OPTIMAL NEUTRON LARMOR PRECESSION MAGNETS

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ABSTRACT: Spectroscopic techniques based on Larmor precession of particle spins require that for all trajectories of a diverging beam the path integral of the modulus of the magnetic field must be a constant. The amount of precession performed by each spin is then a function of the particle energy only. For cylinder magnets this homogeneity condition can be expressed as a variational problem, the analytical solution of which is presented in this paper. This solution describes the optimal field shape (OFS) to obtain the best possible homogeneity for a given magnet length. In practice the ideal shape can be obtained by superposing a series of solenoids of different lengths. For practical magnet lengths this homogeneity is generally not good enough (say $\Delta f/f = 10^{-4}$) so that in-beam correction coils have to be envisaged. Their optimal design can also be performed analytically using OFS, including corrections for the line integral differences caused by the finite beam divergence. We present the solution together with a method to implement it in practice using discrete in-beam current distributions. The resulting magnet has a homogeneity of 10^{-6} , so that the Larmor precession angle is still well defined after 10^4 turns. This avoids tedious correction procedures needed for presently existing magnets and opens new fields of applications of Larmor precession spectroscopy methods.

1. INTRODUCTION: Neutron Larmor precession spectroscopy techniques use the particle spin as a velocity (energy) indicator. They are based on the condition that for all neutrons having the same energy E , the amount of precession performed while traversing a magnetic field region is identical. The classical picture of Larmor precession is applicable and the equation of motion of the magnetic moment $\underline{m}$ of a neutron traversing a magnetic field induction $\underline{B}$ is:

$$\frac{d\mathbf{m}}{dt} = \gamma \mathbf{m} \times \mathbf{B} \quad (1.1)$$

v_z being the constant neutron velocity along a beam axis z and γ the neutron gyromagnetic constant ($\gamma = 1.8 \cdot 10^8$ radian/sec. Tesla), the time and z dependence can be exchanged so that the z variation of $\underline{m}$ reads:

$$\frac{d\mathbf{m}}{dz} = \gamma / v_z (\mathbf{m} \times \mathbf{B}) \quad (1.2)$$

Initiating the precession with $\underline{m}$ perpendicular to $\underline{B}$ for all neutron velocities, which corresponds to the experimental situation, we require that this situation remains the same along the entire precession path. In other words the spatial variations of $\underline{B}$ have to satisfy: $\partial(\underline{m} \cdot \underline{B})/\partial t = 0$, and we look for the conditions imposed on $\underline{B}$ and the expression for the total amount of Larmor precession performed in a given field distribution. Clearly the field $\underline{B}$ has to keep the same direction or vary very slowly as experienced by the moving neutron. Thence the amount of precession performed by a given neutron while traversing a time independent magnetic field region will be a measure of its energy. The amount of Larmor turns N accumulated over a length L across $\underline{B}$ by a neutron of energy E will be proportional to the line integral along the path of the neutron of the modulus of $\underline{B}$:

$$N + (\gamma/\sqrt{E}) \int \|\underline{B}\| dl \quad (1.3)$$

If the relation between N and the neutron energy is to be unique, the $\|\underline{B}\|$ line integrals have to be exactly identical for all possible neutron trajectories within the beam, whether they are parallel or inclined with respect to the beam axis (within the angular beam divergence). If this condition is not fulfilled, neutrons of energy E , which undergo the same spectroscopic event but travel through different sections of the magnet, may end-up with different Larmor phase angles. This effect blurs the final spin polarization and appears as a resolution limiting effect. If the field does not strictly point in the same direction everywhere along the integration path L and shows too rapid variations as compared to $\omega = \gamma B$ the Larmor frequency, we will have to check the proportionality between N and $\int \|\underline{B}\| dl$ by solving the exact equation of motion (1.1).

Although Larmor precession techniques have been in use for more than a decade now, no systematic study of the best suited magnetic field shapes has so far been undertaken. In the present paper we present optimization calculations which will allow to build precession magnets good enough to enable Larmor spectroscopy techniques such as neutron spin echo¹, an analogue to NMR spin echo², to produce better data and to reach their intrinsic resolution potential. The resolution limitation of inelastic NSE arises from the fact that the relationship between the measured polarization and the spectroscopic quantity to be determined is only simple and linear provided the total amount of Larmor precession is limited to about $N = 10^4$. A precession magnet for inelastic NSE should therefore be strong enough to guarantee a path independent number of turns up to $N = 10^4$. The homogeneity should be such that for a given energy, the deviation in N is not seen by the final spin analyzer, which imposes an average relative homogeneity of 10^{-6} to the precession field. With realistic spectral and angular beam divergence distributions, this allows for maximum relative integral deviations of a few 10^{-6} . The above conditions are very restrictive and sufficient for other Larmor precession techniques than NSE too. To show that the above conditions can indeed be fulfilled is the object of the present paper.

