2 c}{8} \\ &= N_0 \frac{1}{6\pi^2 \sigma_p^2 \sigma_z V_{\text{thresh}}^3 (R_i C_m)^3} (|I_{\text{cath}}| T)^3 \\ &\quad \begin{aligned} T &< R_i C_m \\ T &< R_m C_m \end{aligned} \end{aligned} \quad (8)$$

in which N_0 is the nodal density.

For an anode at (X, Y, Z) the signs of the current and the activating function are reversed. This results in two excitation regions at one internodal distance from the electrode. At their center the activating function is dominated by the first and second term of (3), respectively. Thus for small values of I_{an} and T one has approximately

$$f(x, y, z, \lambda) \cong \phi(x, y, z \pm \lambda) = \frac{|I_{\text{an}}|}{4\pi\sqrt{(\sigma_p\sigma_z)}\sqrt{(x-X)^2 + (y-Y)^2 + (z-Z \pm \lambda)^2}\sigma_p/\sigma_z} \quad x, y, z \approx X, Y, Z \mp \lambda. \quad (9)$$

Fig. 5. The activating function for monopolar stimulation in a nerve bundle. Isovalues are in volts. For an explanation of the symbols see text and Table I. $\sigma_p = 0.1 \Omega^{-1} \text{m}^{-1}$, $\sigma_z = 0.5 \Omega^{-1} \text{m}^{-1}$, $\sigma_s = 2000 \Omega^{-1} \text{m}^{-2}$, $\sigma_e = 0.1 \Omega^{-1} \text{m}^{-1}$, $a = 250 \mu\text{m}$, $r = 100 \mu\text{m}$, $I_{\text{cath}} = 20 \mu\text{A}$, $\lambda = 1000 \mu\text{m}$.

The nodes excited in one region correspond to fibers that also have an excited node in the corresponding region on the opposite side of the electrode, since the excitation centers are exactly two node distances apart. The average number of fibers stimulated by an anode, N_{an} , is approximately

$$N_{\text{an}} \cong N_0 \frac{1}{48\pi^2 \sigma_p^2 \sigma_z V_{\text{thresh}}^3 (R_i C_m)^3} (|I| T)^3 \quad \begin{aligned} T &< R_i C_m \\ T &< R_m C_m \end{aligned} \quad (10)$$

This is one eighth of the number of fibers stimulated by a cathode with the same stimulus strength $|I| T$ [see (8)].

B. The Activating Function of a Tripole

Flanking a cathode by two anodes, each delivering half the current of the cathode, results in an oval-shaped excitation region around the cathode that is strongly restricted in its width. This region therefore has a relatively large length/width ratio. Fig. 6 shows the isovalue-lines of the activating function. The electrodes are placed in the upper right corner and designated $+$ (anodes) and $-$ (cathode).

As for monopolar stimulation an excitation region is found around the cathode. Four additional excitation regions are created as a result of the presence of the two anodes, further to be referred to as "anodal stimulation regions." Two are seen in the plot one internodal distance to the left of the anodes while the other two regions are one internodal distance to the right (not shown). In each of these zones the field of one of the anodes is dominant. Very near the cathode the activating function is dominated by the potential field imposed by the cathodal current. At the center of the anodal stimulation regions the activating function is dominated by the field due to the

Fig. 6. The activating function for tripolar stimulation in a nerve bundle. Isovalues are in volts. $\sigma_p = 0.1 \Omega^{-1} \text{m}^{-1}$, $\sigma_z = 0.5 \Omega^{-1} \text{m}^{-1}$, $\sigma_s = 2000 \Omega^{-1} \text{m}^{-2}$, $\sigma_e = 0.1 \Omega^{-1} \text{m}^{-1}$. $a = 250 \mu\text{m}$. $r = 0/100/200 \mu\text{m}$. $I_{\text{cath}} = 20 \mu\text{A}$, $I_{\text{an}} = 10 \mu\text{A}$. $\lambda = 1000 \mu\text{m}$.

single anodal currents. At very low stimulus strength the contributions of the separate electrodes are given by (8) and (10). For the total configuration of one cathode and two anodes one finds

$$\begin{aligned} N_{\text{cath}} &= K(|I_{\text{cath}}|T)^3 & T < R_i C_m \\ N_{\text{an}} &= 2K(|I_{\text{an}}|T)^3/8 = N_{\text{cath}}/32 & T < R_m C_m \end{aligned} \quad (11)$$

with

$$K = N_0 / (6\pi^2 \sigma_p^2 \sigma_z V_{\text{thresh}}^3 (R_i C_m)^3).$$

Stimulation due to the presence of the anodes plays a minor role in this configuration as long as stimulus strength is small. For a threshold value in the activating function of 0.1 V (e.g., $V_{\text{thresh}} \approx 20 \text{ mV}$, $R_i C_m \approx 150 \mu\text{s}$, $T \approx 30 \mu\text{s}$, see hatched area) $N_{\text{an}}/N_{\text{cath}} \approx 0.15$. The volume of the excitation region is comparable to the same volume at monopolar stimulation with a threshold value of 0.2 V (compare Figs. 5 and 6).

