ac loss measurements have been performed on a superconducting transformer. The transformer is a part of a 25 kA thermally switched superconducting rectifier operating at a frequency of 0.1 Hz. The loss measurements have been automatized by means of a microcomputer sampling four relevant signals and processing these data to reliable loss numbers. Results were obtained at amplitudes of the secondary current between 5 and 15 kA at various current rates between 5 and 30 kA s^{-1}.

ac loss measurements on a superconducting transformer for a 25 kA superconducting rectifier

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Keywords: superconductivity, transformer, rectifier, ac loss

High frequency applications of superconductivity are often not feasible because of the ac losses in the multifilamentary superconductors especially in the case of 50 (60) Hz public utility. However, in the low frequency region some applications are possible. The thermally switched high current superconducting rectifier is a low frequency device (0.1 Hz). The activities in this field relate to the performance of a 25 kA, 1.5 kW superconducting rectifier system. This system consists of a series connection of three individual 25 kA, 0.5 kW rectifiers.[1,2] Such a superconducting rectifier consists of a current step-up transformer (37 A → 25 kA) and superconducting switches and acts as a current supply for high current sc coils. The designer of a sc rectifier has to take account of the fact that the efficiency of the rectifier is not determined for a great part by the ac losses in the superconductors. Therefore these losses have to be studied, calculated and measured.

The rectifier-system has three sc transformers, each capable of meeting the requirements of current (25 kA) and power (0.5 kW). They will be different among themselves concerning the superconductors used: internal geometry, matrix composition, twist lengths etc. The primary and secondary windings of the first transformer have been made with in-stock copper matrix NbTi-conductors. The lay-out of the transformer, the ac loss measurements and a comparison to calculated hysteretic loss will be presented in this paper. The next step will be to decrease the ac loss in a subsequent transformer by using very fine filament material (hysteretic loss) in a mixed matrix (coupling loss) and by optimizing the cable geometry and twist lengths in the strand and the 25 kA cable for the secondary circuit. A third step may be to reach a lower limit of ac losses at 0.1 Hz.

The transformer

The centre tap transformer consists of a system of concentric solenoids being three primary coils and two secondary coils in one layer. The longitudinal section is shown in Fig. 1. The coupling coefficient of this air-core transformer is 0.965.

The inductances of the primary and secondary windings are determined by the required power of the rectifier and the available power supply.[1] As mentioned before the conductors used for this first 25 kA transformer were in-stock NbTi in copper matrix conductors.

The transformer data are collected in Table 1.

The windings have been vacuum impregnated with STYCAST 2850 FT epoxy. Between the layers thin copper wires parallel to the field act as heat drains. The contribution of these copper wires and other construction materials to the ac loss can be disregarded.

Loss measurements

During real operation of the superconducting rectifier the primary windings are exposed to a symmetrical trapezoidal current. The currents in the two secondary coils are periodically swept between zero and the already achieved current.[1] The actual ac loss of the transformer in such a situation can only be measured during operation of the rectifier itself. This will be done later.

It is important, however, to estimate these losses with results of separate loss measurements. For this

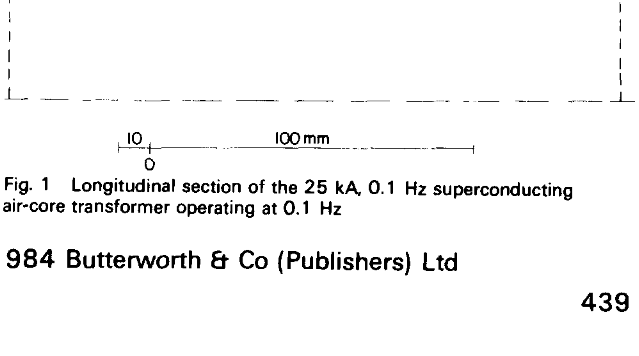

Fig. 1 Longitudinal section of the 25 kA, 0.1 Hz superconducting air-core transformer operating at 0.1 Hz