2. OPTIMAL FIELD SHAPE FOR PRECESSION MAGNETS: We start calculating the optimal field shape in cylinder geometry followed by the determination of the current distributions required to obtain the desired field shape. In-beam current distributions needed to remove at the same time the residual magnet inhomogeneities and the pathlength differences due to finite beam divergence will then be considered. Finally the overall response of a Larmor precession spectrometer equipped with optimized magnets must be calculated by solving the spin equation of motion across the complete apparatus numerically. This is needed to verify that field wiggles due to finite conductor dimensions or fringe fields of ancillary devices such as flippers do not perturb precession or main coil homogeneity. The result is a feasibility demonstration of the above optimization calculations, leading towards an optimal use of Larmor precession.

We start from a rotation symmetric magnetic field induction $\underline{B}(r,z)$ the rotation axis being z , and r the radial coordinate. Furthermore we suppose no currents flowing for $r < R$. Using Maxwell's equations the line integral of the modulus of $\underline{B}$ over the length L can be written to lowest order as a series expansion:

$$\int \|\underline{B}\| dz = \int B_z(0,z) dz + \frac{1}{8} r^2 \int \{ (\partial B_z(0,z)/\partial z)^2 / B_z(0,z) \} dz - \frac{1}{4} r^2 \int \{ \partial^2 B_z(0,z)/\partial z^2 \} dz \quad (2.1)$$

in the vicinity of the z axis for $r < R$. This approximation is sufficient to obtain the required accuracy. For less symmetric fields, similar expressions can be worked-out, but the coefficients of the r^2 terms are always larger³. For example for a transverse field distribution having a symmetry plane the r^2 coefficients are four times larger. We now look for a symmetric solution on the z axis $B_z(0,z) > 0$ in the finite interval $-1/2L < z < 1/2L$, such that $B_z = 0$ and $\partial B_z/\partial z = 0$ at the boundary points $z = -1/2L$ and $z = 1/2L$. These vanishing field and zero gradient conditions favour better flipper operation at the points where precession is started and stopped. They also guarantee minimum magnetic interaction between the various components of the spectrometer, so that the field in the sample and flipper areas can be controlled independently of the precession fields⁴. The optimal field shape shall be the solution(s) of minimizing:

$$\frac{1}{8} r^2 \int \{ (\partial B_z(0,z)/\partial z)^2 / B_z(0,z) \} dz - \frac{1}{4} r^2 \int \{ \partial^2 B_z(0,z)/\partial z^2 \} dz \quad (2.2)$$

Note that for symmetric continuous $B_z(0,z)$ functions over this interval the second term integrates to zero because of the boundary conditions. Hence there is no such solution which satisfies (2.2) = 0 exactly since the first term always is positive and $\neq 0$.

Let us search for a continuous solution first, minimizing the first term of (2.2). Suppose that $f(z) = B_z(0,z)$ is the OFS, then for any small perturbation $h(z)$ to

$$f(z) \text{ such that: } h(-1/2L) = h(1/2L) = 0 = \int_{-1/2L}^{1/2L} h(z) dz \quad (2.3)$$

the deviation ($'$ denotes the z derivative)

$$\int_{-1/2L}^{1/2L} \{ (f+h)^2 / (f+h) \} dz - \int_{-1/2L}^{1/2L} (f^2/f) dz = \int_{-1/2L}^{1/2L} \{ 2h'(f/f) - h(f/f)^2 \} dz + \text{order } (h^2) \text{ terms} \quad (2.4)$$

must be of order h^2 . This implies that for any $h(z)$ satisfying (2.3):

$$\int_{-1/2L}^{1/2L} 2h'(f/f) dz - \int_{-1/2L}^{1/2L} h(f/f)^2 dz = 0$$

Remembering the third condition of (2.3) this implies:

$$2(f/f)' + (f/f)^2 = 0, \quad \text{or } \pm A^2 \quad (2.5)$$

with A a positive constant. The $+A^2$ solution leads to an unphysical solution containing exponential functions. Let us develop the $-A^2$ possibility:

$$2(f/f)' + (f/f)^2 = -A^2 \quad (2.6)$$

can be solved easily by setting $g = f/f$, which yields

$$g = A \tan 1/2 A (C - z), \quad C \text{ being an integration constant, leading to } f(z) = D \cos^2 1/2 A (C - z), \quad D \text{ is a constant} \quad (2.7)$$

