On the mechanical behaviour of thin perforated plates and their application in silicon condenser microphones

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Abstract

In this paper an alternative approach to the modelling of plates with a large number of holes is presented. By means of plate theory, it is shown that perforated plates can be modelled by conventional orthotropic plates with modified elastic properties. The modification of the elastic constants is derived by equalizing the strain-energy of the perforated and the orthotropic plate. The model obtained is then compared with previous methods and applied in the electromechanical simulation of a silicon micromachined microphone structure. The microphone structures are simulated numerically, using an algorithm based on finite differences.

Keywords: Silicon microphones; Perforated plates; Finite differences; Electromechanical simulation

1. Introduction

In the development of sensors, silicon micromachining technology has proven to be very useful. Mechanical devices with dimensions that were impossible to manufacture with conventional technologies are now feasible. Furthermore, by introducing the new technology it is sometimes possible to enhance the functionality and the performance of the sensors. A good example of this is the development of the silicon condenser microphone (Fig. 1). The condenser microphone basically consists of a parallel-plate capacitor, where one plate is sensitive to the sound pressure.

The static bias voltage required over the plates is 50–200 V for conventional microphones, which is normally achieved by using an electret in the air gap of the microphone. By the introduction of silicon micromachining, it has become possible to reduce the air gap between the two plates from $> 30 \mu\text{m}$ to $1\text{--}3 \mu\text{m}$ [1,2], thereby reducing the required bias voltage to 5–15 V, which can be applied externally with a small battery and a voltage multiplier. The electret can therefore be omitted, and the long-term drift problems normally associated with these devices can be avoided.

The downscaling, however, also means that control of the air flow in the gap, which directly influences the upper cut-off frequency of the microphone, becomes a critical design criterion. In order to reduce sufficiently the flow resistance in the gap, a large amount of holes is required in the backplate.

Fig. 1. Cut-view of a silicon condenser microphone [2].

If the thickness of the backplate is comparable with the thickness of the membrane, a reduction of the rigidity, due to the holes, may have a significant influence on the behaviour of the microphone. In the field of microsensors this problem has not yet been considered in detail, mainly due to limited use of perforated membranes. For silicon condenser microphones, it has until now been common to use the simple assumption that the elastic properties of the perforated plate could be described by a solid plate with a thickness reduced by the same fraction as the surface area occupied by the holes [3].

In this paper, an alternative, more complex, model of the perforated plate is presented, and the differences from the simple model are investigated, both for single plates and for plates in a microphone structure. The simulations of the microphone structure are compared with measurements on a silicon nitride condenser microphone. Since the theory is of a general nature, it can be useful in any application containing a perforated plate.

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2. Deflection of perforated plates

Calculating the mechanical parameters of a perforated plate proves to be difficult. It is well known that stress concentrations appear around holes in deflected plates, however, the only accurate way to obtain this stress distribution is to insert the boundary conditions of a free edge on the edges of each hole [4]. This is already a complex task for a square plate with one hole, and if the plate contains several hundred holes, this method must in reality be considered impossible.

A different approach is to model the perforated plate by a solid plate with pseudo-elastic constants and mass. This theory has previously been applied for large-scale mechanical constructions [5,6]. The idea is to equal the strain-energy of a small element in a solid plate with the energy of a similar element in a perforated plate. For simplicity it is assumed that the holes in the perforated plate are squares, and that they are placed in a regular pattern (Fig. 2).

From the definition of the stress-strain relations in an orthotropic plate, the bending moments are given by [4]:

$$\begin{aligned} M_x &= \frac{h^3}{12} \left(C_{11} \frac{\partial^2 w}{\partial x^2} + C_{12} \frac{\partial^2 w}{\partial y^2} \right) \\ M_y &= \frac{h^3}{12} \left(C_{21} \frac{\partial^2 w}{\partial x^2} + C_{22} \frac{\partial^2 w}{\partial y^2} \right) \\ M_{xy} &= \frac{h^3}{6} C_{44} \frac{\partial^2 w}{\partial x \partial y} \end{aligned} \quad (1)$$