With an array of n intrafascicular electrodes transverse to the fiber direction, the technique of tripolar stimulation with three adjacent electrodes out of the array, offers a means of creating $n - 2$ excitation regions. Due to their restricted width the mutual overlap between these regions is negligible. The only substantial overlap in stimulation is seen between two tripoles that share one electrode as an anode (when the cathodes are twice the interelectrode distance apart). Anodal stimulation due to this anode is common to both tripoles. For a threshold value of 0.1 V the number of nodes stimulated by the common anode is approximately 7.5% of the number stimulated by each tripole.

C. The Influence of the External Medium on Monopolar Stimulation

The electrical field induced inside the nerve bundle does depend on the external conductivity of the surrounding medium. Fig. 7 shows one extreme case: the activating function for monopolar stimulation of a fiber bundle immersed in a highly conducting medium. The resistance of the perineurium has been neglected. Compared to Fig. 5, the field of the activating function has changed. Due to the presence of the highly conducting surroundings, the potential has zero value at the periphery of the bundle.

Fig. 7. The activating function for monopolar stimulation in a homogeneous nerve bundle, immersed in a good conducting medium. Isovalues are in volts. $\sigma_p = 0.1 \Omega^{-1} \text{m}^{-1}$, $\sigma_z = 0.5 \Omega^{-1} \text{m}^{-1}$, $\sigma_s = \infty$, $\sigma_e = \infty$. $a = 250 \mu\text{m}$. $r = 100 \mu\text{m}$. $I_{\text{cath}} = 20 \mu\text{A}$. $\lambda = 1000 \mu\text{m}$.

Therefore, along this surface, also the activating function has zero value. In Fig. 7 the volume, contained by an isovalue-surface of the activating function with a given positive value, is smaller than the corresponding volume in Fig. 5. As a result, in the situation of Fig. 5, more fibers will be stimulated with the same stimulus strength.

The activating function for monopolar stimulation of an insulated nerve, the other extreme case, is shown in Fig. 8. No current can pass the boundary of the cylinder and therefore the isopotential lines are perpendicular to this surface. One can deduce that, as a result, the isovalue-lines of the activating function are perpendicular to this edge too. Comparing the volumes of the excitation regions in Figs. 5, 7, and 8, the conclusion is that the stimulus strength needed to excite a certain amount of fibers, increases with the value of the external conductivity.

The average number of fibers stimulated depends on the radial position of the electrode. A cathode placed in the middle of an electrically insulated nerve will, at low currents, excite nodes within an ellipsoid centered at the electrode position and (8) will be valid. This changes when the electrode is placed at the surface of the nerve bundle (Fig. 9). For low stimulus strength the excitation region can be regarded as being the half of a ellipsoidal excitation zone of a virtual cathode, delivering twice the real cathode current ($I_{\text{virtual}} = 2I_{\text{cath}}$). The other half is situated within the insulating medium which contains no excitable nerve fibres. For an electrode at the periphery:

$$\begin{aligned} N_{\text{cathodal}} &\approx K(|I_{\text{virtual}}|T)^3/2 = 4K(|I_{\text{cath}}|T)^3 & T < R_i C_m \\ & & T < R_m C_m \end{aligned} \quad (12)$$

Therefore the average threshold for an electrode at the surface of the nerve bundle will be lower than for an electrode centered within the nerve bundle by approximately a factor $4^{1/3} \approx 1.6$. When the nerve is immersed in a medium of high conductivity the threshold for an electrode at the surface will actually be very high since almost all the current leaks into the surroundings.

D. The Influence of the External Medium on Tripolar Stimulation

In contrast to monopolar stimulation as described in the previous section, the excitation region arising from tri-

Fig. 8. The activating function for monopolar stimulation in a homogeneous nerve bundle, electrically insulated from its surroundings. Isovalues are in volts. $\sigma_\rho = 0.1 \Omega^{-1} \text{m}^{-1}$, $\sigma_z = 0.5 \Omega^{-1} \text{m}^{-1}$, $\sigma_s = 2000 \Omega^{-1} \text{m}^{-2}$, $\sigma_e = 0 \Omega^{-1} \text{m}^{-1}$, $a = 250 \mu\text{m}$, $r = 100 \mu\text{m}$, $I_{\text{cath}} = 20 \mu\text{A}$, $\lambda = 1000 \mu\text{m}$.

Fig. 9. The excitation area of an electrode at the edge of an electrically insulated nerve can be regarded as the half of an ellipsoid-shaped excitation area belonging to a virtual electrode with twice the real current. The other half of this area is situated at the external medium and plays no role in the excitation process.

polar stimulation is hardly influenced by the conductivity of the surroundings of the cylinder (Figs. 6, 10, and 11). Size as well as position of the stimulation zone is nearly unaffected. This condition arises because the net source current is zero. For a monopole the far field potential of the primary source (see Appendix) will show a r^{-1} dependence. The far field potential of a tripole has a quadrupolar character (r^{-3} dependence) and therefore falls off more rapidly. The potential of the tripole primary source

Fig. 10. The activating function for tripolar stimulation in a homogeneous nerve bundle immersed in a good conducting medium. Isovalues are in volts. $\sigma_\rho = 0.1 \Omega^{-1} \text{m}^{-1}$, $\sigma_z = 0.5 \Omega^{-1} \text{m}^{-1}$, $\sigma_s = \infty$, $\sigma_e = \infty$, $a = 250 \mu\text{m}$, $r = 0/100/200 \mu\text{m}$, $I_{\text{cath}} = 20 \mu\text{A}$, $I_{\text{an}} = 10 \mu\text{A}$, $\lambda = 1000 \mu\text{m}$.

monopole. As a consequence the secondary sources will be relatively reduced with increasing multipole order. Since the influence of the inhomogeneities and thereby of the external conductivity is expressed by these secondary sources one expects the results noted above.