This is a physical solution which we rewrite, taking the boundary conditions into account and setting $B_z(0,0) = B_0$

$$B_z(0,z) = B_0 \cos^2 \pi z/L = B_0/2 (1 + \cos 2\pi z/L) \quad (\text{OFS}) \quad (2.8)$$

The integrals in (2.1) then write:

$$\int_{-1/2L}^{1/2L} B_z(0,z) dz = 1/2 B_0 L \quad \text{and} \quad \int_{-1/2L}^{1/2L} \{ (\partial B_z(0,z)/\partial z)^2 / B_z(0,z) \} dz = 1/2 B_0 L (2\pi/L)^2$$

which leads to the inhomogeneity formula for the $\cos^2$ field shape

$$\eta_{\text{OFS}} = \left(\int \|\underline{B}\|(r,z) dz - \int B_z(0,z) dz \right) / \int B_z(0,z) dz = 1/2(\pi r/L)^2 \quad (2.9)$$

It can easily be verified that this field shape gives indeed better homogeneity than for example other standard bell-shaped curves.

The optimal $\cos^2$ field shape can be reproduced in practice by simple current distributions exhibiting line integral inhomogeneities for parallel trajectories

which agree with the theoretical inhomogeneity expression (2.9). The solution to this inverse problem is described in the next section. It can be implemented with small diameter magnets which have a length of about .8 L, L being the length of the field distribution.

Since the theoretical inhomogeneity corresponds to the shape of a field it is independent of the diameter of any current distribution as in the formula valid for simple solenoids (see below). The important consequence is that we do not have to use solenoids of large diameters to make them homogeneous, but instead use the smallest diameters compatible with the beam size thus considerably reducing costs.

Magnets producing an optimal field shape (OFS) can also be made shorter than simple solenoids at comparable homogeneity. Because in OFS magnets the inhomogeneities are less localized than in solenoids, their corrections are easier to implement. Unlike simple solenoids, for which the homogeneity increases with increasing length and diameter, the OFS homogeneity improves faster with length and does not depend on its diameter which can therefore be made small enough to reduce lateral stray fields to such a low level that it does not influence the field in the other magnet.

An approximate formula for the line integral homogeneity of simple solenoids was given by Mezei⁵. The relative line integral inhomogeneity due to the radial field increase as one moves away from the axis is given by $\eta_{\text{SOL}} \approx r^2/2DL_S$, with r the beam radius, D the solenoid diameter and L_S its length. If we set $L_S = .8L$ and $D = .08$ Meter, we can compare with the homogeneity formula (2.9) and form the equal length and beam diameter ratio $\eta_{\text{SOL}}/\eta_{\text{OFS}} \approx 2L_S$. For reasonable η values L_S has to be larger than 1 Meter so that OFS is at least two times better than a solenoid of same diameter. For $\eta_{\text{OFS}} = 5 \cdot 10^{-4}$ (beam diameter .04 Meter) a magnet length of 1.7 M is required whereas a solenoid needs to have a length of 5 Meters to reach this homogeneity.

Let us now come back to the first solution of (2.5) which after setting $g = f'/f$, leads to the differential equation: $2g' + g^2 = 0$ the solution of which is:

$$g = (k + 1/2z)^{-1} \text{ leading to: } f = C (k + 1/2z)^2 \quad (k \text{ and } C \text{ are constants and}) \quad (2.10)$$

$$B_z(0,z) = B_0(1 - 2L/L)^2$$

This solution has a finite derivative at $z = 0$, and since $\partial^2 B_z / \partial z^2 \neq 0 \forall z$ the second term of (2.2) is nonzero and exactly compensates the first term, so that for (2.10) equation (2.2) gives a minimum of exactly 0. The corresponding field shape is discontinuous at $z = 0$, it gives perfect homogeneity for all neutron paths parallel to the z axis and this even for very short magnets. The limitations here possibly arise from too fast field variations at $z = 0$ for small Larmor frequencies. The practical implementation too is more involved than for the smoother $\cos^2$ shape. Such rapidly varying field shapes needs high current densities and probably an in-beam current sheet to reproduce the discontinuity at $z = 0$ accurately enough. This is not really a problem since a few in-beam correction coils are required anyhow to remove the pathlength differences due to finite angular divergence. Calculations have nevertheless convinced us that the implementation of field shape (2.10) is worth further investigation the result of which will be described elsewhere.