where h is the plate thickness, $C_{11} = C_{22}$, $C_{12} = C_{21}$ and C_{44} are material constants, $w(x, y)$ is the deflection function, and x, y are the primary directions of the plane of the plate. The strain energy dV_{bs} from bending of a small element $dx dy$ of the plate is then [4]:

$$\begin{aligned} dV_{bs} &= -\frac{1}{2} \left[M_x \frac{\partial^2 w}{\partial x^2} + M_y \frac{\partial^2 w}{\partial y^2} \right. \\ &\quad \left. - \left(M_{xy} \frac{\partial^2 w}{\partial x \partial y} + M_{yx} \frac{\partial^2 w}{\partial y \partial x} \right) \right] dx dy \\ &= \frac{h^3}{24} \left[C_{11} \left(\frac{\partial^2 w}{\partial x^2} \right)^2 + C_{22} \left(\frac{\partial^2 w}{\partial y^2} \right)^2 \right. \\ &\quad \left. + 2C_{12} \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} + 4C_{44} \left(\frac{\partial^2 w}{\partial y \partial x} \right)^2 \right] dx dy \end{aligned} \quad (2)$$

In addition to this, the contribution from eventual built-in stress should be included. This energy dV_{st} can be expressed by the following equation [4]:

$$dV_{st} = \frac{1}{2} (N_x \epsilon_x + N_y \epsilon_y) dx dy \quad (3)$$

where N_x , N_y , ϵ_x and ϵ_y are built-in forces and strains in the middle plane of the plate. By use of the force-strain relations [4] and $N_x = N_y = \sigma h$, the equation can be rewritten as

Fig. 2. Perforated plate with square holes in a regular pattern.

$$\begin{aligned} dV_{st} &= \frac{1}{2hE} [N_x(N_x - \nu N_y) + N_y(N_y - \nu N_x)] dx dy \\ &= \frac{h(1-\nu)\sigma^2}{E} dx dy \end{aligned} \quad (4)$$

where E is Young's modulus, ν is Poisson's ratio and σ is the built-in stress. It should be noted that this energy only depends on the built-in stress, since it is assumed that no strains are induced in the middle plane by the deflection. The total strain-energy of a bent element in a solid orthotropic plate is then

$$\begin{aligned} dV_{st} &= dV_{bs} + dV_{st} = \frac{h(1-\nu)\sigma^2}{E} dx dy \\ &\quad + \frac{h^3}{24} \left[C_{11} \left(\frac{\partial^2 w}{\partial x^2} \right)^2 + C_{22} \left(\frac{\partial^2 w}{\partial y^2} \right)^2 + 2C_{12} \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} \right. \\ &\quad \left. + 4C_{44} \left(\frac{\partial^2 w}{\partial y \partial x} \right)^2 \right] dx dy \end{aligned} \quad (5)$$

Considering the basic element in the perforated plate (Fig. 2), it can be seen that the element may be regarded as one small plate, with sidelength $b-2a$, and four beam tips with length a . By calculating the strain-energy of these parts [6], it can be shown that the strain-energy dV_{bp} from bending of the basic element with sidelength b is

$$\begin{aligned} V_{bp} &= \frac{Eh^3}{24} \left[\left\{ \frac{b(b-2a)}{1-\nu^2} + \frac{a(b-2a)^2}{b} \right\} \left(\frac{\partial^2 w}{\partial x^2} \right)^2 \right. \\ &\quad \left. + \left(\frac{\partial^2 w}{\partial y^2} \right)^2 \right] + \frac{2\nu b(b-2a)}{1-\nu^2} \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} + \frac{1}{1+\nu} \\ &\quad \times \left[2b(b-2a) + \frac{12Ka(b-2a)}{bh^3} \right] \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 \end{aligned} \quad (6)$$

where K is a constant defined by

$$K = \begin{cases} \frac{1}{3} \left(1 - 0.63 \frac{b-2a}{h} \right) (b-2a)^3 h & \text{for } h > b-2a \\ \frac{1}{3} \left(1 - 0.63 \frac{h}{b-2a} \right) (b-2a) / h^3 & \text{for } h < b-2a \end{cases} \quad (7)$$