Table 1. Transformer data

primary inductance	$L_p = 10.5$ H
secondary inductance (shortcircuited)	$L_s = 20.7$ μH
coupling coefficient	$k = 0.965$
current amplification	670
inner diameter	16 cm
outer diameter	20 cm
length	16 cm
volume	5 l
primary turns	9472
conductor	0.28 diameter MCA 367 filament 9.7 μm diameter Cu/NbTi = 1.25
maximum current	77 A at 3 T
NbTi volume	134 cm^3
conductor volume	302 cm^3
secondary turns	12
conductor	24 strand Rutherford
sizes	12 x 2 mm^2
strand	1.05 mm diameter MCA 367 filaments 36.5 μm diameter Cu/NbTi = 1.25, 25 mm twist
maximum current	26 kA at 3 T
NbTi volume	65 cm^3
conductor volume	146 cm^3
maximum field at the windings	2.6 T at 25 kA

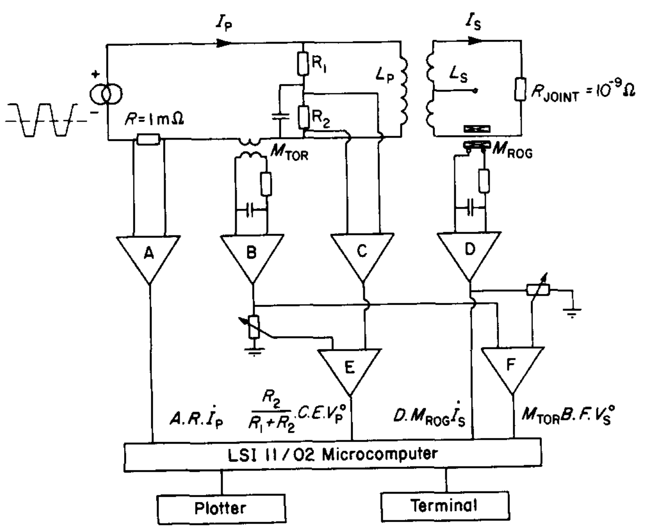

Fig. 2 The measuring arrangement to determine the ac losses of a sc transformer

reason the transformer secondary is short-circuited with a low ohmic joint ($\approx 10^{-9}$ Ω) while the central tap is not connected. The primary coil is connected to a current supply capable of producing a trapezoidal current with an adjustable current rate and a frequency of 0.1 Hz in the primary and thus in the secondary windings as well (Fig. 2). The operating frequency of 0.1 Hz and the trapezoidal shape of the currents preclude a measuring method with a lock-in amplifier. Therefore a microcomputer operating as a joule meter was used. A heating element at the secondary conductor can switch off the secondary current. A heating element is necessary in order to measure the ac loss when the secondary current is not symmetric but asymmetric between zero and its peak value. This heating element causes a small secondary resistance because of the copper matrix. The asymmetric secondary current is obtained if the primary current is increased slowly while the secondary current is switched off once every period during the flat part of the trapezoidal current. It is not desirable to shorten the secondary circuit with a highly resistive superconducting switch with a CuNi matrix conductor, because this would make this method of measuring the ac loss impossible since the contributions to the ac loss of the switch conductor and the transformer conductor cannot be separated.

The secondary short-circuited transformer is described by two equations:

$$V_p = L_p\dot{I}_p - M\dot{I}_s + V_p^R \quad (1)$$

$$V_s^R = M\dot{I}_p - L_s\dot{I}_s \quad (2)$$

in which V_p^R and V_s^R are the voltage components in the primary and secondary circuit which give rise to the ac losses. It should be noted that the self-inductances L_p and L_s and the current ratio of the transformer are a function of the currents because the windings are made from superconducting material and the magnetic field varies between zero and 2.6 T. This dependence is a result of the magnetic energy content of the filaments which is current (field) dependent. Therefore, the formula for the self-inductance is:

$$L(I) \equiv \frac{2}{\mu_0 I^2} \int_{\text{space}} B^2(I)\,\mathrm{d}V \quad (3)$$