3. CURRENT DISTRIBUTION FOR OFS MAGNETS: To solve the inverse problem of finding the current distributions which reproduce OFS along the beam axis z : $B_z(0,z) = B_0 \cos^2 \pi z/L$ in the best way we start with a number M of superposed and concentric solenoid layers of radii r_i and decreasing lengths l_i , i varying from 1 to M , all carrying the same current density. The initial guess for the set of M values of l_i is taken in such a way that the shape of the current distribution is close to the field shape to be produced. The r_i are calculated for given wire diameters and insulator thicknesses, starting from R the radius of the innermost layer. The optimum set of l_i values for the lengths of the superposed coils are then determined numerically by a least square routine minimizing the difference between OFS and the exactly calculated field from the M superposed solenoids. Typically $M=10$ superposed solenoids are sufficient to properly reproduce the $\cos^2$ shape.

The next step is to look for the best inner diameter of such a composite coil system. In order to reduce the stray field we require the smallest possible diameter. Since the homogeneity is not directly related to the magnet diameter, the latter can be kept as small as allowed by the neutron beam diameter (typically .04 Meter). If the coils are made superconducting sufficient space for cryostat walls and thermal insulation has to be left between beam and coils. The smallest practical superconducting coil diameter with a room temperature bore for the above beam diameter seems to be of the order of .08 Meter. In the following we will focus on a pair of superconducting magnets designed for a thermal neutron Triple Axis Spectrometer with Spin Echo (TASSE)⁶, at least as far as precise design details are concerned. Extension towards stronger fields suitable for the higher neutron energies of recent spallation sources is straightforward. In fact there is a more fundamental lower limit for the inner solenoid diameter. Since the overall current distribution of the M solenoids necessarily varies stepwise along z , despite the use of low current (20 Amps) and small wire diameters (.0002 Meter), there will be wiggles in the radial component of the off-axis field at positions corresponding to $z = l_i$. The effect of these current density steps increases as the neutron passes closer to the wires and for given current density steps there will be a minimum value for the innermost diameter of the magnet. Relation (1.3) may therefore not hold anymore when the field variations as seen by the neutron become comparable to the Larmor frequency. To evaluate the depolarization effect of such field vector

wiggles precisely one has to integrate the equation of motion of the individual spin (1.1) across a realistic field distribution calculated exactly from the current distribution. We performed such calculations numerically for various neutron trajectories and came to the conclusion that the practical limit of .08 M for the inner coil diameter can only be used if the number of superposed solenoids is made larger than the initial value of 10 needed to obtain a sufficiently good field shape on the axis. With 20 layers the OFS approximation is only slightly better but the wiggles of the radial field component at the outer beam radius are sufficiently weak not to be seen by the neutron spins at any envisageable energy. In fact there are no real wiggles at the $l_i = l_j$ values but rather inflexion points of the radial component distribution. We also checked that the field interaction of OFS magnets with such a diameter is indeed vanishingly small for all spectrometer settings, the fringe fields do not perturb the homogeneities. Table 1 shows exact numerical line integral calculations for various neutron paths across the real magnet, including the reference integral along the magnet axis. It can be seen that the inhomogeneity for parallel trajectories corresponds indeed to the value calculated using the OFS inhomogeneity formula (2.9).

OFS magnet homogeneities depend on their lengths only, for practical lengths η_{OFS} is limited to $\sim 10^{-4}$. Standard angular beam divergencies introduce pathlength inhomogeneities of the same order so that further corrections are required.