As mentioned above, the strain-energy induced by the built-in stress dV_p is considered to be independent of the plate deflection. Therefore, it may be assumed that Eq. (4) can be used by simply reducing it by the same factor with which the element area is reduced by the holes:

$$dV_{sp} = \left(1 - \frac{4a^2}{b^2} \right) \frac{h(1-\nu)\sigma^2}{E} dx dy \quad (8)$$

The total strain-energy for the bent element in Fig. 2 is therefore

$$V_p = \frac{Eh^3}{24} \left\{ \left[\frac{b(b-2a)}{1-\nu^2} + \frac{a(b-2a)^2}{b} \right] \left\{ \left(\frac{\partial^2 w}{\partial x^2} \right)^2 + \left(\frac{\partial^2 w}{\partial y^2} \right)^2 \right\} + \frac{2\nu b(b-2a)}{1-\nu^2} \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} + \frac{1}{1+\nu} \times \left\{ 2b(b-2a) + \frac{12Ka(b-2a)}{bh^3} \right\} \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 \right\} + \frac{h(b^2-4a^2)(1-\nu)\sigma^2}{E} \quad (9)$$

By equalizing this expression with Eq. (5) for an orthotropic solid element with sidelength b , the pseudo material constants of the perforated plate can be derived:

$$C_{11} = C_{22} = \frac{E}{b^2} \left\{ \frac{b(b-2a)}{1-\nu^2} + \frac{a(b-2a)^2}{b} \right\} \quad (10)$$

$$C_{12} = C_{21} = \frac{\nu E(b-2a)}{b(1-\nu^2)} \quad (11)$$

$$C_{44} = \frac{E}{4b^2(1+\nu)} \left\{ 2b(b-2a) + \frac{12Ka(b-2a)}{bh^3} \right\} \quad (12)$$

It is now possible to use these modified elastic constants in a normal calculation, meaning that a solid plate is assumed to have the elastic properties of a perforated plate. The same can be done with the built-in stress. It may be seen from Eq. (8) that the strain-energy from stress has been reduced by a factor $(1-4a^2/b^2)$. This can be translated into a solid plate having a built-in stress of $\sqrt{(1-4a^2/b^2)}\sigma$.

3. Plate analysis

The deflection of a plate with built-in stress, subjected to an external lateral load $q(x,y)$, is described by the following differential equation [4]:

$$\frac{\partial^2 M_x}{\partial x^2} - 2 \frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} + \sigma h \sqrt{1 - \frac{4a^2}{b^2}} \nabla^2 w = -q \quad (13)$$

By substitution of Eq. (1) and Eqs. (10)–(12) in Eq. (13), a fourth degree differential equation of the deflection $w(x,y)$ is obtained. An analytical solution to this equation with the boundary conditions of built-in edges is not known.

A numerical method suitable for solving this equation is finite differences, which has previously been applied for orthotropic plates [7]. By discretization of the function $w(x,y)$ in an $n \times n$ grid, the discrete version of Eq. (13) can be used to write a linear equation for each point on the grid, thereby producing a linear equation system of order n^2 of the form

$$\bar{A} \cdot \bar{w} = \bar{q} \quad (14)$$

By solving this system, the deflection function for any arbitrary load function $q(x,y)$ can be found.

If the resonance frequency of the plate is to be determined, an additional load from the inertial mass must be added, yielding

$$\Psi w + \mu \frac{\partial^2 w}{\partial t^2} = -q(x,y,t) \quad (15)$$

where Ψ is an operator for the actions on $w(x,y,t)$ on the left side of Eq. (13), and μ is the surface density of the plate. When analysing a perforated plate, the surface density must be reduced by the hole fraction.

Eq. (15) can be solved using the iterative implicit method [8], with a discrete equation of the form

$$\left(\bar{A} + \frac{\mu}{\Delta_t^2} \right) \cdot \bar{w}_{i+1} = -\bar{q}_i + \frac{\mu}{\Delta_t^2} (2\bar{w}_i - \bar{w}_{i-1}) \quad (16)$$

where Δ_t is the discretization constant in the time domain.