The measured current amplification shows this phenomenon (Fig. 3). A secondary current of 5 kA ensures that the current amplification is constant within 1% and the inductances become constant and equal to their 'copper' values. In this case it is correct to use the transformer equations (1) and (2). Substitution of (2) in (1) gives:

$$V_p = \left(L_p - \frac{M^2}{L_s}\right)\dot{I}_p + \frac{M}{L_s} V_s^R + V_p^R \quad (4)$$

The total ac loss per period time T of the transformer is described by:

$$Q_{TOT} = \int_0^T V_p I_p \,\mathrm{d}t \quad (5)$$

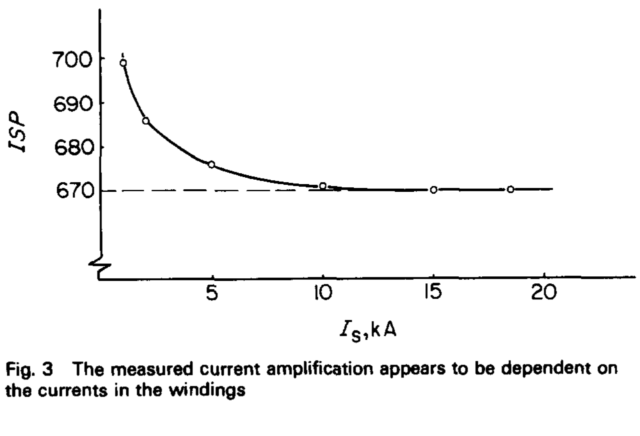

Fig. 3 The measured current amplification appears to be dependent on the currents in the windings

The first term in (4) is the largest and represents the components of the primary voltage that is without loss. Integration of this term does not contribute to the losses, since:

$$\int_0^T I_p \dot{I}_p \, dt = 0 \qquad (6)$$

Therefore, the sensitivity of the measuring method is largely increased if the primary voltage is compensated by $\alpha \dot{I}_p$:

$$Q_{TOT} = \int_0^T (V_p - \alpha \dot{I}_p) I_p \, dt = \int_0^T V_p^O I_p \, dt \qquad (7)$$

With $\alpha = L_p - M^2/L_s$ the compensation is perfect and the shape of the loss voltage can be studied directly. This loss voltage is fully resistive if the inductances in (7) are considered as the circuit inductance including the magnetisation terms of the sc wires themselves (see (3)). Similar arguments hold for the ac loss of the secondary windings:

$$Q_{SEC} = \int_0^T V_s^R I_s dt = M \int_0^T \left(\dot{I}_p - \frac{L_s}{M} \dot{I}_s + \beta \ddot{I}_s \right) I_s \, dt$$

$$= \int_0^T V_s^{RO} I_s dt; \quad \beta \sim L_s/M \qquad (8)$$

Equation (8) can be transformed by partial integration to:

$$Q_{SEC} = M \int_0^T I_p (I_s - \dot{I}_s - \gamma \dot{I}_p) \, dt; \quad \gamma \sim M/L_s \qquad (9)$$

Hence, the secondary loss can be measured in several ways. It appears from (7) and (8) that four signals V_p^O, I_p, V_s^{RO} and $\dot{I}_s$ have to be measured. These signals are composed, amplified, filtered and sampled by a LSI 11/02 microcomputer (Fig. 2). Ten periods, each of 1000 samples, are averaged. dc levels of the amplifiers do not affect the measurements because V_p^O, V_s^{RO} and $\dot{I}_s$ are made symmetric by the computer, before integration starts. This is correct since both the primary and secondary currents are symmetric. After this the data are processed to the final loss numbers. The equations for Q_{TOT} and Q_{SEC} in this practical case are:

$$Q_{TOT} = \frac{R_1 + R_2}{R_2 \, R \, A \, C \, F} \int_0^T [V_p - \alpha' \dot{I}_p] \, I_p dt \qquad (10)$$

$$Q_{SEC} = \frac{M}{M_{TOR} M_{ROG} \, B \, D \, E}$$

$$\int_0^T \left[(\dot{I}_p - \beta' \dot{I}_s) \int_0^T \dot{I}_s \, dt' \right] dt \qquad (11)$$