4. IN-BEAM CORRECTION COILS: The only way to further improve the irrotational OFS solution described above is to introduce currents in the neutron beam. Fortunately most conductors are rather transparent to neutrons. Rewriting Maxwell's equations including an unknown current distribution $J(r,z)$ and generalizing the path integrals to oblique trajectories we shall minimize the line integral again. We derive the formalism of in-beam corrections for general cylinder field shapes, having in mind their application to OFS magnets and classical solenoids, and why not to an improved correction of already existing magnets. We consider current distributions applied to field regions where the main field B is essentially oriented along the cylinder axis, i.e. $|B_r| \ll |B_z|$, so that $B \sim B_z$ is significantly stronger than the one generated by the correction coil field induction B^c (strong field limit). The modulus of the total field B^t can then be written as

$$|B^t| = \{ (B_z + B_z^c)^2 + (B_r + B_r^c)^2 \}^{1/2} \quad (4.1)$$

$$= |B| + B_z^c + (B_r B_r^c + 1/2 (B_r^c{}^2 + B_z^c{}^2)) / |B|$$

$B_z^c(r,z)$ is exactly the term proportional to the correction current density we wish to introduce in order to correct for the line integral variations of $|B|$. To obtain this simple field superposition, we minimize the third term which first requires that for the main field, $|B_r| \ll |B_z|$ which limits the application of corrections to the central high field part of the magnet. More quantitatively, we impose that the radial to longitudinal component field ratio $|B_r|/|B_z|$ should be small enough not to produce field integral errors larger than 10^{-6} . For a $\cos^2$ shape field for example, the residual integral error is $1/2(\pi r/L)^2$ so that the relative integral error will have to be less than $2(L/\pi r)^2 \cdot 10^{-6}$. This sets an upper limit to the ratio $(B_r/B_z)^2$. From this limit we can deduce the maximum central z range, $|z| < L_C$ for the application of in-beam corrections:

$$B_r/B_z < [2(L/\pi r)^2 \cdot 10^{-6}]^{1/2} \quad (\text{for } \cos^2 \text{ shape fields}) \quad (4.2)$$

For other field shapes this condition has to be adapted. Second, B_r^c and B_z^c being both proportional to $J(r,z)$, the correction current distribution we are looking

for, we need to minimize $\int J^2(r,z) / |B|^2 dz$ which completes the minimization of the third term of (4.1) and establishes the framework of the strong field approximation. The uncorrected field modulus is of the form (compare 2.1) $|B(r,z)| = B_z(0,z) + r^2 f(z)$ which defines $f(z)$. Hence the radial part of the correction current must be of order r^2 , i.e. $J(r,z) = r^2 J'(z)$ and we need to find the longitudinal (z) shape of this current distribution $j(r) = \partial J(r,z)/\partial r = r J'(z)$. In order to take oblique trajectories into account and to correct for their longer pathlength we express the Larmor precession angle $\phi = 2\pi N$ should be the same for all trajectories and depend on the neutron energy only.

$$\phi + \gamma / v_z \int_A^B |B^t| dz = \gamma / v_z \int_A^B \{ B_z(0,z) + r^2 f(z) + B_z^c(r,z) \} dz$$

for a general trajectory connecting point $A(x_A, y_A, -L/2)$ to $B(x_B, y_B, L/2)$.

We calculate $\int_A^B B_z^c(r,z) dz$ using Maxwell's relation $\nabla \times B = \mu_0 J$ since the trajectories now traverse current density J .

Using $x(z) = x_A + v_x/v_z z$ and $y(z) = y_A + v_y/v_z z$, we calculate a loop integral along a general trajectory and back along the z axis. The current term then reads:

$$\mu_0 \int_A^B \int_0^{r(z)} r J'(z) dr dz = \mu_0/2 \int_A^B r^2(z) J'(z) dz$$

This yields an expression for $\int_A^B B_z^c(r,z) dz$ leading to an explicit formula for the Larmor phase angle along a general trajectory across the magnet:

$$\phi + \gamma / v_z \int_A^B \{ C(z) + r^2(z) [f(z) - \mu_0/2 J'(z)] \} dz \quad (4.4)$$

taking the z components of the main coil together with the constant contribution of the correction coils $C(z) = B_z(0,z) + 1/L \int_A^B B_z^c(0,z) dz$. $J'(z)$ is the z -dependent part of the correction current distribution we are looking for. Note that r is a

function of z now. For parallel trajectories $\partial r/\partial z = 0$ and the solution of our problem is given by:

$$\int_A^B [f(z) - \mu_0 J'(z)] dz = 0 \quad (4.5)$$

We may then replace $J'(z)$ by a delta function at any value z_0 within the limits imposed by the strong field condition (4.2). The correction is then given by $j(r) = rJ'(z_0)$ with:

