Teaching problem-solving in physics: A course in electromagnetism

J. H. P. van Weeren, F. F. M. de Mul, and M. J. Peters
Twente University of Technology, Department of Applied Physics, Enschede, The Netherlands
H. Kramers-Pals
Twente University of Technology, Department of Chemical Engineering, Enschede, The Netherlands
H.J. Roossink
Twente University of Technology, Centre of Educational Research and Development, Enschede, The Netherlands

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In order to improve the teaching and learning situation in the course Electromagnetism for first-year students at the Twente University of Technology, this course has been reconstructed. The main activity in the reconstruction has been directed towards developing means and instructional procedures helpful to the students in learning how to tackle problems in electromagnetism. Therefore a model of the process of solving such problems has been developed and a system of actions and methods appropriate to this problem solving has been derived from the model. On the basis of the learning and instruction theory of Gal'perin this system has been incorporated in the instructional procedures of the course. The results of the reconstructed course as shown in the examination scores have improved compared to previous courses.

I. INTRODUCTION

For everyone teaching science it is a well-known fact that very often students are not able to solve problems although they have all the necessary knowledge. We had to deal with such a situation in the course Electromagnetism for first-year students at the Twente University of Technology (THT), the Netherlands. A large number of students had great difficulties in solving electromagnetism problems and therefore did not reach the intended level of mastery of the subject matter.

In order to improve this situation a cooperative project of the Department of Applied Physics and the Centre for Educational Research and Development of the THT was started in 1977. It is called "Problem solving in Electromagnetism."

The aim of the project is

—to develop means which could be helpful to the students in learning how to tackle electromagnetism problems;

—to reconstruct the course Electromagnetism in order to include the developed means in the instructional procedures.

A similar project on problem solving concerning a thermodynamics course in the Department of Chemical Engineering of THT was started in 1975 by Mettes *et al.*[1] The results of that project seem to be applicable not only to the solving of thermodynamics problems but more generally to the solving of problems which are usually dealt with in the teaching of natural sciences. We found that the problems used in the thermodynamics and the electromagnetism courses are of the same type. Therefore we thought it appropriate to use results of the thermodynamics project in the electromagnetism project.

In Sec. II a short description of the electromagnetism course is given. The theoretical framework of the electromagnetism project is described briefly in Sec. III.

In order to be able to improve the teaching we need a description of good methods of solving problems. Therefore a descriptive model of the process solving electromagnetism problems was developed in the project. This is described in Sec. IV.

From the descriptive model a prescriptive model was drafted and then a system of heuristics useful in the training of students in problem solving deduced (Sec. V). The instructional procedures used in the reconstructed course are described in Sec. VI. In Sec. VII an evaluation of results of teaching in the reconstructed course is given. Section VIII contains conclusions.

II. THE COURSE "ELECTROMAGNETISM"

The course Electromagnetism at THT is a part of the curriculum of the first-year students[2] of all technological departments.[3] The book *Fundamental University Physics* by M. Alonso and E. J. Finn in a Dutch translation[4] and a separate collection of selected problems is used in the course. The instructional objectives of the course include skills in solving rather difficult electromagnetism problems.

The teaching period spans eight weeks of two to four lecture hours a week (depending upon the requirements of the individual departments). In this period the students can also exercise the solving of electromagnetism problems in classes of about 20 students during two hours a week.

III. THEORETICAL FRAMEWORK OF THE PROJECT

A. Theory of Gal'perin

It is frequently seen that courses which have been carefully (re)constructed and being successful in the first years after the (re)construction show a decline in success in the course of time. A reason for this may be that the construction of the course has lacked a firm theoretical base and that the success of the course has been carried merely by the enthusiasm of the teacher and has declined after some years.

In our opinion it is necessary to have a theory of learning and instruction as a basis for course (re)construction activi-

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ties. The theory of Gal'perin (Talyzina[5]) has been chosen as a basis from the existing theories of learning and instruction.

An important reason for this choice is that in this theory—contrary to other theories—the actions of the student in handling the subject matter to be learned are the main constituents. The aim of the electromagnetism course is to enable students to handle the subject matter, i.e., to act in rather complex problem situations. Therefore a learning and instruction theory which emphasizes the actions of the student is very useful. Another reason is that it, more than other theories, seems to apply also to rather complex learning processes as are dealt with in university teaching. For the construction of the electromagnetism course we used as a theoretical framework the theory of Gal'perin combined with contributions of Talyzina[5] and Landa.[6] The main elements of this theoretical framework are

(1) A basis of orientation: This basis is the set of elements of knowledge and a set of rules that one has at his disposal in directing his actions. This basis must be as complete as possible and generally applicable. A complete basis of orientation consists of (a) the subject matter of the course, in an operational form; (b) general and subject specific methods of thinking (heuristics).

(2) Stage-by-stage formation of mental actions: In the first stage of the learning process the student uses the basis of orientation and performs new actions, i.e., actions in problem solving unknown to him. The actions are performed by the student on paper without dropping any of the required action links. This stage is called the *materialized* form. After having reached the level of correctly acting in this stage the student passes on to the next stage, the *verbal* form, explaining his actions to the teacher or other students. Having mastered this form he will pass on to the *mental* form. Changes in the occurrence of actions and action links will take place in connection with the passing of these stages. A number of actions and action links will be dropped and thus a shortening in the process of solving the problem will take place. An important characteristic of this method of learning by the student is that in the first stage he becomes acquainted with the coherence and consequences of the application of all the actions and their links in written and thus observable form, and thus can have optimum control of his own activities. The advantage for the teacher is that it is rather easy to give feedback as the actions in the first stage are as much as possible external and hence observable.