The measuring error of the final results depends on the accuracy of the amplifiers *A–F*; the shunt resistor *R*; voltage divider R_1, R_2; the mutual inductances of the transformer *M*, the primary pick-up coil M_{TOR} and the Rogowski coil M_{ROG}. The estimated accuracies of the results are 4 and 7% for the total and secondary losses respectively. A typical output of the computer showing the four relevant signals is shown in Fig. 4.

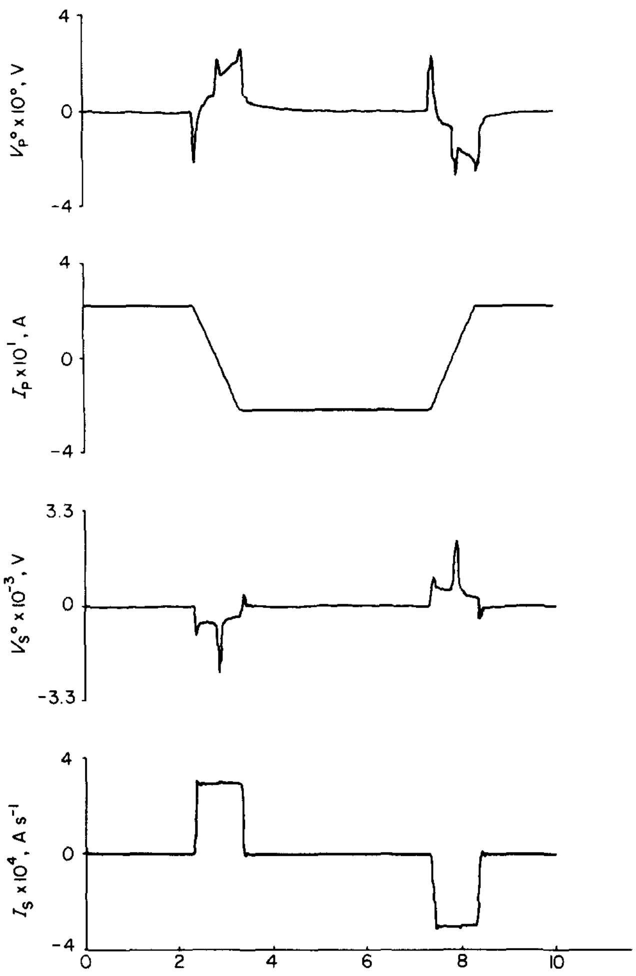

Fig. 4 Computer output showing the four relevant loss signals being the compensated primary and secondary voltages V_P^{RO} and V_S^{RO}, the primary current I_P and the secondary current rate $\dot{I}_S$

Results

Symmetric secondary current. The maximum current in the secondary windings (central tap not connected) is 26.4 ± 0.6 kA. The current amplification of the transformer is then 670. The total and secondary losses of the transformer were measured at current amplitudes of 5, 10 and 15 kA in the secondary of the

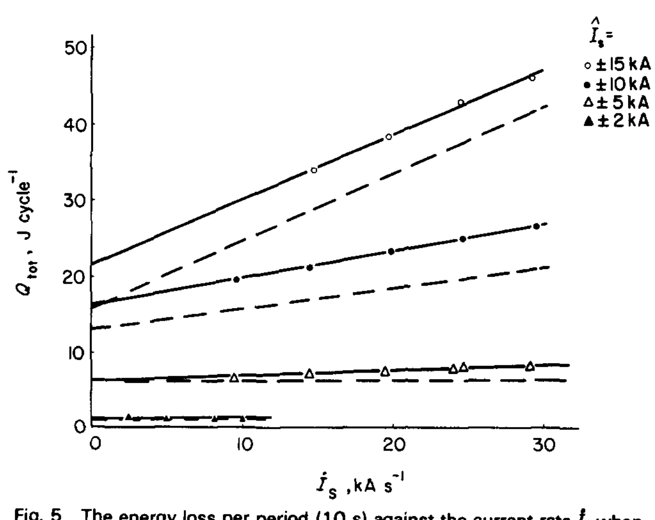

Fig. 5 The energy loss per period (10 s) against the current rate $\dot{I}_s$ when applying a symmetrical trapezoidal current to both the primary and secondary windings. The solid lines are the total losses, the dashed lines the secondary losses

