OPTIMIZING THE CONDUCTOR DIMENSIONS FOR A 10-13 T SUPERCONDUCTING DIPOLE MAGNET

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Abstract

Most of the dipole magnet design studies are focused on the quality of the dipole field. However, to be of practical interest certain requirements have to be considered. For instance, to facilitate the winding of the coil end part, the top angle of the inner layer of the coils should be limited and the cable used for this layer has to be kept thin. An essential requirement is also that the peak field in a coil layer must not exceed the maximum field of the superconductor as a function of the current density. In this paper a method is described to design the conductor dimensions and lay-out for high field dipole magnets subject to a number of requirements and using a measured $J_{\max}(B)$ relation for the superconductor.

Introduction

The design of superconducting dipole magnets is usually based upon a small set of parameters. This set includes for instance the dipole field, the beampipe diameter, the field homogeneity and perhaps the outer dimensions of the magnet. In most cases the use of (cold) iron is inevitable and the inner radius of an iron yoke is specified, based on the value of the dipole field. Considering these specifications, a lay-out of conductors can already be designed. However, such a lay-out is often in conflict with aspects of feasibility, and properties as transport current, cable dimensions, etc. are not within practical limits. The design method described in this report takes into account additional requirements, to produce a practical solution. This design is focused on the determination of the conductor dimensions for an 11.5 tesla dipole magnet(*).

Another important aspect of the design is the peak field in a coil layer. It must not exceed the maximum field of the superconductor as a function of the current density. This implies that already in the design process the $J_{\max}(B)$ relation of the superconductor should be taken into account in order to yield a realistic solution.

The constraints in this study, considering a 2-dimensional structure, are summarized below.

- The winding configuration of the dipole is a shell-type $\cos\phi$ (see fig. 1), in order to optimize the use of superconductor. To reduce cost and work the number of layers should be kept to a minimum.
- The prescribed homogeneity gives an upper bound to the relative value of the multipoles. Considering fabrication tolerances an upper bound of 10^{-4} can just be met.
- If the volume of superconductor is to be minimized, all layers should simultaneously reach their critical current density with respect to the maximum field in the layer. This also increases the safety of the magnet in case of a quench.
- The different layers of the magnet have to carry the same transport current which should not exceed 20 kA in order to remain practical.
- Special attention is asked for the magnet end geometry which follows from the optimized structure. A thin cable and a large bending radius of the cable will facilitate the winding and it avoids cable damage due to sharp bends.

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-The number of strands and also the diameter of the strands should remain within a practical range.

-The external shape of the coils has to enable a suitable mechanical support and prestress. This implies e.g. that the top angles of the layers should diminish in the outward direction.

In practice these constraints can not be simultaneously satisfied and as a consequence reasonable values have to be specified. In our case the following values were used:

- cable thickness : < 3 mm,
- top angle of layers : ≤ 70 degrees,
- number of strands per cable : ≤ 36 .

The employed $J_{\max}(B)$ relation, specifying the average current density, was obtained from measured strand data of a commercially available Nb_3Sn superconductor taking into account margins for (1) the compaction factor of the cable, (2) the insulation thickness and (3) safety. These margins of 13%, 16% and 10% resp. rendered the following $J_{\max}(B)$ relation for the cable ($B \in [8, 13]$ T):

$$J_{\max}(B) = -9.02 \ln(B) + 25.3 \quad [10^8 \text{ A/m}^2] \quad (1)$$

This corresponds to a current density of 1373 A/mm^2 at 10 tesla in the non-Cu part of the strands. To simplify the calculation the following assumptions were used:

- the optimization is 2 dimensional; no end effects are considered,
- only one beampipe is considered,
- the iron surrounding the magnet has a circular bore, is assumed to extend infinitely and is unsaturated (μ_r is infinite),
- the cables in the layers are produced from the same superconductor; the same $J_{\max}(B)$ relation holds for all layers,

Fig. 1. Schematic view of 1/4 of the cross section of a magnet showing the 2 coil layers and the relevant variables: B: peak field in a layer; w: width of a layer; J: overall current density and ϕ : angle of a circular segment.