B. Procedure for course construction

On the basis of Gal'perin's theory Talyzina developed a procedure for course construction. This procedure involves the construction of two subprograms which are parts of the basis of orientation:

(1) The first subprogram describes all the knowledge concerning the course.

(2) The content of the second subprogram is a description of the rational actions and methods of thinking which are satisfactory to acquire and to use this knowledge. This subprogram can be divided into two parts: (a) actions and methods constituting types of thinking which are specific for the subject matter in question; and (b) logical actions and methods of thinking not dependent on specific subject matter.

Talyzina notices that the construction of the second subprogram is difficult because usually the actions and methods specific for the subject matter have not been formulated explicitly and also because teachers in general are not able or experienced enough to formulate explicitly all the relevant actions and methods.

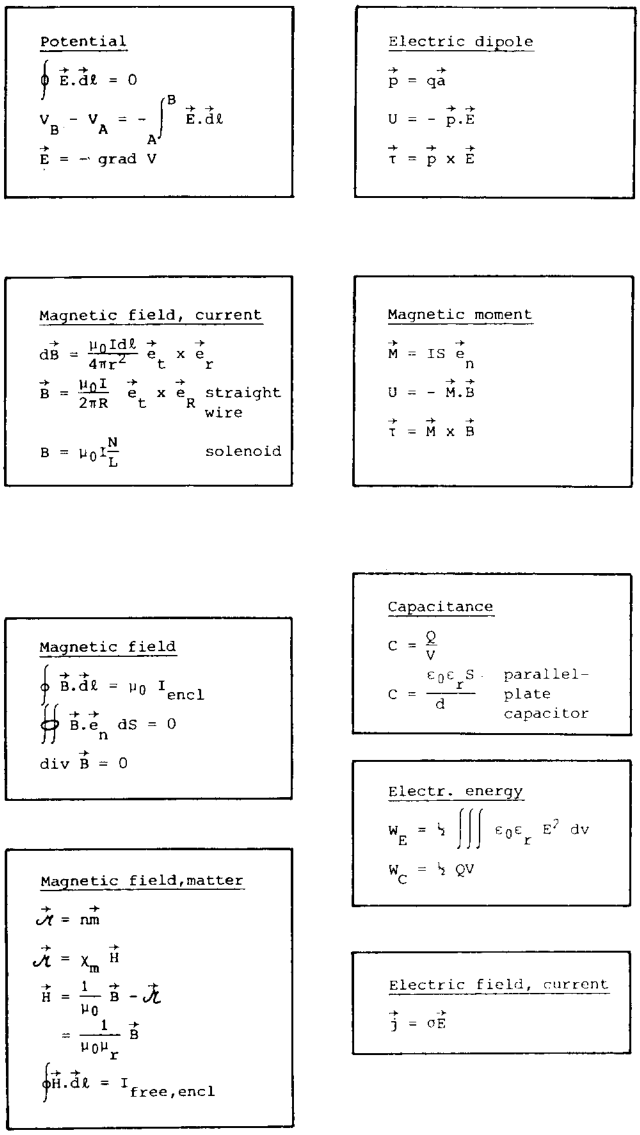
Both the second subprogram, which from now on we will indicate with PAM (program of actions and methods), and the first subprogram have to be made available to the student in a written form so that they can be used by the student as an appropriate basis of orientation.

IV. CONSTRUCTION OF PAM FOR SOLVING ELECTROMAGNETISM PROBLEMS

For the construction of PAM we used three sources of information (cf. Mettes[1]):

(a) Heuristic schemes for problem solving in related fields. We call fields related if the problems of the fields in question are of the same type.

(b) Protocols recorded on audio tape of problem-solving behavior in interviews of teachers and students tackling



Potential

$\oint \vec{E}.\vec{d\ell} = 0$

$V_B - V_A = -\int_A^B \vec{E}.\vec{d\ell}$

$\vec{E} = -\text{grad } V$

Electric dipole

$\vec{p} = q\vec{a}$

$U = -\vec{p}.\vec{E}$

$\vec{\tau} = \vec{p} \times \vec{E}$

Magnetic field, current

$d\vec{B} = \frac{\mu_0 I d\ell}{4\pi r^2} \vec{e}_t \times \vec{e}_r$

$\vec{B} = \frac{\mu_0 I}{2\pi R} \vec{e}_t \times \vec{e}_R$ straight wire

$B = \mu_0 I \frac{N}{L}$ solenoid

Magnetic moment

$\vec{M} = IS\,\vec{e}_n$

$U = -\vec{M}.\vec{B}$

$\vec{\tau} = \vec{M} \times \vec{B}$

Magnetic field

$\oint \vec{B}.\vec{d\ell} = \mu_0 I_{encl}$

$\oiint \vec{B}.\vec{e}_n\, dS = 0$

$\text{div } \vec{B} = 0$

Capacitance

$C = \frac{Q}{V}$

$C = \frac{\varepsilon_0 \varepsilon_r S}{d}$ parallel-plate capacitor

Electr. energy

$W_E = \frac{1}{2} \iiint \varepsilon_0 \varepsilon_r E^2\, dv$

$W_C = \frac{1}{2} QV$

Magnetic field,matter

$\vec{\mathcal{M}} = n\vec{m}$

$\vec{\mathcal{M}} = \chi_m \vec{H}$

$\vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{\mathcal{M}}$

$= \frac{1}{\mu_0 \mu_r}\vec{B}$

$\oint \vec{H}.\vec{d\ell} = I_{free,encl}$

Electric field, current

$\vec{j} = \sigma\vec{E}$

Fig. 1. Part of the KR chart used in the reconstructed course Electromagnetism for first-year students at THT.

electromagnetic problems typical for the course.