E. Approximation of the Activating Function in the Close Neighborhood of an Electrode

The approximate equations (8), (10), and (11) for the stimulated number of fibres were derived under the "local" assumptions expressed in (7) and (9). The question that arises is to what degree these assumptions are valid.

When n electrodes are placed inside the bundle the potential field can be written as a sum of the primary and secondary fields of the separate electrodes. The primary field of an electrode (source) is the field of the electrode placed in an infinite homogeneous medium and the secondary field (boundaries) represents the modification of the field due to the inhomogeneities:

$$\phi(x, y, z) = \sum_{i=1}^n [\phi_{\text{prim},j}(x, y, z) + \phi_{\text{sec},j}(x, y, z)]. \quad (13)$$

The primary field of electrode number j , conducting current I_j , and being placed at position (X_j, Y_j, Z_j) , is

$$\phi_{\text{prim},j}(x, y, z) = \frac{I_j}{4\pi\sqrt{\sigma_\rho\sigma_z} \sqrt{(x - X_j)^2 + (y - Y_j)^2 + (z - Z_j)^2\sigma_\rho/\sigma_z}}. \quad (14)$$

at the boundary of the cylinder will therefore be small and localized compared to the corresponding potential for a

Consequently, the activating function for one specific electrode (nr. 1) can be written as

$$\begin{aligned} f(x, y, z, \lambda) = & \frac{-2I_1}{4\pi\sqrt{\sigma_\rho\sigma_z} \sqrt{(x - X_1)^2 + (y - Y_1)^2 + (z - Z_1)^2\sigma_\rho/\sigma_z}} \\ & + \frac{I_1}{4\pi\sqrt{\sigma_\rho\sigma_z} \sqrt{(x - X_1)^2 + (y - Y_1)^2 + (z - Z_1 - \lambda)^2\sigma_\rho/\sigma_z}} \\ & + \frac{I_1}{4\pi\sqrt{\sigma_\rho\sigma_z} \sqrt{(x - X_1)^2 + (y - Y_1)^2 + (z - Z_1 + \lambda)^2\sigma_\rho/\sigma_z}} \\ & + \sum_{j=2}^n [-2\phi_{\text{prim},j}(x, y, z) + \phi_{\text{prim},j}(x, y, z - \lambda) + \phi_{\text{prim},j}(x, y, z + \lambda)] \\ & + \sum_{j=1}^n [-2\phi_{\text{sec},j}(x, y, z) + \phi_{\text{sec},j}(x, y, z - \lambda) + \phi_{\text{sec},j}(x, y, z + \lambda)]. \end{aligned} \quad (15)$$

Fig. 11. The activating function for tripolar stimulation in a homogeneous nerve bundle electrically isolated from its surroundings. Isovalues are in volts. $\sigma_p = 0.1 \Omega^{-1} \text{m}^{-1}$, $\sigma_z = 0.5 \Omega^{-1} \text{m}^{-1}$, $\sigma_s = 2000 \Omega^{-1} \text{m}^{-2}$, $\sigma_e = 0 \Omega^{-1} \text{m}^{-1}$. $a = 250 \mu\text{m}$. $r = 0/100/200 \mu\text{m}$. $I_{\text{cath}} = 20 \mu\text{A}$, $I_{\text{an}} = 10 \mu\text{A}$. $\lambda = 1000 \mu\text{m}$.

The right-hand side of this equation contains five terms. The first term is the primary field of the first electrode multiplied by a factor -2 . The second and third term are the primary fields of two imaginary "twins" of the first electrode placed one internodal distance away from this electrode. The fourth term is the influence of the primary fields of the other electrodes and the fifth term is the influence of all the secondary fields.

In the close neighborhood of electrode 1 the first term will dominate and when this electrode is a cathode, (7) will be valid. In this case the contributions of the second, third, fourth, and fifth term can be neglected. The distance from the electrode where the approximation starts to fail depends on the magnitude of the separate terms.

It can be easily verified that at distances from the electrode of $0.1 \cdot \lambda$ along the fibre direction and of $0.1 \cdot \sqrt{\sigma_p/\sigma_z} \cdot \lambda$ transverse to the fibers the contribution of the second and third terms together is 10% of the contribution of the first term. In the situations shown (Figs. 4–8, 10, 11) these distances are 100 and $45 \mu\text{m}$, respectively.

The fourth term depends on the positions of the separate electrodes. For monopolar stimulation this term is zero ($n = 1$) but for the tripolar configuration studied the influence of the anodes on the primary cathodal field near the central electrode is calculated to be also 10% at a distance of $0.1 \cdot d$ transverse to the fibers and at a distance of $0.1 \cdot d \cdot \sqrt{\sigma_z/\sigma_p}$ along the fibre direction (d is the interelectrode distance, $d \ll \lambda$). In the tripolar situations shown these distances are 10 and $22 \mu\text{m}$, respectively.

The fifth term expresses the contributions of the secondary fields. No general rule can be given on its magnitude. The secondary fields not only depend on the values of the conductivities of the tissues but also on the number of the electrodes and their positions relative to the boundaries of the fiber bundle. The influence of these parameters was assessed in Sections III-C and III-D. When the nerve is imbedded in body tissue whose conductivity matches the intraneural conductivities relatively well the fifth term will have a low contribution, especially when tripolar stimulation is used (and provided the resistance of the epineural sheath is negligible, see "Discussion").