-the maximum field in a layer is assumed to occur at the top angle at the inner radius of the layer (see fig. 1). Finally, the beampipe radius is 25 mm and the inner radius of the iron is 100 mm.

Maximum field in a dipole layer

The amount of computation time necessary for an optimization will in general be dominated by the evaluation of the optimization function. In this study the calculation of the multipoles and the maximum field in a layer will dominate the evaluation. The contribution of circular current segments surrounded by a circular iron screen can be quickly calculated using analytical expressions. The maximum field of circular layers however can only be obtained by numerical integration, which is very time consuming, or it can be expressed in a series. It is possible to derive that the components of the magnetic field B at position (r, θ) , due to a circular current segment bounded by a_1 and a_2 in the radial direction and ϕ_1 and ϕ_2 in the azimuthal direction, have the form:

$$B_{\text{comp}} \propto \sum_{n=1}^{2k-1} (\sin(n\phi_2) - \sin(n\phi_1)) G(n\theta) \frac{r^{n-1}}{n(2-n)} \left[a_2^{2-n} - a_1^{2-n} \right]$$

$$r \leq a_1, k \rightarrow \infty \quad (2)$$

and

$$B_{\text{comp}} \propto \sum_{n=1}^{2k-1} (\sin(n\phi_2) - \sin(n\phi_1)) G(n\theta) \frac{r^{-n-1}}{n(2+n)} \left[a_2^{2+n} - a_1^{2+n} \right]$$

$$r \geq a_2, k \rightarrow \infty \quad (3)$$

where the function $G(n\theta)$ equals $\sin(n\theta)$ for $\text{comp} = r$ and $\cos(n\theta)$ for $\text{comp} = \theta$. For $a_1 \leq r \leq a_2$ (2) and (3) must be combined. According to "Dirichlet's test" these series are convergent if the limit $n \rightarrow \infty$ of the term T_n with r, a_1 and a_2 in (2) and (3) is 0, since $\sin(n\theta)G(n\theta)$ is bounded. Let us analyze (2), a similar analysis can be applied to (3):

$$T_n = \frac{r^{n-1}}{n(2-n)} \left[a_2^{2-n} - a_1^{2-n} \right] = \frac{r}{n(2-n)} (p_2 - p_1), \quad (4)$$

where $p_1 = a_1/r$ and $p_2 = a_2/r$. Clearly T_n has a limit 0, so (2) converges. The shape of T_n in (4) raises the presumption that (2) converges as $1/n$. This becomes clear when writing T_n for large k :

$$\sum T_n \approx \sum 1/n^2 = \frac{\pi^2}{6} - \frac{\text{const}}{k+1} \approx \frac{\pi^2}{6} - 1/k. \quad (5)$$

It appears that an extrapolation of B using the 16th term and the 21st term already agrees within 6 digits with a numerical integration of the field with a large number of subdivisions. Since (4) can be quickly evaluated and θ is a constant, the computation of B is very fast. In practice an accuracy of 3 to 4 digits is sufficient.

The optimization function

The optimization is in fact a multivariate minimization subject to linear bounds. A suitable algorithm is the "quasi-Newton", for which routines are available (NAG). The minimization function that was used has the form:

$$F = g_1 \sum_n \left[\frac{B_n}{B_1} \right]^2 + g_2 \sum_L \left[\frac{\hat{B} - B_{\text{max}}(J)}{B_{\text{max}}(J)} \right]^2, \quad (6)$$

where n denotes the multipole number, L the layer, $\hat{B}$ the maximum field occurring in a layer and $B_{\text{max}}(J)$ the maximum field as a function of J . The constants g_1 and g_2 are used to scale the terms to equal size. Optimization functions including terms with e.g. the thickness or top angle of the inner layer cable were also investigated. These functions failed to produce good solutions, let alone systematic solutions. The simplest

optimization functions produced the best solutions.