(c) Heuristic schemes for problem solving in electromagnetism found in the literature.

The problems of the electromagnetism course are specification problems, i.e., it is required to specify a given electromagnetic system with one or more unknown characteristics of the system. For the linking up of the given and the required system characteristics in the problem solving a number of relationships between quantities is needed. In the electromagnetism course a set of relations being of special importance as seen from the physical subject matter or from their use in problem solving is build up. The relations of this set are called *key relations*.[7]

During the course a set of key relations (KR) is made available in parts to the students on a so-called KR chart. This chart is an appropriate form of the first subprogram with respect to the instruction. As example a part of the KR chart is shown in Fig. 1.

As the problems of the thermodynamics and the electromagnetism course are both of the specification type it is appropriate to take as a starting point the model of Mettes *et al.*,[1] which describes the process of solving thermodynamics problems. In order to design a model describing the phases of the process of solving electromagnetism problems we carried out some experiments in which students as well as teachers tried to solve problems representative for those of the electromagnetism course. They were asked to think aloud while solving a problem and this was recorded on tape. Protocols of this problem-solving behavior were transcribed and subsequently analyzed.

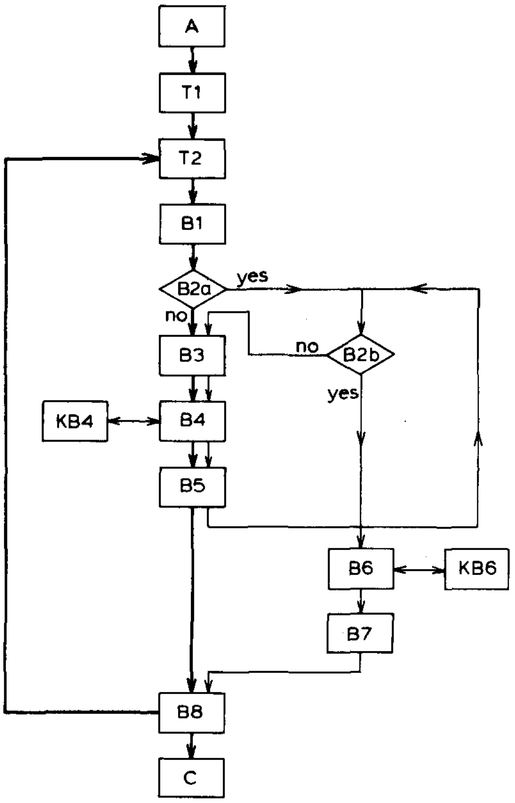
In the analysis of the protocols we made use of the descriptive model for solving thermodynamics problems. This analysis revealed some differences in behavior in solving thermodynamics and electromagnetism problems. It turned out that in the problems of the electromagnetism course emphasis is placed more on the problem transformations, while in the problems of the thermodynamics course attention was paid more to the execution and the results of standard operations. Because of these differences an adaptation of the model for a description of the process of solving electromagnetism problems was required. The adapted model is shown in Fig. 2.

The frequencies of transition between phases in the problem-solving process as deduced from the protocols of the teachers and students are shown in Fig. 3. From this figure it can be seen that all phases, except B2, which is obviously an implicit phase, are passed which is an indication for the usefulness of the model.

Having designed the descriptive model (Fig. 2) we developed from this a prescriptive model providing a program of actions and methods (PAM) useful in the training of problem solving. PAM was constructed by filling the respective categories of the descriptive model with actions and methods specified in detail in order to ensure a systematic and effective problem-solving process. In the construction, literature on the application of heuristics in the solving of problems of electromagnetism (Dubovskaja[8]; Marples[9]) was used.

PAM itself turns out to be too detailed and extensive for presentation to students. Therefore the next step is to construct an orientation basis for solving actions by deducing a more restricted system of heuristics from PAM. This is described in Sec. V.

First, we again look at Fig. 3 from which it can be concluded that students more than teachers go through almost all phases. Teachers shorten the solving process more than



Phases

A : Reading of the text and figures of the given problem;

T1 : Orientation analysis:
- making a sketch of the given situation
- constructing a physical description of the problem situation, in electromagnetic terms
- further analysis after partial solution of the problem;

T2 : Planning analysis:
- looking for possible combinations of given characteristics of the situation and of relations between them
- choosing a way of approaching the problem, even without having a clear view on the whole path to the solution
- evaluating the process of solving the problem, as performed so far
- changing a previously made plan;

B1 : Choosing a (sub-)problem to solve;

B2 : Establishing whether a standard routine is relevant (B2a) and/or available (B2b) for the solution of the problem chosen in B1;

B3 : Searching for possibly suitable problem transformations, and performing a corresponding analysis:
- further analysis of the problem situation
- searching for suitable key relations
- putting forward assumptions and approximations
- establishing the validity of the relations selected, for the given problem situation
- establishing the usefulness of those relations;

B4 : Performing problem transformations:
- specifiying a key relation for the given situation
- successively transforming the required quantity;

B5 : Establishing the result of the performed problem transformation;

B6 : Performing standard (numerical) operations:
- arithmetic calculating, integrating, differentiating, substituting using S.I.-units, etc.;

B7 : Establishing the result of the standard operation;

B8 : Evaluating and integrating intermediate results or the final result by judging them with regard to previous estimations or expectations, extrapolations of the given problem situation or of the required quantity, dimensions and units, and procedures applied with other problems;

C : Final phase of the solving process.

The phases KB4 and KB6 are control phases.

Fig. 2. Model used in the description of the process of solving electromagnetism problems.