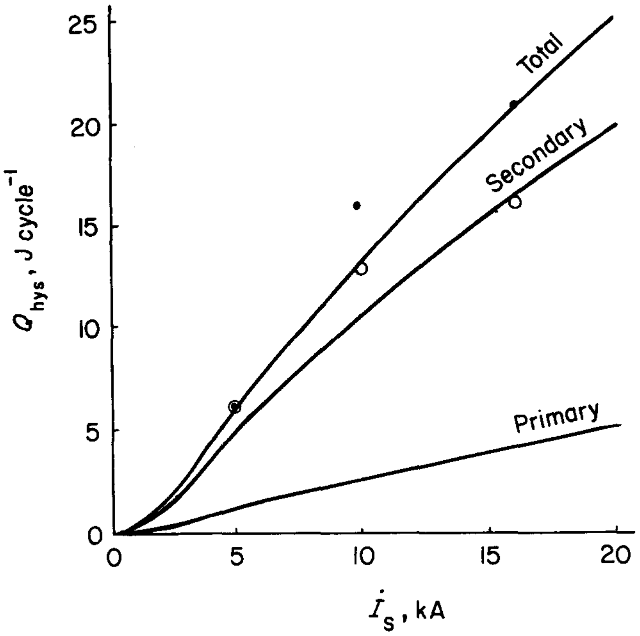

Fig. 6 The measured hysteretic loss in the total transformer (●) and in the secondary windings (O) are compared to the calculated losses (lines)

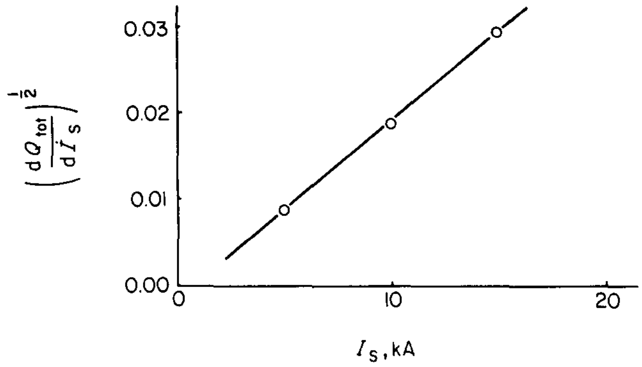

Fig. 7 The square root of the slopes of the loss curves in Fig. 5 versus the secondary current show that the matrix losses are proportional to $I_s^2 \dot{I}_s$

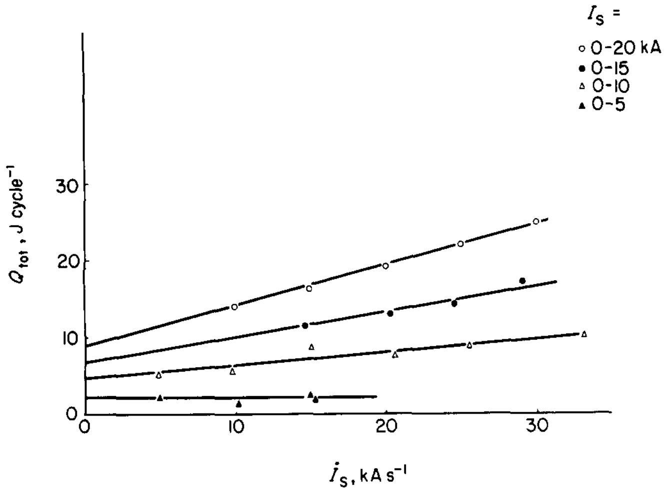

Fig. 8 The total energy loss per period when the secondary current is asymmetrical and swept between zero and the peak current $\hat{I}_s$