F. Recruitment Order of Monopolar and Tripolar Stimulation

For greater stimulus strength (8) is no longer valid. The first and second term on the right-hand side of (3) can no

Fig. 12. The activating function for monopolar stimulation in a nerve bundle containing thin motor fibers. Isovalues are in volts. $\sigma_p = 0.1 \Omega^{-1} \text{m}^{-1}$, $\sigma_z = 0.5 \Omega^{-1} \text{m}^{-1}$, $\sigma_s = 2000 \Omega^{-1} \text{m}^{-2}$, $\sigma_e = 0.1 \Omega^{-1} \text{m}^{-1}$. $a = 250 \mu\text{m}$. $r = 100 \mu\text{m}$. $I_{\text{cath}} = 20 \mu\text{A}$. $\lambda = 400 \mu\text{m}$.

Fig. 13. The activating function for tripolar stimulation in a nerve bundle containing thin motor fibers. Isovalues are in volts. $\sigma_p = 0.1 \Omega^{-1} \text{m}^{-1}$, $\sigma_z = 0.5 \Omega^{-1} \text{m}^{-1}$, $\sigma_s = 2000 \Omega^{-1} \text{m}^{-2}$, $\sigma_e = 0.1 \Omega^{-1} \text{m}^{-1}$. $a = 250 \mu\text{m}$. $r = 0/100/200 \mu\text{m}$. $I_{\text{cath}} = 20 \mu\text{A}$, $I_{\text{an}} = 10 \mu\text{A}$. $\lambda = 400 \mu\text{m}$.

longer be neglected and the region of excitation becomes a function of the internodal distance λ . In this section the influence of smaller values of λ will be studied.

Fig. 12 shows the activating function for monopolar stimulation of fibers having a small λ ($= 400 \mu\text{m}$). Comparing the excitation regions in Figs. 5 and 12 bounded by isovalue-surfaces of the activating function equal to 0.1 V, one clearly distinguishes differences in their size. The volume of the excitation zone for the larger value of λ is much larger. Due to the r^{-3} dependence of the far field of a tripole, tripolar stimulation is restricted to a small region of which the size is less dependent on λ . This is clearly seen in Fig. 13. This figure shows the activating function for tripolar stimulation of fibers having a small internodal distance ($\lambda = 400 \mu\text{m}$). The volume of the hatched area bounded by an isoactivating value of 0.1 V is now very similar to the same zone in Fig. 6.

The number of fibers stimulated is related to the volume of the excitation zone, and since the dependence of this volume on λ changes with the electrode configuration used, recruitment orders for tripolar and monopolar stimulation are expected to be different. The ratio of the probabilities of exciting a thick fiber relative to a thin fiber is large for monopolar stimulation, but decreases using tripolar stimulation. Consequently, application of the tripole yields a more natural recruitment order.

IV. DISCUSSION

In the model discussed in this paper the nerve fiber is represented by a passive electric network (Fig. 1). A more accurate representation can be obtained when the mem-

brane is modelled using the Frankenhaeuser-Huxley equations or the model developed by Chiu *et al.* [4], [6]. Moreover, while the ohmic resistance of the myelin-sheath is high, its underlying internodal membrane as well as its capacity can play a role in the stimulation process [8], [12], [13]. Due to these leakage currents stimulus strength needed for excitation will be higher than predicted.

Equation (2) has been derived under the conditions $T < R_i C_m$ and $T < R_m C_m$ (see Appendix). Rutten *et al.* [20] stimulated the personal nerve of rat and for the term $R_i R_m C_m / (2R_m + R_i)$ measured a value of about 70 μ s. This leads to $R_i C_m > 140 \mu$ s and $R_m C_m > 70 \mu$ s which means that the conditions of (2) are certainly fulfilled when $T < 70 \mu$ s. An exact value of V_{thresh} has not been given. The present authors suggest a value of approximately 20 mV to be adequate but the exact value can depend upon the duration of the stimulus pulses T [12].

In the second part of the model it was assumed that the tissues were homogeneous. However, the microscopic structure of the tissues reveals inhomogeneities which will locally disturb the potential field. These inhomogeneities have a typical size of about 10 μ m (the average fiber diameter). Beyond a scale of about 40 μ m the influence of the microscopic structure on the potential field will probably be negligible. Furthermore, the conductivities of the fiber bundle are not unambiguously known. For the toad sciatic nerve Tasaki [25] measured $\sigma_z = 0.4 \Omega^{-1} \text{m}^{-1}$, $\sigma_p = 0.01 \Omega^{-1} \text{m}^{-1}$ but no values were measured for mammalian nerve. For the spinal cord of a cat Geddes and Baker reported $\sigma_z = 1.2 \Omega^{-1} \text{m}^{-1}$, $\sigma_p = 0.13 \Omega^{-1} \text{m}^{-1}$ [7]. The range of values used by other authors in model studies is large. Roth and Wikswo estimated $\sigma_z = 5 \Omega^{-1} \text{m}^{-1}$, $\sigma_p = 0.03 \Omega^{-1} \text{m}^{-1}$ and $\sigma_e = 0.25 \Omega^{-1} \text{m}^{-1}$ whereas for example Veltink *et al.* used $\sigma_z = 0.5 \Omega^{-1} \text{m}^{-1}$, $\sigma_p = 0.08 \Omega^{-1} \text{m}^{-1}$ and $\sigma_e = 0.1 \Omega^{-1} \text{m}^{-1}$ [18], [27].