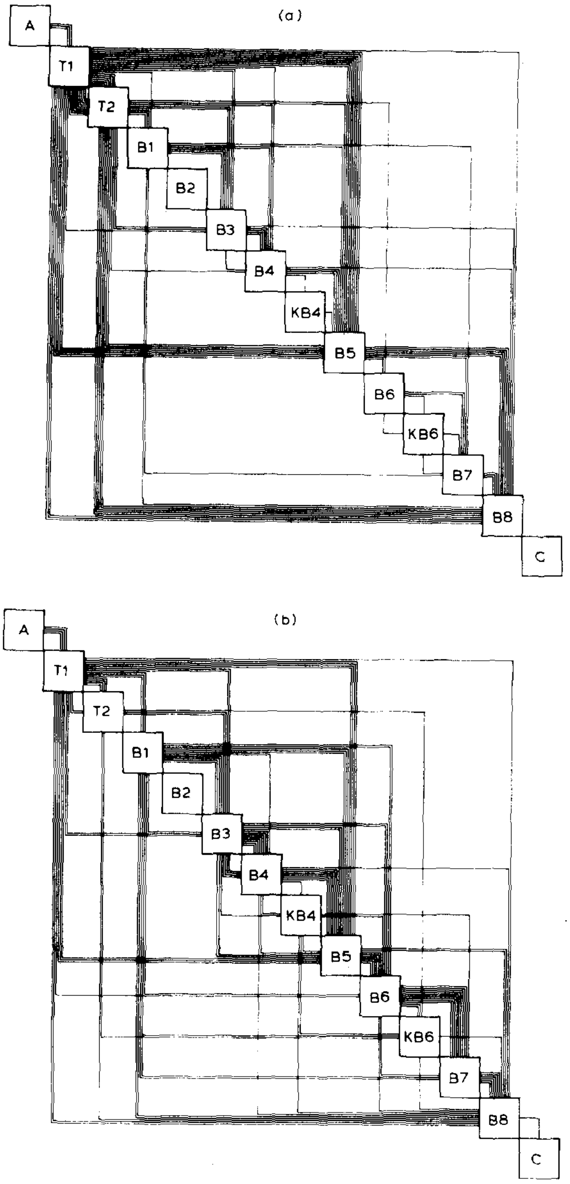


Fig. 3. Frequencies of transition between phases in the protocols of three teachers (a) and four students (b). Each line connecting phases represents a transition of which the direction is always clockwise.

students, while for students the analyzing phases play a much less central role in the whole process. We also conclude that students are frequently caught in phases B3, B4, and B5, which may indicate a trial-and-error situation. Comparing the analyses of the protocols of the successful problem solvers, on the one hand, and of those who were not successful, on the other, it can be concluded that the successful problem solvers, more frequently than the unsuccessful ones, use the analyzing phases (T1 and T2) with respect to both the frequencies of transition to these phases and the time spent on these phases (see Fig. 4).

From the foregoing we conclude that:

(a) The role of the analyzing phases in the process of problem solving is very important. Much attention has to be paid to this point in the teaching and learning of problem solving.

(b) In general, nontrained students do not make explicit plans about how to tackle a given problem. In the training of students attention has to be paid to an adequate planning in problem solving.

(c) Teachers must be continuously aware of their shortening of the problem-solving process, undoubtedly due to previous experience which may actually prevent a useful communication with the students.

V. SAP CHART

The system of heuristics deduced from PAM is called SAP (systematic approach to problem solving). It is available to students on what is called a SAP chart. Students can apply this system of heuristics when exercising problem solving and teachers can refer to it when giving feedback to students. The SAP chart is an appropriate form of the second subprogram with respect to the instruction.

In deciding what to include in SAP and thus what to take care of in the instructional procedures we have to take into account that the phases of the process of problem solving turned out to have different weights with respect to the frequency of transition to and time spent in the phases. Also we have to take into account that students already have experience in using a number of phases due to previous problem-solving activities. In view of the conclusions in Sec. IV we decided to emphasize the actions of analyzing, mapping, and concluding. Therefore we described heuristics on the SAP chart with respect to the following items:

(a) Analyzing [cf. T1 and T2 (in Fig. 2)]. The SAP chart contains a number of directions with respect to both a general and a subject specific analysis of problem situations with given system characteristics.

(b) Mapping (cf. T2 and B3). Making a plan for the solving of a problem implies a number of actions: looking for relations between given system characteristics, choosing a promising entry to the solving process, searching for possible problem transformations.

(c) Concluding (cf. B8 and C). At the end of the solving of a problem one should again consider the way the problem has been solved, looking for possible directions which may be useful for the solving of other problems.

The action of mapping seems to be similar to one of the elements of the model of the process of solving mechanics problems by an expert as described by Larkin and Reif.[10] They found that the expert problem solver after having analyzed the problem does not immediately construct a detailed mathematical description of the problem. Instead he first constructs a qualitative "physical description" of the problem, and then uses this description to assure that there will be no intractable difficulties in applying his selected method. Afterwards he elaborates the mathematical details of his solution.

In instruction, the learning of making plans can be structured by teaching the students to map out a route of solving the problem before actually carrying out the problem transformations in detail. In mapping the route one indicates step by step which key relations, results of the analysis or results of previous steps can be combined. One of the functions of the instruction is therefore to offer to the student a specific strategy for applying the set of the obtained results of the analysis, on the one hand, and the set of the key relations, on the other, in his search for a way to solve the problem by means of successive transformations.