$$J'(z_0) = \delta(z - z_0) / \mu_0 \int_A^B f(z) dz, \quad |z_0| < L_c \text{ clearly depending on the shape of the precession field. This is similar to the correction coils introduced by Mezei}^7, \text{ except that here we draw the conclusion that they should be positioned inside the magnet rather than at the edges, for the sake of the strong field approximation. To obtain } J'(z) \text{ for oblique trajectories too, we substitute } r^2(z) = x_A^2 + x_B^2 + [(v_x^2 + v_y^2)/v_z^2] z^2 \text{ in (4.4) which gives, remembering that all functions of } z \text{ have to be even and substituting for } E + v_x^2 + v_y^2 + v_z^2$$

$$\varphi + \gamma/\sqrt{E} \left\{ \int_0^B [2C(z) - (v_x^2 + v_y^2)/v_z^2 z^2 \{ \mu_0 J(z) - 2f(z) \}] dz \right\} \\ (1 - (v_x^2 + v_y^2)/v_z^2)^{-1} \quad (4.6)$$

This expression is of the form $\varphi = (A - D\alpha)/(1 - \alpha)$, with $\alpha = (v_x^2 + v_y^2)/v_z^2$. We want φ to depend on the neutron energy E only, not on the direction under which it travels. Expressing therefore the independence of φ with respect to α leads to the condition $D/A = 1/1 = 1$ or explicitly:

$$\int_0^B [2C(z) dz] = \int_0^B [2f(z) - \mu_0 J(z)] dz \quad (4.7)$$

$\int_0^L J^2(r, z) / |B|^2 dz$ minimal and conditions (4.5) and (4.7) now describe our variational problem completely. Using a similar perturbation technique than for the analytical OFS solution above we obtain $J'(z)$. The general form of the solution is:

$J'(z) = 1/\mu_0 (X z^2 - Y) C^2(z)$ where X and Y are constants. The shape of $J'(z)$ for the case of an OFS magnet can be seen in figure 1.

Practically the continuous solution $J(r, z)$ will have to be discretized to be implemented inside the magnet. This means that we will dispose along the magnet axis, and within $|z| < L_c$ limits given by the strong field limit, a small number of identical spiral coils representing a current density $j(r)$ at distances calculated to reproduce the function $J'(z)$ best. The calculation of these optimal positions for the spirals is performed numerically along the formalism described above, but in a discrete manner taking a given number (8 in this case) of spiral sheets. The solution is an optimal current distribution reducing the remaining maximum inhomogeneity to 2.10^{-6} for all parallel and oblique neutron trajectories as can be seen in table 1.

This table contains the results of exact line integrals for an OFS magnet of dimensions:

$L = 1.7$ Meter, $D = .08$ Meter, beam diameter $.038$ Meter.

Table 1. EXACT LINE INTEGRALS FOR OFS MAGNET

Starting point of neutrons ($X_A, Y_A = 0, Z_A = 0$)			End point ($X_B, Y_B, Z_B = 2M$)			
X_A METERS	X_B	Y_B	LINE INTEGRAL TESLA-METER	DIFFERENCE	LINE INTEGRAL TESLA-METER	DIFFERENCE
0.0000	0.0000	0.0000	1.000224	REFERENCE	NEUTRON 1.000732	REFERENCE
0.0095	0.0000	0.0000	1.000263	0.000039	1.000732	0.000000
0.0095	0.0095	0.0000	1.000327	0.000103	1.000732	0.000000
0.0095	0.0190	0.0000	1.000471	0.000247	1.000732	0.000001
0.0095	0.0000	0.0095	1.000302	0.000078	1.000732	0.000000
0.0095	0.0000	0.0190	1.000419	0.000195	1.000733	0.000001
0.0095	-0.0095	0.0000	1.000276	0.000052	1.000732	0.000000
0.0095	-0.0190	0.0000	1.000368	0.000144	1.000732	0.000000
0.0190	0.0000	0.0000	1.000380	0.000156	1.000733	0.000001
0.0190	0.0095	0.0000	1.000471	0.000247	1.000733	0.000001
0.0190	0.0190	0.0000	1.000640	0.000416	1.000734	0.000002
0.0190	0.0000	0.0095	1.000419	0.000195	1.000733	0.000001
0.0190	0.0000	0.0190	1.000537	0.000313	1.000733	0.000001
0.0190	-0.0095	0.0000	1.000368	0.000144	1.000732	0.000000
0.0190	-0.0190	0.0000	1.000434	0.000210	1.000732	0.000001
OFS alone				OFS with in-beam corrections		

For a solenoid of similar dimensions, $L_S = 1.7$ Meter, $D = .08$ Meter, $J'(z)$ (fig 2.) shows more pronounced peaks near $z/L = .4$, and higher current densities, since the overall correction is larger.