The conductance of the epineurium of frog sciatic nerve was measured by Weerasuriya *et al.* who found $\sigma_s = 20 \Omega^{-1} \text{m}^{-2}$ [28]. They did not report the thickness of the epineurium but assuming a value of 10 μ m, one calculates a specific resistance of $5 \times 10^3 \Omega \text{m}$. An epineural sheath with this resistance will almost act as an insulator which is in contrast to the finding of Tasaki who found in his measurements of σ_p that the resistance of the epineurium could almost be neglected [25]. The present authors used mostly $\sigma_z = 2000 \Omega^{-1} \text{m}^{-2}$, assuming a sheath thickness of 10 μ m and a specific resistance of 50 Ωm , which is about twice the specific resistance of fat [7].

Electrical field strength transverse to the fibres using tripolar stimulation can be very high. The field strength in the middle between the cathode and an anode (Fig. 6) was calculated to be $4 \times 10^3 \text{ V/m}$. Eickhorn *et al.* [5] exposed frog sciatic nerves to strong transverse electrical fields generated by condenser discharges (field strengths 17×10^3 to $100 \times 10^3 \text{ V/m}$, time constants 0.2–8.0 ms) and found that these fields could bring about a block of the action potential. This block could last for up to 30 min after which a slow, exponential restitution was observed. For smaller field strengths of short duration the conduction block has not to be total and shows a recovery time

Fig. 14. To cover all the nodes of Ranvier using only tripolar stimulation, one electrode array is insufficient. More arrays are needed. The distance between successive arrays should not be larger than four times the distance between adjacent electrodes.

of several seconds. This blocking of the action potential suggests that strong transverse electrical fields cause a diminished excitability of the nodal membrane. This conductivity block is believed to be induced by a breakdown of the ionic gradients across the cell membrane due to electroporation. As a consequence a stimulus pulse can temporarily decrease the excitability of the nodes of Ranvier, thereby affecting the response to succeeding pulses. This effect may be a serious drawback of tri(multi)polar stimulation techniques but needs experimental verification.

The phenomenon that recruitment order can be influenced by electrode geometry has also been discussed by Veltink *et al.* [27], who showed that on the average an intrafascicular electrode yields a more orderly recruitment than an extraneural one. In this paper it is shown that this effect can be enhanced using multipolar stimulation techniques.

Design of a Stimulation Device

By use of the model, several electrode configurations are studied. It is concluded that the properties of stimulation with a linear tripole, a controlled width of the stimulation region and an orderly recruitment, make it an attractive stimulation method (except for possible electroporation effects).

In Fig. 6 the length of the central excitation region is smaller than the internodal distance of the nerve fibers. Therefore some of the fibers passing this zone will have no node of Ranvier within it. With one array of electrodes inside a fascicle, not all the fibers can be excited. More arrays are needed or tripolar stimulation has to be combined with monopolar stimulation. For a realistic situation (Fig. 6) with an appropriate stimulus strength (threshold condition $f(x, y, z, \lambda) > 0.1 \text{ V}$) the length of the excitation region is about three times the distance between two adjacent electrodes. For a series of electrode arrays along the nerve (Fig. 14), the distance between two successive arrays should not be larger than three times the electrode distance in one array. Almost all the nodes of Ranvier are then covered by an excitation region.

V. APPENDIX

A. Dynamic Response of the Electric Network
Equivalent of a Nerve Fiber to a Short Stimulus Pulse

The dynamic response of the nerve fibre described by (1) can be written in matrix-notation:

$$K\bar{V}_n + \tau \frac{d\bar{V}_n}{dt} = M\bar{V}_e \quad (16)$$

using

$$Q = R_i/R_m + 2, \quad \tau = R_i C_m,$$

$$K = \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & Q & -1 & 0 & 0 & \cdot \\ \cdot & -1 & Q & -1 & 0 & \cdot \\ \cdot & 0 & -1 & Q & -1 & \cdot \\ \cdot & 0 & 0 & -1 & Q & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix},$$

$$V_n(t) = \begin{bmatrix} \cdot \\ V_{n-2} \\ V_{n-1} \\ V_n \\ V_{n+1} \\ \cdot \end{bmatrix}$$

$$M = \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & -2 & 1 & 0 & 0 & \cdot \\ \cdot & 1 & -2 & 1 & 0 & \cdot \\ \cdot & 0 & 1 & -2 & 1 & \cdot \\ \cdot & 0 & 0 & 1 & -2 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix},$$

$$V_e(t) = \begin{bmatrix} \cdot \\ V_{e,n-2} \\ V_{e,n-1} \\ V_{e,n} \\ V_{e,n+1} \\ \cdot \end{bmatrix} \quad (17)$$

With the boundary conditions $\bar{V}_n(t \rightarrow -\infty) = \bar{V}_e(t \rightarrow -\infty) = 0$, the solution of this equation is

$$\bar{V}_n(t) = \int_{-\infty}^t \exp\left[-\frac{K}{\tau}(t-t')\right] M V_e(t')/\tau dt'. \quad (18)$$