Mapping out a route can be done in three ways:

(a) Forward reasoning going from the given to the re-

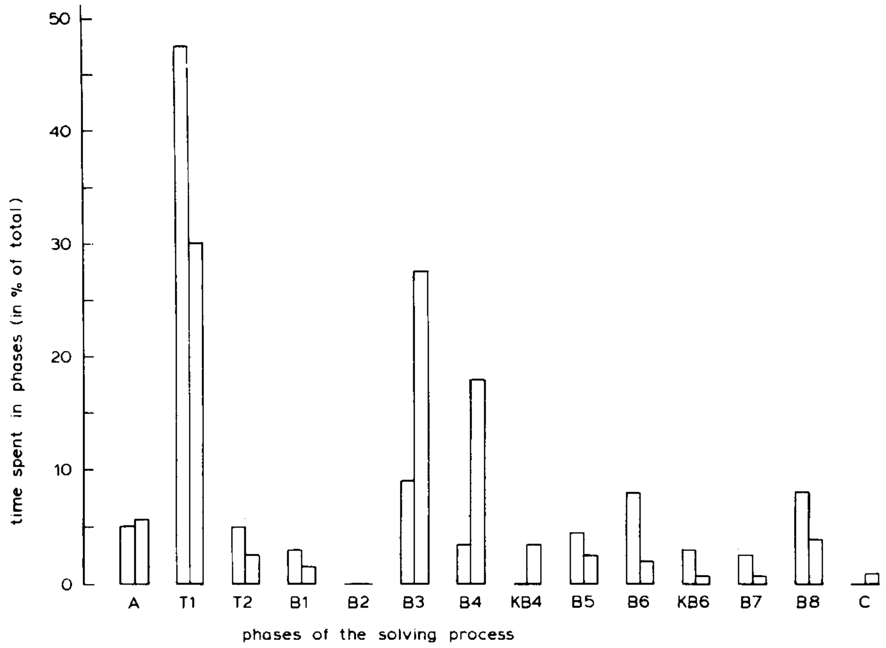


Fig. 4. Time (in %) spent in the phases during the solving of an electromagnetism problem. The left bars represent the successful problem solvers, the right bars the unsuccessful ones.

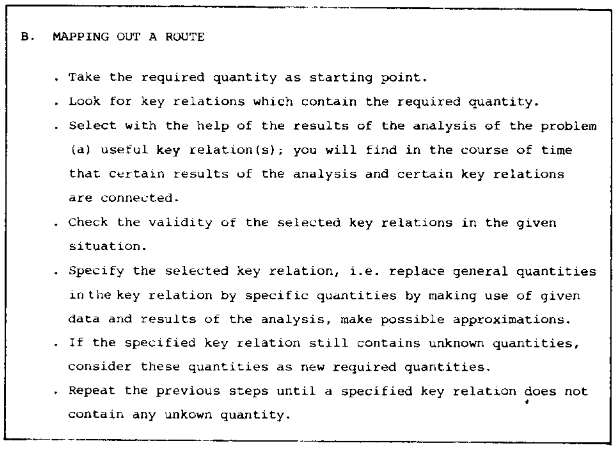
quired system characteristics.

(b) Backward reasoning starting with the required characteristic.

(c) Alternating reasoning meeting somewhere between.

The aim of the searching process is to find a suitable key relation, relating known and unknown quantities. The process is repeated until the required quantity (in case a) or the given characteristics (in case b) has been reached. Backward reasoning (Marples[9]; De Mul[11]) is useful because in this way the problem solver always knows the starting point of the route to be mapped out.

On the SAP chart a heuristic with respect to backward reasoning is given. As example this part of the SAP chart is shown in Fig. 5. In order to facilitate feedback to be given by the teacher to the student it suggested that the student should make use of a simple short-hand notation for mapping out the route of solving the problem (De Mul[11]).


B. MAPPING OUT A ROUTE

- Take the required quantity as starting point.
- Look for key relations which contain the required quantity.
- Select with the help of the results of the analysis of the problem (a) useful key relation(s); you will find in the course of time that certain results of the analysis and certain key relations are connected.
- Check the validity of the selected key relations in the given situation.
- Specify the selected key relation, i.e. replace general quantities in the key relation by specific quantities by making use of given data and results of the analysis, make possible approximations.
- If the specified key relation still contains unknown quantities, consider these quantities as new required quantities.
- Repeat the previous steps until a specified key relation does not contain any unkown quantity.

Fig. 5. Part of the SAP chart with the heuristic for mapping out a route of solving the problem.