The optimization

The relevant variables for the optimization are indicated in fig. 1. As a first step only the multipoles B_3 and B_5 are optimized with cables modeled as circular segments for a 2-layer structure. This provides a fast and reasonable approximation of the final structure and it is very instructive. As many variables as possible are eliminated using the above requirements. For example, a dipole field of 11.5 tesla means that the current density in one of the layers can be eliminated as variable. As a result of the optimization two classes of solutions are found, one with top angles for inner/outer layer of about 70/45 degrees resp. and another with top angles of about 50/80 degrees resp. The last class requires more superconductor and is difficult to support mechanically. Therefore, attention was focused on the first class of solutions. An interesting investigation is to vary the desired dipole field B_d and to observe the relevant properties of the dipole: volume of superconductor (or rather area in 2-D), top angle of the layers, transport current and the ratio of the widths of the two layers. Fig. 2 shows the result when B_d is varied between 9 and 13 tesla for the $B_{\text{max}}(J)$ relation of (1) without the mentioned margins. The top angles of the layers exhibit almost no variation, therefore instead of the area of superconductor fig. 2 shows the total width of the layers. Clearly, a dipole with two layers of equal width is not necessarily the best solution. The inclusion of the $B_{\text{max}}(J)$ relation in the optimization function precludes scaling of a solution for higher fields. Rather a strong increase of the transport current and the total width of the layers is found for higher B_d .

Fig. 2. Effect of the desired dipole field on the transport current (I), the ratio of width of the two layers (w_1/w_2) and the total width of the two layers (w_t).

As a next step in the optimization the ratio of width of the two layers w_2/w_1 is taken as a parameter and varied for several values of the desired dipole field B_d . No spacers are considered and relation (1) is used for $B_{\text{max}}(J)$. Solutions are obtained for which the properties of the resulting dipole vary systematically. Figs. 3 and 4 show two of these properties. In most of the figures discontinuities occur when the solution changes from one class to the other (see above). This can be biased by varying the starting values of the variables. Some solutions of the second class are also shown in order to illustrate the effect on the properties. The thickness of the thinnest side of the inner cable, t_1 , is obtained by using the

requirement that the cable in the outer layer is to be made from 36 strands (to reduce the thickness). The ratio of thickness and width of the outer cable can then be approximated. Fig. 3 shows that t_1 increases when the width of the outer layer increases relative to the width of the inner layer. This is due to the demand that the inner and outer cable must carry the same transport current. Fig. 4 shows the top angle of the inner layer versus w_2/w_1 . This is an important property since it determines the smallest bending radius of the (thickest) cable in the magnet end. The decrease of this angle for higher fields is due to the inclusion of $B_{max}(J)$ in the optimization function. A top angle of more than 70 degrees is not acceptable but the use of spacers causes a decrease of the top angle as will be discussed in the next section. Considering all the properties as a function of w_2/w_1 it becomes clear that a compromise must be made and a certain range of w_2/w_1 must be selected that meets the requirements. In our case the amount of superconductor is of minor importance as is shown in fig. 5. Only for very small values of w_2/w_1 the amount of superconductor increases with more than 5% relative to $w_2/w_1 = 1$.

Fig. 3. Thickness of the thinnest side of the inner coil cable t_1 , versus w_2/w_1 for various values of the dipole field.

Fig. 4. Top angle of the inner coil layer versus w_2/w_1 for various values of the dipole field.

Optimization with spacers

The next step in the optimization is to take into account higher multipoles. In this study the emphasis was put on the first 4 multipoles (B_3 - B_9), as higher multipoles tend to be small due to their definition³. To optimize B_7 and B_9 , at least two spacers must be added². This increases the number of variables and more solutions are found. However, the solutions still vary systematically with the parameter w_2/w_1 . The

solution most suitable to our requirements was selected and the circular current segments were transformed into keystone cables. In the resulting structure the multipoles were optimized again with a code that integrates the contributions of the cables. The final solution differs only slightly from the previously selected one. Fig. 6 shows the definitive solution. The relevant properties of this structure are listed in Table 1.

Table 1. Relevant properties of the final solution.