4. Quasistatic microphone analysis

The quasistatic deflection of the two thin plates in the condenser microphone structure in Fig. 1 is given by:

$$D_m \nabla^4 w_m - \sigma_m h_m \nabla^2 w_m = p_{ac} + p_{el} \quad (17a)$$

$$\Psi w_b = -p_{el} \quad (17b)$$

where D_m , σ_m , h_m and $w_m(x,y)$ are the rigidity, built-in stress, thickness and deflection function of the membrane, p_{ac} is the acoustic sound pressure, $w_b(x,y)$ is the deflection function of the backplate and $p_{el}(x,y)$ is the electrostatic load from the bias voltage V_b across the plates, given by

$$p_{el} = \frac{\epsilon_0}{2(d - w_m - w_b)^2} V_b^2 \quad \kappa \approx 1 - \frac{4a^2}{b^2} \text{ for } d < 2a \quad (17c)$$

with ϵ_0 and d being the permeability and thickness of the air gap between the two plates, and where κ is a constant accounting for the reduction of the electrostatic force due to the holes in the backplate.

By solving the two non-linear coupled differential equations defined by Eqs. (17), it is possible to gain important

information about quasistatic microphone parameters, like critical bias voltage and maximum allowable sound pressure (MASP).

A numerical solution of this complex problem can be obtained by using an iterative finite-difference method, where Eq. (17a) is first solved with the initial condition $w_b(x,y) = 0$. The solution for $w_m(x,y)$ is then used in Eq. (17b) to find the new $w_b(x,y)$. This iteration is repeated until the specified relative precision is acquired. Since the function $p_{cl}(x,y)$ is monotonically increasing, for increasing $w_m(x,y)$ and $w_b(x,y)$, and since only one solution to Eq. (17a) and Eq. (17b) exists for the given boundary conditions, it can be proved that the iteration will only converge towards one value. However, if the bias voltage V_b is too high, the system becomes unstable, and no solution exists. This can be interpreted as a collapse of the structure, and the maximum possible voltage where this can be avoided is named the critical bias voltage. A similar value exists for the quasistatic sound pressure p_{ac} .

The quasistatic open-circuit sensitivity S of the condenser microphone is defined as the electrical output signal, divided by the change Δp of the sound pressure. Assuming that the electrical charge in the microphone is constant, and that the deflections are small compared with the height of the air gap, the sensitivity may be defined as

$$S = S_m S_c \frac{V_b}{C_0} = \frac{\Delta C}{\Delta p} \frac{V_b}{C_0} \quad S_m = \frac{\Delta w}{\Delta p} \quad S_c = \frac{\Delta C}{\Delta w} \quad (18)$$

where V_b is the bias voltage, C_0 is the capacitance of the microphone without an acoustic load, S_m is the mechanical sensitivity, and S_c is the capacitive sensitivity. The term $\Delta C / \Delta p$ can be determined by solving Eqs. (17) for two different sound pressures p_1 and p_2 and using the following equation:

$$\frac{\Delta C}{\Delta p} = \left| \frac{\int_0^L \int_0^L \frac{\kappa \epsilon_0}{d - w_{m1} - w_{b1}} dy dx - \int_0^L \int_0^L \frac{\kappa \epsilon_0}{d - w_{m2} - w_{b2}} dy dx}{p_1 - p_2} \right| \quad (19)$$

where L is the sidelength of the microphone.

5. Results

The numerical methods described above were implemented with a C-program on an HP-9000 computer system. Due to the assumed symmetry, only one quarter of each plate was simulated.

Simulations of perforated plates with built-in edges have been carried out in order to investigate the influence of the holes. In Fig. 3 and Fig. 4 the first resonance frequency and mechanical sensitivity (centre deflection divided by the load) for two different materials (stress-free silicon (Si) and silicon nitride (SiN) with tensile stress (50 MPa)), are shown. Since the results were normalized to a plate having no holes,

Fig. 3. Normalized first resonance frequency for two perforated plates.