transformer while the current rate was varied between 5 and 30 kA s^{-1}. The energy loss per period for the total and the secondary loss are shown in Fig. 5. The secondary current is swept between $-\hat{I}_s$ and $+\hat{I}_s$. The period time is 10 s. The hysteretic loss Q_H was determined from this figure by extrapolation to $\dot{I}_s = 0$. The values obtained for the hysteretic losses of the total and the secondary of the transformer compared to calculated losses (discussed below) are shown in Fig. 6. It appears that about 75% of the hysteretic loss comes from the filaments in the secondary of the transformer.

The matrix loss Q_M assuming this to be additional to the hysteretic loss is found by subtraction of the measured loss from the determined hysteretic loss: $Q_M(I_s, \dot{I}_s) = Q(I_s, \dot{I}_s) - Q_H(I_s, 0)$. The current or field dependence of the matrix losses can be shown from the slopes of the loss curves (Fig. 7). The matrix loss Q_M appears to be proportional to $I_s^2 \dot{I}_s$ or B^3/T.

Asymmetric secondary current. Finally, the results are given in a situation where the secondary current is not swept symmetrically between $-\hat{I}_s$ and $+\hat{I}_s$ but asymmetrically between zero and $\hat{I}_s$ while the primary current is still symmetric (Fig. 8). In this case the losses at comparable current changes (± 10 kA equals 0-20 kA) are somewhat lower as might be expected because of the smaller area of the magnetization loop due to the field offset at the filaments. The latter situation is more realistic for the true rectifier operation.

Comparison with calculated hysteretic losses

The filaments in the primary and secondary windings are exposed to an almost transverse magnetic field that is in phase with the currents through the windings.

This means that in the absence of an external bias field the critical current density J_c in the filaments is highly dependent on the local magnetic field B in the windings. As a consequence the hysteretic loss of the filaments can only be calculated with a formula that takes into account the field dependent critical current density and the local magnetic field in the windings that varies strongly with position and time. Usually two formulae are used to calculate the hysteretic losses in a filament with diameter d in the case of a constant current density. One formula describes the loss below the penetration field $B_p = \mu_0 J_c d/\pi$ and shows the loss to be proportional to the third power of the field. The second one describes the loss above B_p and shows a linear field dependence.[3] The penetration field B_p at 2.5 T is 13 mT for the primary filaments and 50 mT for the secondary filaments. Consequently, the hysteretic loss below the penetration field B_p can be disregarded in this case. Above B_p the magnetization M at a constant critical current density is given by:

$$M = \frac{2d}{3\pi} J_c \tag{12}$$

If J_c is field dependent

$$M(B) = \frac{2d}{3\pi} J_c(B) \tag{13}$$

The $J_c(B)$ at low fields can be well described using the KIM relation $J_c(B) = J_0 B_0/(B_0 + |B|)$. The local hysteretic loss per unit volume per cycle now becomes:

$$q_H(B) = 4 \int_0^B M(B)\,dB$$

$$= \frac{8}{3\pi} d\, J_0 B_0 \ln\left[1 + |B|/B_0\right] \quad \text{Jm}^{-3} \tag{14}$$

The contribution of the transport current to the hysteretic loss is taken into account with a general factor:

$$q_H(B, I) = q_H(B)\,[1 + 1/3\,(I/I_c)^2] \quad \mathrm{Jm^{-3}} \tag{15}$$

The ratio of the transport current to the critical current I/I_c is different for the primary and secondary windings (see Table 1). The hysteretic loss of the transformer can now be found by detailed computer calculations of the local magnetic field in the windings. The volume occupied by the windings is divided into N rings with an average magnetic field B_n and a certain volume of filaments V_n. The loss per cycle of the complete transformer is found by addition of the losses in the separate rings:

$$Q_H = \sum_{n=1}^{N} q_n(B, I)\;V_n \quad \mathrm{J} \tag{16}$$

The critical current density of the MCA wires used fits the KIM relation $J_c(B) = 1.25/(1.25 + |B|)10^{10}\ \mathrm{Am^{-2}}$ reasonably well. A comparison between the calculated loss and the measured data was already given in Fig. 6. It appears that the ratio of secondary and total hysteretic loss is 77% which is in good agreement with the measurements.