The response to stimulation with a short rectangular current pulse is:

$$\bar{V}_e(t) = 0 \quad \bar{V}_n(t) = 0 \quad (t < 0)$$

$$\bar{V}_e(t) = \bar{V}_{e,T} = \begin{pmatrix} \cdot \\ V_{e,n-1,T} \\ V_{e,n,T} \\ V_{e,n+1,T} \\ \cdot \end{pmatrix}$$

$$\bar{V}_n(t) = (I - e^{-(K/\tau)t})K^{-1}M\bar{V}_{e,T} \quad (0 < t < T)$$

$$\bar{V}_e(t) = 0$$

$$\bar{V}_n(t) = e^{-(K/\tau)(t-T)}(I - e^{-(K/\tau)T})K^{-1}M\bar{V}_{e,T} \quad (t > T). \quad (19)$$

Without proof

$$q = \frac{1}{2}Q - \sqrt{Q^2/4 - 1}$$

$$\bar{V}_n(t)|_{0 < t < T} = \{I - e^{-(K/\tau)t}\} \left\{ \frac{Q-2}{Q-2q} \right.$$

$$\left. \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & 1 & q & q^2 & q^3 & \cdot \\ \cdot & q & 1 & q & q^2 & \cdot \\ \cdot & q^2 & q & 1 & q & \cdot \\ \cdot & q^3 & q^2 & q & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} - I \right\}$$

$$\begin{bmatrix} \cdot \\ V_{e,n-2,T} \\ V_{e,n-1,T} \\ V_{e,n,T} \\ V_{e,n+1,T} \\ \cdot \end{bmatrix}. \quad (20)$$

The time derivatives at the start of the stimulus pulse are given by

$$\left. \frac{d^m(\bar{V}_n)}{dt^m} \right|_{t=0} = \tau^{-m}(-K)^{m-1}M\bar{V}_{e,T}. \quad (21)$$

The first two derivatives are

$$\left. \frac{d}{dt} \begin{bmatrix} \cdot \\ V_{n-2} \\ V_{n-1} \\ V_n \\ V_{n+1} \\ \cdot \end{bmatrix} \right|_{t=0} = \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & -2 & 1 & 0 & 0 & \cdot \\ \cdot & 1 & -2 & 1 & 0 & \cdot \\ \cdot & 0 & 1 & -2 & 1 & \cdot \\ \cdot & 0 & 0 & 1 & -2 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \begin{bmatrix} \cdot \\ V_{e,n-2,T} \\ V_{e,n-1,T} \\ V_{e,n,T} \\ V_{e,n+1,T} \\ \cdot \end{bmatrix} R_i C_m$$

$$\left. \frac{d^2}{dt^2} \begin{bmatrix} \cdot \\ V_{n-2} \\ V_{n-1} \\ V_n \\ V_{n+1} \\ \cdot \end{bmatrix} \right|_{t=0} = \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & 2Q+2 & -Q-2 & 1 & 0 & \cdot \\ \cdot & -Q-2 & 2Q+2 & -Q-2 & 1 & \cdot \\ \cdot & 1 & -Q-2 & 2Q+2 & -Q-2 & \cdot \\ \cdot & 0 & 1 & -Q-2 & 2Q+2 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \begin{bmatrix} \cdot \\ V_{e,n-2,T} \\ V_{e,n-1,T} \\ V_{e,n,T} \\ V_{e,n+1,T} \\ \cdot \end{bmatrix} (R_i C_m)^2. \quad (22)$$

The rise of the membrane potential at the end of the stimulus is to a first-order approximation

$$\Delta \bar{V}_n \cong T \left. \frac{d}{dt} \begin{bmatrix} \cdot \\ V_{n-2} \\ V_{n-1} \\ V_n \\ V_{n+1} \\ \cdot \end{bmatrix} \right|_{t=0} = T \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & -2 & 1 & 0 & 0 & \cdot \\ \cdot & 1 & -2 & 1 & 0 & \cdot \\ \cdot & 0 & 1 & -2 & 1 & \cdot \\ \cdot & 0 & 0 & 1 & -2 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \begin{bmatrix} \cdot \\ V_{e,n-2,T} \\ V_{e,n-1,T} \\ V_{e,n,T} \\ V_{e,n+1,T} \\ \cdot \end{bmatrix} / R_i C_m \quad (23)$$

$T < R_i C_m$
 $T < R_m C_m$

B. Derivation of the Potential Field

The potential in the cylinder induced by a point current source at position $(r, 0, 0)$ inside the cylinder (Fig. 2), can be written as a superposition of a primary field, ϕ_{prim} , and a secondary field, ϕ_{sec} . The primary field is the field induced by the source as if it was placed in an infinite homogeneous medium with the same conductivities as the

inside of the cylinder. The secondary field represents the modification of the field due to the inhomogeneities:

$$\phi_i = \phi_{\text{prim}} + \phi_{\text{sec}}$$

$$= \frac{I}{4\pi\sqrt{\sigma_\rho\sigma_z}\sqrt{(x-r)^2+y^2+z^2\sigma_\rho/\sigma_z}} + \phi_{\text{sec}}. \quad (24)$$