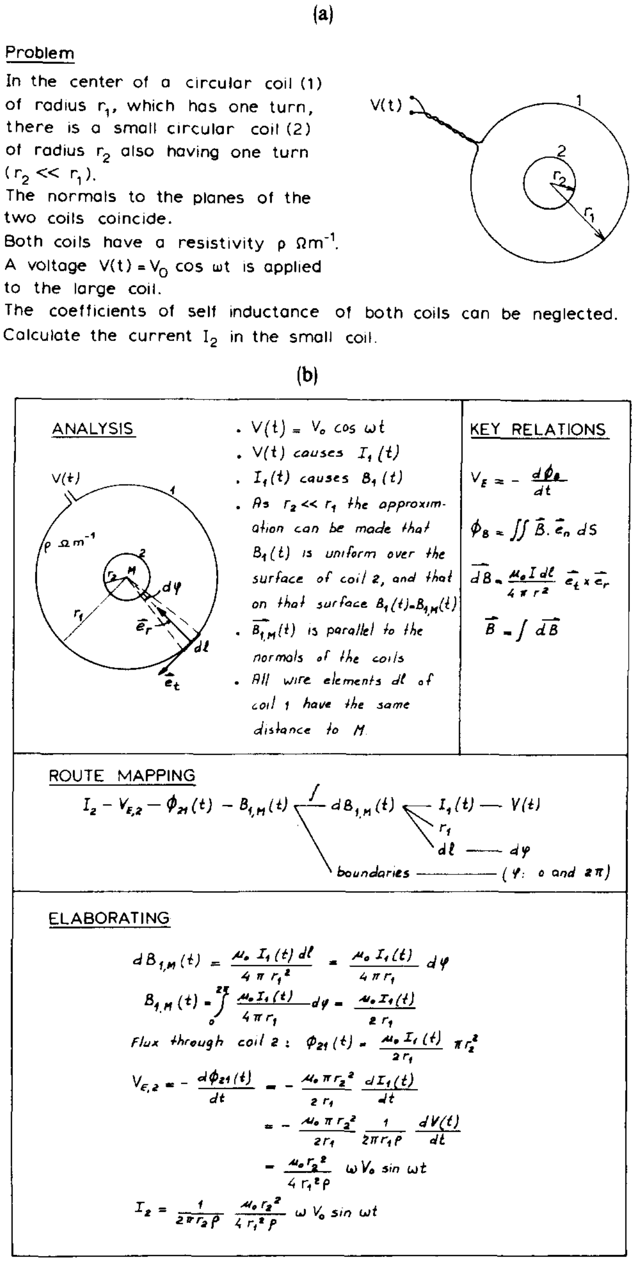
VI. INSTRUCTIONAL PROCEDURES

The two main functions of the instruction are the presentation of a basis of orientation and the training of the students in solving problems.

The knowledge and actions specific for the subject and the heuristics are presented in the lectures. All examples and applications contained in the course are discussed in the lectures as if they were problems to be solved. The teacher shows how to solve these problems systematically but does not usually show the complete working out. He thereby refers frequently to the SAP chart and the KR chart and also regularly uses the heuristics when explaining concepts and laws in the lectures.

In classes of about 20 students (the group of students is divided into subgroups of about 20 students) the student learns to solve problems by individual exercise and application of the system of heuristics. The student is asked to use the SAP and KR charts and to work as much as possible according to the system of heuristics. In the first sessions he has to do the problem solving completely on a special worksheet which has a layout in accordance with the heuristics. An example of such a sheet with a worked-out problem is shown in Fig. 6. The layout of the worksheet facilitates the feedback by the teacher to the individual student. The teacher observes the work of each student on the worksheet. He quickly determines in which phase of the solving process the student possibly has problems and looks for mistakes being made and tries to find the causes. He then gives directions to the individual student referring to SAP and KR chart. If necessary he gives explanations. He avoids, however, showing the student how to solve the problem because such an activity does not suit an adequate learning process. The student has to solve the problem by himself. However, if the feedback given several times by the teacher does not help a student in solving the problem the teacher can provide the student with a sheet with the problem worked out according to the system of heuristics.

Following the theory of Gal'perin concerning the stage-



(a)

Problem

In the center of a circular coil (1) of radius r_1, which has one turn, there is a small circular coil (2) of radius r_2 also having one turn ($r_2 << r_1$).
The normals to the planes of the two coils coincide.
Both coils have a resistivity ρ Ωm^{-1}.
A voltage $V(t) = V_0 \cos \omega t$ is applied to the large coil.
The coefficients of self inductance of both coils can be neglected.
Calculate the current I_2 in the small coil.

(b)

ANALYSIS

- $V(t) = V_o \cos \omega t$
- $V(t)$ causes $I_1(t)$
- $I_1(t)$ causes $B_1(t)$
- As $r_2 << r_1$ the approximation can be made that $B_1(t)$ is uniform over the surface of coil 2, and that on that surface $B_1(t) = B_{1,M}(t)$
- $\overrightarrow{B_{1,M}}(t)$ is parallel to the normals of the coils
- All wire elements $d\ell$ of coil 1 have the same distance to M.

KEY RELATIONS

$$V_E = -\frac{d\phi_B}{dt}$$

$$\phi_B = \iint \vec{B}.\vec{e}_n \, dS$$

$$\overrightarrow{dB} = \frac{\mu_0 I \, d\ell}{4\pi r^2} \vec{e}_t \times \vec{e}_r$$

$$\vec{B} = \int \overrightarrow{dB}$$

ROUTE MAPPING

$I_2 - V_{E,2} - \phi_{21}(t) - B_{1,M}(t) \xleftarrow{\int} dB_{1,M}(t) \leftarrow I_1(t) - V(t)$
r_1
$d\ell - d\varphi$
boundaries — (φ: 0 and 2π)

ELABORATING

$$dB_{1,M}(t) = \frac{\mu_0 I_1(t) \, d\ell}{4\pi r_1^2} = \frac{\mu_0 I_1(t)}{4\pi r_1} d\varphi$$

$$B_{1,M}(t) = \int_0^{2\pi} \frac{\mu_0 I_1(t)}{4\pi r_1} d\varphi = \frac{\mu_0 I_1(t)}{2 r_1}$$

Flux through coil 2 : $\phi_{21}(t) = \frac{\mu_0 I_1(t)}{2 r_1} \pi r_2^2$

$$V_{\varepsilon,2} = -\frac{d\phi_{21}(t)}{dt} = -\frac{\mu_0 \pi r_2^2}{2 r_1} \frac{dI_1(t)}{dt}$$

$$= -\frac{\mu_0 \pi r_2^2}{2 r_1} \frac{1}{2\pi r_1 \rho} \frac{dV(t)}{dt}$$

$$= \frac{\mu_0 r_2^2}{4 r_1^2 \rho} \omega V_0 \sin \omega t$$

$$I_2 = \frac{1}{2\pi r_2 \rho} \frac{\mu_0 r_2^2}{4 r_1^2 \rho} \omega V_0 \sin \omega t$$