The final corrected homogeneity (table 2.) obtained is ten times worse than for OFS.

Table 2. EXACT LINE INTEGRALS FOR SOLENOID MAGNET

Starting point of neutrons ($X_A, Y_A = 0, Z_A = 0$)			End point ($X_B, Y_B, Z_B = 2M$)			
X_A	X_B	Y_B	LINE INTEGRAL TESLA-METER	DIFFERENCE	LINE INTEGRAL TESLA-METER	DIFFERENCE
METERS						
0.0000	0.0000	0.0000	1.17125	REFERENCE	NEUTRON 1.173650	REFERENCE
0.0095	0.0000	0.0000	1.17139	0.00014	1.173652	0.000002
0.0095	0.0095	0.0000	1.17158	0.00033	1.173654	0.000004
0.0095	0.0190	0.0000	1.17207	0.00082	1.173658	0.000008
0.0095	0.0000	0.0095	1.17153	0.00028	1.173653	0.000003
0.0095	0.0000	0.0190	1.17195	0.00070	1.173659	0.000009
0.0095	-0.0095	0.0000	1.17147	0.00022	1.173658	0.000008
0.0095	-0.0190	0.0000	1.17184	0.00059	1.173656	0.000006
0.0190	0.0000	0.0000	1.17182	0.00057	1.173653	0.000003
0.0190	0.0095	0.0000	1.17207	0.00082	1.173657	0.000007
0.0190	0.0190	0.0000	1.17263	0.00138	1.173670	0.000020
0.0190	0.0000	0.0095	1.17195	0.00070	1.173654	0.000004
0.0190	0.0000	0.0190	1.17239	0.00114	1.173663	0.000013
0.0190	-0.0095	0.0000	1.17184	0.00059	1.173655	0.000005
0.0190	-0.0190	0.0000	1.17214	0.00089	1.173655	0.000005
Solenoid alone			with optimized in-beam corrections			

Therefore OFS magnets not only yield better intrinsic homogeneity, but owing to their ideally distributed inhomogeneities, they can also be corrected much more efficiently to reach final homogeneities which surpass similarly corrected solenoids by one order of magnitude.

5. CONCLUSION: To summarize, OFS allows the use of significantly shorter magnets while our novel correction techniques enables line integral homogeneities which surpass previous values by an order of magnitude. The use of larger beam cross sections and larger angular divergencies, the effect of which can now be corrected for, allow better beam intensities. Because the line integral homogeneity achieved are as good as needed, the ultimate and intrinsic resolution limits of Larmor techniques, in particular inelastic NSE spectroscopy, even at high incident energies, can be reached. Resolution corrections due to imperfect magnets shall be pointless. OFS magnets present extremely reduced stray fields as compared to the large solenoids used before, thus solving the problem of the magnetic interaction between the two precession magnets used in NSE.

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- ² E.L. Hahn, "Spin Echoes", Phys. Rev. vol. 80, p 580, 1950.
- ³ J. Hayter and R. Pynn, "On the production of Homogeneous Field Integrals for Neutron Spin Echo", JLL internal scientific report, 81HA5, 1981
- ⁴ In standard techniques precession start/stop occurs in low field regions, so that the optimal field shape is the one which increases to a maximum (and symmetrically drops back) over the shortest possible distance yet minimizing (2.2). The ideal field would of course be a strong, short and strictly constant field. Abrupt (in the sense of Larmor frequency) field ends could be used as flippers. We considered such designs; the superconducting field separation sheets are the delicate part.
- ⁵ F. Mezei in reference 1, Workshop Proceedings p 184
- ⁶ C.M.E. Zeyen, "A Neutron Spin Echo Device to improve the Energy Resolution of Triple Axis Spectrometers", in Neutron Scattering, Conference Proceedings, A.I.P. pp101-110, 1982
- ⁷ F. Mezei. in reference 1, Workshop Proceedings p 186