Discussion

If the currents in both the primary and secondary coils of the transformer are symmetric the filaments are exposed to an almost transverse field which is in phase with the transport currents in the windings that generate this field. The direction of the magnetic field vector at the filaments is fixed but the magnitude of the field varies with the transport current ($B \propto I$).

However, in the case of realistic operation of the transformer in a rectifier, the currents in both parts of the secondary windings are not symmetric but asymmetric and opposite in direction. This means that the magnetic field vector rotates around the filaments while its magnitude is still proportional to the currents in the windings, see Fig. 9. The angle of rotation is different at all windings which extremely complicates a theoretical analysis of the ac losses.

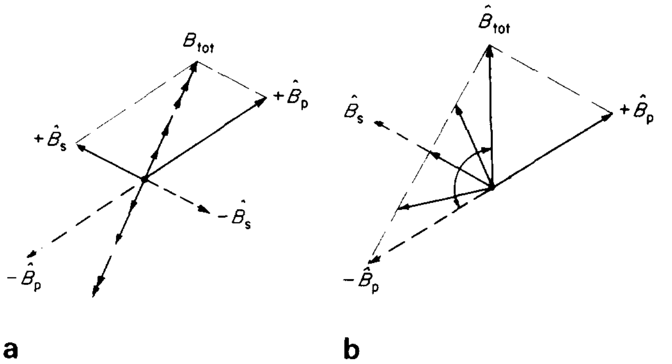

Fig. 9 If both the primary and secondary currents in the transformer are symmetric, the direction of the transverse magnetic field at a filament is fixed (a). Otherwise the field vector rotates (b)

Another complication is the presence of two different conductors in the transformer. These are a single wire for the primary windings and a cabled conductor for the secondary windings of the transformer. Moreover, both conductors are not loaded with the same relative current I/I_c. Especially the secondary windings are exposed to large gradients of the magnetic field and large field and current rates up to 3 T s^{-1} and 30 kA s^{-1} respectively.

Measuring the total loss of the transformer all these effects are mixed up and can hardly be distinguished from each other. Keeping this in mind there is a reasonable agreement between calculated and measured hysteretic losses. The agreement concerning the ratio of the hysteretic loss in the primary and secondary windings is remarkable. The exactness of the correction factor $1 + 1/3(I/I_c)^2$ representing the contribution of the transport current to the hysteretic loss if the magnetic field and transport current are in phase, is questionable in spite of the results. The hysteretic losses in the secondary windings per volume superconductor is a factor eight larger than those in the primary windings. This is mainly caused by the thick filaments in the secondary windings (ϕ 36.5 μm). It can be concluded that the filament hysteretic losses of the transformer which are found by extrapolation of the losses to zero current rate can be well calculated.

The situation is completely different for the matrix losses. They were observed to be proportional to $B^2\dot{B}$. Mostly the dependence of the matrix losses on the magnetic field B is found to be $B\dot{B}$ although only few measurements of ac losses on complete coils or transformers with transport currents are known.[4–6] In many cases these results were obtained with helium boil-off methods with limited accuracy and have been done in a single-step ramp field with a single peak value of the magnetic field B and various field rates. A $B\dot{B}$ dependence is assumed and the time constant of the eddy currents is calculated according to $Q = (2\pi/\mu_0)\,B\dot{B}\,\tau\ \mathrm{Jm^{-3}}$.[3] Moreover, these experiments are carried out with medium field magnets. In the present case the maximum field at the windings of the transformer is 2.6 T. In other words, considerable volumes of superconductor with mainly self-field conditions as well as transverse field conditions are combined in one device. Hence it is important to realize that the mechanisms producing the observed