The secondary field can be regarded as originating from imaginary current sources at the boundary of the cylinder. No extra sources are created inside. Therefore ϕ_{sec} has to fulfill the equation of continuity:

$$\nabla \cdot J = 0 \Leftrightarrow -\sigma_\rho \left(\frac{\delta^2 \phi_{\text{sec}}}{\delta x^2} + \frac{\delta^2 \phi_{\text{sec}}}{\delta y^2} \right) - \sigma_z \frac{\delta^2 \phi_{\text{sec}}}{\delta z^2} = 0. \quad (25)$$

After a proper coordinate transformation this changes into the Laplace equation:

$$x^* = \sqrt{\sigma_z/\sigma_\rho} x$$

$$y^* = \sqrt{\sigma_z/\sigma_\rho} y$$

$$z^* = z \quad (26)$$

$$\left(\frac{\delta^2}{\delta x^{*2}} + \frac{\delta^2}{\delta y^{*2}} + \frac{\delta^2}{\delta z^{*2}} \right) \phi_{\text{sec}} = \Delta^* \cdot \phi_{\text{sec}} = 0 \quad (27)$$

Using a cylindrical coordinate system centered in the bundle,

$$\rho^* = \sqrt{x^{*2} + y^{*2}}$$

$$\theta = \arctan(-y/x) \quad x > 0$$

$$= \arctan(y/x) + \pi \quad x < 0$$

$$= \pi/2 \quad x = 0, y < 0$$

$$= 3\pi/2 \quad x = 0, y > 0. \quad (28)$$

Equation (21) can be written as

$$\left(\frac{\delta^2}{\delta \rho^{*2}} + 1/\rho^* \frac{\delta}{\delta \rho^*} + \frac{\delta^2}{\rho^{*2} \delta \theta^2} + \frac{\delta^2}{\delta z^2} \right) \phi_{\text{sec}} = 0. \quad (29)$$

Separation of variables leads to the general solution

$$\begin{aligned} \phi_{\text{sec}} = & \int_{-\infty}^{\infty} \left(\sum_{n=0}^{\infty} (a_n(k) I_n(k\rho^*) + b_n(k) K_n(k\rho^*)) \right. \\ & \cdot \cos(n\theta) + \sum_{n=1}^{\infty} (c_n(k) I_n(k\rho^*) \\ & \left. + d_n(k) K_n(k\rho^*)) \sin(n\theta) \right) e^{ikz} dk \quad (30) \end{aligned}$$

where $I_n(k\rho^*)$ and $K_n(k\rho^*)$ are the modified Bessel functions of the first and second kind of order n . Symmetry of the system implies

$$\phi_{\text{sec}}(\theta) = \phi_{\text{sec}}(-\theta). \quad (31)$$

Therefore,

$$c_n(k) = d_n(k) = 0. \quad (32)$$

Due to the limit

$$\lim_{x \rightarrow 0} K_n(x) = \infty \quad (33)$$

the terms with $K_n(k\rho^*)$ supply a physical unrealistic solution. This results in

$$b_n(k) = 0. \quad (34)$$

The primary field, using a coordinate system ρ', θ', z centered at the point current source (Fig. 2), can also be written as an expansion of Bessel functions. With the use of

$$\rho'^* = \sqrt{\sigma_z/\sigma_\rho} \rho' \quad (35)$$

$$\frac{1}{\sqrt{\rho'^2 + z^2}} = \frac{1}{\pi} \int_{-\infty}^{\infty} K_0(k\rho) e^{ikz} dk. \quad (36)$$

The result is

$$\phi_{\text{prim}} = \frac{I}{4\pi^2 \sigma_\rho} \int_{-\infty}^{\infty} K_0(k\rho'^*) e^{ikz} dk. \quad (37)$$

This can be rewritten in the coordinate system (ρ, θ, z) by means of the Bessel function relations:

$$\begin{aligned} (\rho < r) \quad K_n(k\rho') \cos(n\theta') \\ = K_n(kr) I_0(k\rho) + \sum_{m=1}^{\infty} (K_{n+m}(kr) \\ + K_{n-m}(kr)) I_m(k\rho) \cos(m\theta) \quad (38a) \end{aligned}$$

$$\left. \frac{d}{d\rho} \left(a_n(k) I_n(k\rho^*) + \frac{I}{4\pi^2 \sigma_\rho} (2 - \delta_n) K_n(k\rho^*) I_n(kr^*) \right) \right|_{\rho=a} = \sigma_e \left. \frac{d}{d\rho} (e_n(k) K_n(k\rho)) \right|_{\rho=a} \quad (44a)$$

$$a_n(k) I_n(ka^*) + \frac{I}{4\pi^2 \sigma_\rho} (2 - \delta_n) K_n(ka^*) I_n(kr^*) - e_n(k) K_n(ka) = -\sigma_e/\sigma_s \left. \frac{d}{d\rho} (e_n(k) K_n(k\rho)) \right|_{\rho=a}. \quad (44b)$$

$$(\rho > r) \quad K_n(k\rho') \cos(n\theta)$$

$$\begin{aligned} = I_n(kr) K_0(k\rho) + \sum_{m=1}^{\infty} (I_{n+m}(kr) \\ + I_{n-m}(kr)) K_m(k\rho) \cos(m\theta). \quad (38b) \end{aligned}$$