Transport current	[kA]	17.7		
Multipoles	[T]	B_1	11.5	
	[T]	B_3	$-7 \cdot 10^{-5}$	
	[T]	B_5	$-6 \cdot 10^{-4}$	
	[T]	B_7	$-7 \cdot 10^{-4}$	
	[T]	B_9	$4 \cdot 10^{-4}$	
		Inner layer	Outer layer	
Average J_{max}	[A/mm ²]	325	504	
@ max. field	[T]	11.7	9.6	
Top angle	[degrees]	70.5	58.4	
Cable width	[mm]	21.73	17.39	
Cable thickness	[mm]			
incl. insulation		2.26/2.75	1.83/2.21	
excl. insulation		1.98/2.47	1.55/1.93	

Dipole structures for higher fields

In this section a brief discussion of the ins and outs of a 13 tesla magnet will be given, using the superconductor described by (1) and an inner radius of the iron of 150 mm. As a result of the higher field many more ampere turns are needed. For a 2 layer structure this leads to an unacceptably thick inner cable which can also be understood from figs. 2 and 3. Therefore one has to use 4 layers as 3 is impractical because of the interconnections. Another problem is the large number of cables in the outer layer. This increases the cost and fabrication errors of the dipole. The use of 4 layers will also drastically increase the number of variables and as a result the number of solutions becomes very large. In fact, only a few of these solutions are within practical limits. Solutions of 2 concentric 2-layer coils were found, as well as solutions in which the top angle of the first 3 layers decreases in the

Fig. 5. Area of superconductor versus w_2/w_1 for a dipole field of 11.5 tesla.

Fig. 6. Cross sectional view of 1/4 of the magnet showing the conductor lay-out. The dipole field is 11.5 tesla.

Fig. 7. Cross sectional view of 1/4 of the magnet showing the conductor lay-out. The dipole field is 13 tesla.

outward direction and the outer layer has a very large top angle. Fig. 7 shows a solution for 13 tesla that was biased by the requirement of 3 outer layers with almost the same top angle. Then only 2 insert pieces are necessary and it facilitates the mechanical support. The transport current of this particular dipole is 7.2 kA.

The tendency towards a thick inner layer cable is an important problem of high field dipole magnets. A crude estimate of this thickness can be obtained in the following way. Suppose the beam pipe radius r_i is known and the maximum number of strands is 36. If the average width of the cables, $\bar{r}$, and the ratio of current density in the inner and outer layer, J_i/J_o , can be estimated, then the requirement of an equal width of inner and outer layer gives an approximation of the width t_1 of the thinnest side of the cable in the inner coil layer. Fig. 8 shows characteristics of $t_1/\bar{r}$ versus $1/p = J_o/J_i$ with parameter $z = r_i/\bar{r}$. As an example, suppose that an 11.5 tesla magnet must be designed, based upon the 10 tesla LHC reference design. In this reference design $\bar{r} = 17$ mm. If this value is scaled with a factor of 11.5/10 and $J_o/J_i = 1.5$, the result of $t_1 = 2.44$ mm is found. If the ratio of cable widths is not 1

Fig. 8. Characteristics to be used for a crude estimate of the thickness of the cable in the inner coil layer. For an explanation of the symbols refer to the text.

but 21.7/17.4, then $t_1 = 1.96$ mm is found, which is in good agreement with our value listed in Table 1.

Conclusions

A method to obtain systematic solutions for the conductor dimensions and their lay-out was described. From these solutions a selection can be made based on a number of practical requirements. The inclusion of a measured $J_{max}(B)$ relation of the superconductor in order to make the design practical, has a large effect on the final geometry. A dipole magnet with cables of equal width is not necessarily the best solution; it is often better to have a broader inner layer cable. The volume of superconductor appeared to vary only within a few percent in the systematic solutions and is therefore not a relevant criterion. In the case of a 13 tesla dipole magnet and using the $J_{max}(B)$ relation described in this paper, more than 2 layers of conductors must be used in order to keep the design within practical limits. It appears to be difficult to obtain a convenient solution for a 13 tesla magnet, though the number of geometrical variants is very high.

References

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