Fig. 6. (a) An example of an electromagnetism problem. (b) Worksheet with the problem of (a) worked out.

by-stage formation of mental actions the teacher asks the students to use all aspects of the systematic approach and to write down all their actions in the first stage of the excercising. Gradually the students will use the systematic approach more automatically, they will find out that certain results of the analysis and certain key relations are connected. Especially in this stage mutual discussions of the students in the class are stimulated. The students will shorten the solving process. But they still have to write down the steps they are making in solving the problem, so that the teacher can give feedback. In this way the students will proceed more and more to the mental stage of problem solving. However, it is not the aim of the electromagnetism course to enable the student to solve problems completely at the mental stage. Yet the control of the process of solving problems has to be at the mental stage for all students after having completed the course.

Each time important new subject matter is introduced the new elements have to be carefully integrated, i.e., in the analysis new aspects may require attention or new key relations must be introduced. This in principle requires starting again at the first stage in which all the actions have to be written down, but due to the experience of the students with the systematic approach this stage can be passed rather quickly.

In addition to the training in the subgroups, self-study tasks are set. A collection of problems worked out according to SAP is available to the students so that they can check their own results.

VII. RESULTS

Some results of the reconstructed course will be presented here. More detailed results can be found in van Weeren *et al.*[12]

In 1977–78 the course Electromagnetism was given for the first time in the reconstructed form in two of the five technological departments, in 1978–79 in three departments, and in 1979–80 in all five technological departments.

As observed by the teachers it is rather difficult for the students to work systematically and to abandon their behavior in problem solving which was more or less a trial-and-error method. This is certainly the case in the first sessions of the subgroups. In a later stage most students are able to use the system of heuristics reasonably well.

For the teachers it is rather difficult to give feedback in an appropriate way suited to the learning process of the student. The teachers are often tempted to explain how to work out the problem instead of giving suggestions for solving the problem. During the courses all teachers except one gradually found a way of giving appropriate feedback. In one case the teacher decided halfway during the course to organize his classes in a different way from our reconstructed version.

In the evaluation of the course reconstruction the results of the examinations and the time students spent on the study of electromagnetism have been taken as dependent variables, and the entrance qualifications of the students, the course, the department, and the year of the examination as independent variables. With respect to the examination results three different analyses were made.

First, we compared the examination results of the students of all five technological departments who have followed the reconstructed or the unchanged course. The percentages of the participants of the examinations who passed the examinations in the period 1974–75 until 1979–80 for the five departments are given in Fig. 7. The examinations for the departments TN, EL, and TW[3] were identical, those for CT and WB[3] were slightly different. The results shown in Fig. 7 indicate that the reconstructed course, if well executed, gives better examination results than the conventional one.

The data of Fig. 7 are rather rough as we did not take into account the fact that the level of the examinations is not constant over the years, but shows yearly fluctuations, and also the fact that the entrance qualifications of the students differ from one department to another. We made a second, more detailed analysis of the examination results achieved in 1976–77 and 1977–78 by students of the departments TN and CT. These students followed the unchanged course in 1976–77 and the reconstructed course in 1977–78. In the analysis the examination level and the entrance qualifications have been taken into account.

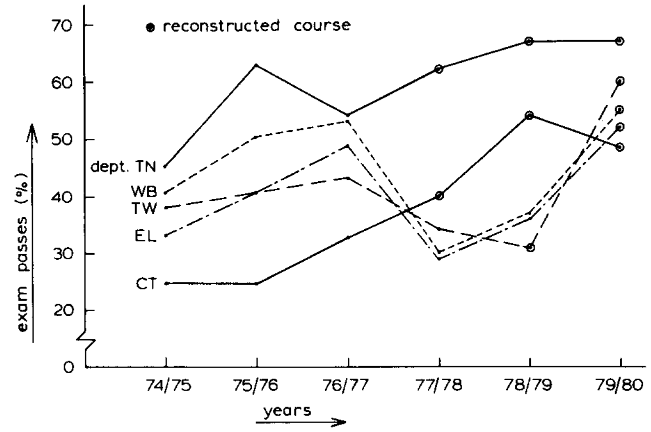


Fig. 7. Percentages of the participants of the examinations which passed the examinations over the period 1974–1975 until 1979–1980. The reconstructed courses are indicated by ⊙ in the figure.

Table I. Mean marks (M) (scale 1–10), standard deviations (S), and number of students (n) for the examinations 1976–77 and 1977–78 of the students of the departments TN, CT, and EL.

year	1976–77			1977–78		
dept.	M	S	n	M	S	n
TN	5.8	1.8	31	5.6	1.9	47
CT	4.8	1.7	27	4.6	2.2	48
EL	5.2	2.1	85	3.8	2.2	78

A correction for the difference in level of the examinations in 1976–77 and 1977–78 was made by transforming the results of 1977–78 of the TN and CT students to the level of 1976–77 with the help of the large reference group of students of the department EL who followed unchanged courses in 1976–77 and in 1977–78. Using the two scales of scores for the examinations of EL students during the two years a transformation equation can be used: a score X in 1977–78 is equivalent to a score Y in 1976–77 if

$$(X - M_x)/S_x = (Y - M_y)/S_y,$$

in which M is the mean score and S the standard deviation of the data of EL students. The TN and CT scores of 1977–78 have been transformed to the level of 1976–77 according to this equation. In Table I the mean marks (M), the standard deviations (S), and the number of students (n) are given for the examinations 1976–77 and 1977–78 of the TN, CT, and EL students. The marks of the final secondary-school examinations in mathematics, physics, and chemistry, which are controlled in the Netherlands by a central examination board, were taken as entrance qualifications. The difference in entrance qualifications were taken into account in an analysis of covariance. In this analysis the dependent variables are the marks for the examinations in electromagnetism, the independent variables are the course (unchanged or reconstructed) and the department (TN or CT). The covariates are the marks for the secondary-school examinations in mathematics, physics, and chemistry. The results of the analysis are shown in Table II.