The result finally is

$$\begin{aligned} (\rho < r) \quad \phi_i(\rho, \theta, z) \\ = \int_{-\infty}^{\infty} \left\{ \sum_{n=0}^{\infty} \left(a_n(k) I_n(k\rho^*) + \frac{I}{4\pi^2 \sigma_\rho} (2 - \delta_n) \right. \right. \\ \left. \left. \cdot K_n(kr^*) I_n(k\rho^*) \right) \cos(n\theta) \right\} e^{ikz} dk \quad (39a) \end{aligned}$$

$$\begin{aligned} (\rho > r) \quad \phi(\rho, \theta, z) \\ = \int_{-\infty}^{\infty} \left\{ \sum_{n=0}^{\infty} \left(a_n(k) I_n(k\rho^*) + \frac{I}{4\pi^2 \sigma_\rho} (2 - \delta_n) \right. \right. \\ \left. \left. \cdot I_n(kr^*) K_n(k\rho^*) \right) \cos(n\theta) \right\} e^{ikz} dk. \quad (39b) \end{aligned}$$

The potential field in the external medium ϕ_e can be written in an equivalent way as ϕ_{sec} in (30). ϕ_e has to be symmetric and has to go to zero at infinity. Therefore and because

$$\lim_{x \rightarrow \infty} I_n(x) = \infty \Rightarrow a_n(k) = 0 \quad (40)$$

the general form of ϕ_e is

$$\phi_e = \int_{-\infty}^{\infty} \sum_{n=0}^{\infty} e_n(k) K_n(k\rho) \cos(n\theta) e^{ikz} dk. \quad (41)$$

The coefficients $a_n(k)$ and $e_n(k)$ are determined by applying two appropriate boundary conditions at the surface of the cylinder. Continuity of radial current demands:

$$\lim_{\rho \rightarrow a} \sigma_\rho \frac{d\phi_i(\rho, \theta, z)}{d\rho} = \lim_{\rho \rightarrow a} \sigma_e \frac{d\phi_e(\rho, \theta, z)}{d\rho} = -J_{\text{surf}}(\theta, z). \quad (42)$$

This current should induce a potential step at the surface:

$$\phi_i(a, \theta, z) - \phi_e(a, \theta, z) = -J_{\text{surf}}(\theta, z)/\sigma_s. \quad (43)$$

Using the orthogonality of the trigonometrical functions, these conditions result in

Solving these equations yields

$$a_n(k) = -\frac{I}{4\pi^2\sigma_p} (2 - \delta_n) I_n(kr^*) \cdot \{ [k\sqrt{\sigma_p\sigma_z} K'_n(ka^*) + \sigma_s K_n(ka^*)] k\sigma_e K'_n(ka) - k\sqrt{\sigma_p\sigma_z} \sigma_s K_n(ka) K'_n(ka^*) \} / \{ [k\sqrt{\sigma_p\sigma_z} I'_n(ka^*) + \sigma_s I_n(ka^*)] k\sigma_e K'_n(ka) - k\sqrt{\sigma_p\sigma_z} \sigma_s I'_n(ka^*) K_n(ka) \} \quad (45a)$$

$$e_n(k) = \frac{I}{4\pi^2\sigma_p} (2 - \delta_n) I_n(kr^*) \cdot k\sqrt{\sigma_p\sigma_z} \sigma_s \{ I_n(ka^*) K'_n(ka^*) - I'_n(ka^*) K_n(ka^*) \} / \{ [k\sqrt{\sigma_p\sigma_z} I'_n(ka^*) + \sigma_s I_n(ka^*)] k\sigma_e K'_n(ka) - k\sqrt{\sigma_p\sigma_z} \sigma_s I'_n(ka^*) K_n(ka) \} \quad (45b)$$

Computation of the field can be done by application of the fast Fourier transform but then our integrals have to be discretised in k . Let Z be the space-domain sampling interval.

$$\phi_i \cong \frac{I}{4\pi\sqrt{\sigma_p\sigma_z}\sqrt{\rho'^2 + z^2}\sigma_p/\sigma_z} + \frac{\pi}{MZ} \sum_{m=-M}^{M-1} \sum_{n=0}^N [a_n(m\pi/MZ) \cdot I_n(\rho^*m\pi/MZ)] \cos(n\theta) e^{izm\pi/MZ} \quad (46)$$

For a line along the z axis, θ and ρ^* are constant. The coefficients $a_n(m\pi/MZ) I_n(\rho^*m\pi/MZ) \cos(n\theta)$ can be calculated and after a fast Fourier transform the potential along this line is known. To make a plot of the activating function in a plane along the length of the cylinder, the potential has to be calculated along a number of lines. To calculate the field of more than one electrode the addition theorem for electrical fields has to be used.

The $m = 0$ terms are a special case because $\lim_{k \rightarrow 0} a_n(k) = \infty$. The coefficients can not be computed straightforward. Using a series expansion of $K_n(x)$ and $I_n(x)$ one finds

$$\lim_{m \rightarrow 0} a_n(m\pi/MZ) I_n(\rho^*m\pi/MZ) = -\frac{I}{4\pi^2\sigma_p n} \left(\frac{r\rho}{a^2} \right)^n \quad (47)$$

This is no solution when $m = 0$, $n = 0$. This case, however, solely determines the offset of the field and is of no influence on the activating function. It is therefore given the value zero.

Accuracy of the calculations depends primarily on M , Z , and N . In our situation M was taken 256 and $a/Z \cong 30$. The appropriate value of N depends largely on the excentricity of the current source. The larger r/a , the larger N has to be chosen to give accurate results. A trade-

off needs to be made between accuracy and computation time.

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