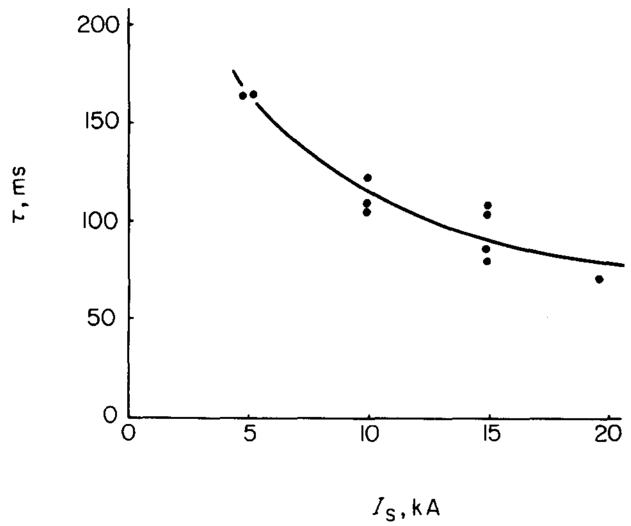

Fig. 10 Comparison of calculated time constant of the eddy currents including a magnetoresistance effect (solid line) and measured time constants (●)

behaviour of the matrix losses are very complicated. The transient field losses of conductors carrying dc transport currents has been treated theoretically by Ogasawara et al.[7] and also led to a different B and $\dot{B}$ dependence of the losses.

The time constant of the eddy currents in the matrix can be determined from the slopes of the voltage curves (Fig. 4) for various currents and current rates. Some 'decrease' of the time constant with the secondary current is observed (Fig. 10). The measured time constant is in agreement with the calculated time constant of the secondary conductor:

$$\tau_s = \frac{1}{2}\,\frac{\mu_0}{\rho_\perp}\left(\frac{l_s^2}{2\pi}\right) \tag{17}$$

in which l_s is the twist length of the strand in the secondary conductor (25 mm) and $\rho_\perp$ the transverse resistivity varying between $1.5 \cdot 10^{-10}$ Ωm at zero current (field) and $2.5 \cdot 10^{-10}$ Ωm at 20 kA (2 T). The secondary Rutherford cable is not filled with solder so that the time constant of the secondary cable equals that of the strand. The time constant of the matrix currents determined from the loss voltages is dominated by the time constant of the secondary strand because of its large twist length of 25 mm compared to the 5 mm twist length of the primary conductor.

The expression generally used to calculate eddy current losses is only valid if the current density in the outer filaments of the conductor does not exceed the critical current density. An increase of losses more than proportional with B^2 may be expected if this situation is achieved or exceeded. The critical field rate is the smallest for the secondary conductor

$$\dot{B}_{\perp c} = \frac{\pi^3}{16 l_s^2}\, J_c \rho_\perp D \tag{18}$$

in which D is the diameter of the strand (1.05 mm), ρ the transverse resistivity ($1.5\text{-}3 \cdot 10^{-10}$ Ωm) and l_s the twist length (25 mm). The critical field rate lies between 2 and 5 T s^{-1}.

The maximum field rate in the transformer during the loss measurements is about 3 T s^{-1} at 30 kA s^{-1} which may be a reason for the measured field dependence of the matrix losses.

Final remarks

The loss measurements presented above have been carried out in order to evaluate a first design of a 25 kA superconducting transformer to be used for a 25 kA, 500 W superconducting rectifier. This rectifier will operate at a frequency of about 0.1 Hz and a maximum current rate in the secondary windings of 10 kA s^{-1}. If the secondary current is then swept between 0 and 25 kA (equal to about -12.5 kA to + 12.5 kA) the maximum loss will be approximately 20 J per cycle, ie 2.0 W.

These data imply that the losses in the transformer reduce the efficiency of the rectifier by 0.5% at the most.

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