From the values of F in the table we can see that the examination results are mainly determined by the secondary-school marks in physics, and by the variables department and course. With respect to these variables it can be stated that (1) the students of the department TN has better examination results than the students of the department CT; (2) the students who had the reconstructed course have better results than the students in the unchanged course. So, the reconstruction of the course is successful in respect to the improving of the examination results.

In a third analysis we looked for a difference in the effect of the reconstructed and the conventional course by comparing the results of the examinations in electromagnetism of the students of four technological departments in 1975–76 and 1979–80. For the purpose of comparison the examination of 1979–80 was made identical to that of 1975–76, except for the CT students for whom the course content was changed too much.

The percentages of passes in the examinations of 1975–76 and 1979–80 can be compared in Fig. 7. In Table III the mean marks (M) and the standard deviations (S) of the two examinations are given. The results of a t test we performed on these data are also mentioned. The values of t in the table indicate that the examination results are improved significantly for two departments due to the reconstructed course provided that the entrance qualifications of the students in 1975–76 and 1979–80 are about the same.

The mean of the marks of each department for the final secondary-school examinations in mathematics, physics, and chemistry of the first-year students in 1975–1976 are slightly lower than those of the first-year students in 1979–1980. The mean differs only by 0.2 to 0.4 on a scale 1 to 10. Therefore we can conclude that the reconstruction of the course is successful.

Table II. Results of an analysis of convariance with as independent variables the transformed marks of TN and CT students, as covariates the marks for the secondary-school examinations in mathematics, physics, and chemistry, and as independent variables the course (unchanged or reconstructed) and the department (TN or CT).

source	df	sum of squares	mean square	F	sign. of F
covariates	3	92.341	30.780	10.98	0.00
mathematics	1	1.671	1.617	0.58	0.45
physics	1	20.672	20.672	7.38	0.01
chemistry	1	0.080	0.080	0.03	0.87
course	1	30.198	30.198	10.77	0.00
department	1	12.974	12.974	4.63	0.03
error variance	144	403.612	2.803	0.08	0.78

Table III. Mean marks (M) (scale 1–10), standard deviations (S), and the results of a t test on these data of the examinations 1975–76 and 1979–80 for the students of the departments TN, EL, TW, and WB.

year	1975–76		1979–80		t test		
dept.	M	S	M	S	df	t	0.05 level sign.
TN	6.0	1.7	6.2	1.8	90	0.55	no
EL	4.4	2.0	5.3	1.8	150	2.84	yes
TW	4.9	1.8	5.8	2.1	68	1.84	yes
WB	5.4	2.1	5.9	2.4	171	1.44	no

In a separate investigation we asked students to keep a record of the time they spend studying. Data on the time spent on the study of electromagnetism indicate that students who followed the reconstructed course did not need more time than those following the conventional course.

VIII. CONCLUSION

In general we can conclude from the results of the examinations that the course Electromagnetism has been improved due to the reconstruction. As mentioned before, we applied in the reconstruction elements of an approach developed for solving problems in thermodynamics with slight modifications. The successful application of this approach in the course Electromagnetism indicates that probably this approach can be used in other fields of natural sciences and technology, and that only small adaptations would be required.

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[1] C. T. C. W. Mettes, A. Pilot, and H. J. Roossink; Third International Congress of the European Association for Research and Development in Higher Education, Klagenfurt, Austria, 1979 (unpublished). (Paper available at OC-THT; P.O. Box 217, Enschede, The Netherlands.)

[2] The level of the curriculum of the first-year students at THT is in general comparable to that of the institutes of technology in the USA with exception of the liberal arts which the Dutch students study already in secondary school.

[3] The Twente University of Technology (THT) has five technological departments: Applied Physics (TN), Chemical Engineering (CT), Applied Mathematics (TW), Civil Engineering (WB), and Electrotechnical Engineering (EL).

[4] M. Alonso and E. J. Finn, *Fundamentele Natuurkunde 2: Electromagnetisme* (Elsevier, Amsterdam, 1978).

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[7] Criteria for a relation to be a key relation: the relation is fundamental, or the relation is very frequently used in solving problems, or the derivation of the relation from other relations is very difficult or time consuming for the students.

[8] V. I. Dubovskaja, *Doklady naucnoy konferencii* (Moscow, 1967), p. 135.

[9] D. L. Marples; Report CUED/C-Educ/Tri, Dept. of Engineering, University of Cambridge (UK), 1974 (unpublished).

[10] J. H. Larkin and F. Reif, Eur. J. Sci. Educ. **1**, 191 (1979).

[11] F. F. M. de Mul, Faraday **45**, 91 (1976) (in Dutch).

[12] J. H. P. van Weeren, H. Kramers-Pals, F. F. M. de Mul, M. J. Peters, and H. J. Roossink, Report no. 39, OC-THT, Enschede (Netherlands), 1979 (in Dutch) (unpublished).

PROBLEM

Show that for a nonrelativistic particle the impulse–momentum theorem follows from the work–energy theorem when the latter is required to be Galilean covariant. Also demonstrate the analogous connection for a system of particles, with the additional assumption that any net work done by internal forces is Galilean invariant. (Solution is on page 756.)

John Sein