Fig. 4. Normalized mechanical sensitivity for two perforated plates.

the specific parameters are not important. Also shown in the Figures is the reduced thickness (RT) model of Bergqvist [3]. It may be seen that no close correlation exists between the simple RT model and the more elaborate strain-energy equalization (SE model). In fact, the SiN plate with tensile stress shows an increase of the resonance frequency, as opposed to the decrease predicted by the RT model. This can be explained by the fact that the mass of the plate decreases faster than the rigidity, thus causing a higher resonance frequency. This is not the case for the stress-free silicon plate, however, the non-linear nature of the SE model becomes evident at higher hole fractions. In Fig. 4, the normalized mechanical sensitivity calculated with the two models (SE and RT) is shown. It may be seen that a significant difference exists both for the Si plate (Si-SE, Si-RT) and the SiN plate (SiN-SE, SiN-RT), but the relative difference is smaller for stressed plates.

Furthermore, the relation was investigated between the holes in the backplate of a condenser microphone and three

Table 1
Dimensions and material parameters of the simulated microphone structure

Thickness of membrane (silicon nitride)	0.25 μm
Thickness of backplate (silicon nitride)	1 μm
Air gap	2 μm
Sidelength	2 mm
Built-in stress (membrane)	100 MPa
Built-in stress (backplate)	50 MPa
Young's modulus	130 GPa
Poisson's ratio	0.2
Permittivity	8.2 ϵ_0

Fig. 5. Normalized critical bias voltage for a microphone with the SE and RT backplate models.

Fig. 6. Normalized maximum allowable sound pressure for a microphone with the SE and RT backplate models.

important parameters, the critical bias voltage, the MASP and the quasistatic sensitivity. The parameters were calculated using both backplate models (SE, RT) in a condenser microphone with the dimensions listed in Table 1. From the results of the critical bias voltage and the MASP in Fig. 5 and Fig. 6, it can be seen that a larger influence of the holes is predicted

Fig. 7. Normalized quasistatic sensitivity for a microphone with the SE and RT backplate models.

Fig. 8. Calculated and measured sensitivities of a silicon condenser microphone.

by the RT model. This is in agreement with the results obtained for the mechanical sensitivity of the backplate (Fig. 4). The spreading of the data is caused by the relative precision of the simulation, which was chosen as 1% in order to give a reasonable trade-off between calculation time and precision. For the quasistatic sensitivity (Fig. 7), however, the dependence on the holes again appears to be opposite for the two models. This may be explained by the fact that a backplate with higher mechanical sensitivity will deflect more due to the electrostatic forces, and thereby cause a higher electrical field in the air gap and subsequently a higher quasistatic sensitivity.

Finally, the dependence of the quasistatic sensitivity on the bias voltage of the microphone has been investigated, and compared to measurements. The microphone was a silicon nitride microphone of the type in Fig. 1, and with the para-

meters of Table 1. The sidelength and the acoustic hole fraction of the microphone were 1.5 mm and 18%, and all measurements were normalized to the sensitivity measured at a bias voltage of 1 V. From Fig. 8 it may be seen that no significant difference of the shape exists between the theory (Theory I) and the measurements (Meas.). By introducing a 50% deviation of the built-in stress (Theory II), which may be induced by the mounting of the microphone, it is possible to achieve a better match between the theory and the measurements. Further differences may be explained by the non-linear deflection properties of the membrane (and the backplate), which are not included in the model.

6. Conclusions

An alternative theory, which was previously developed and validated for larger plates, has been adopted for the analysis of perforated plates in micromechanical sensors. Furthermore, a simulation method for the calculation of the quasi-static properties of silicon condenser microphones, which shows good agreement with measurements, was developed. The model of the perforated plate was compared with a simple reduced-thickness model. From the investigations it can be concluded that the reduced-thickness model predicts a larger influence of the holes, regarding the mechanical sensitivity of the plate. Therefore, the simple model should only be used as a worst-case scenario in estimations (i.e., the conditions of collapse in a microphone). However, when designing and dimensioning a sensor containing perforated plates, the strain-energy equalization may be used to optimize the performance further.

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Biographies

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