Vertical fluxes of sediment in oscillatory sheet flow

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Abstract

Time series of vertical sediment fluxes are derived from concentration time series in sheet flow under waves. While the concentrations $C(z,t)$ vary very little with time for $|z| < 10d_{50}$, the measured vertical sediment fluxes $Q_z(z,t)$ vary strongly with time in this vertical band and their time variation follows, to some extent, the variation of the grain roughness Shields parameter $\theta_{2.5}(t)$. Thus, sediment distribution models based on the pickup function boundary condition are in some qualitative agreement with the measurements. However, the pickup function models are only able to model the upward bursts of sediment during the accelerating phases of the flow. They are, so far, unable to model the following strong downward sediment fluxes, which are observed during the periods of flow deceleration. Classical pickup functions, which essentially depend on the Shields parameter, are also incapable of modelling the secondary entrainment fluxes, which sometimes occur at free stream velocity reversal. The measured vertical fluxes indicate that the effective sediment settling velocity in the high $[0.3 < C(z,t) < 0.4]$ concentration area is typically only a few percent of the clear water settling velocity, while the measurements of Richardson and Jeronimo [Chem. Eng. Sci. 34 (1979) 1419], from a different physical setting, lead to estimates of the order 20%. The data does not support gradient diffusion as a model for sediment entrainment from the bed. That is, detailed modelling of the observed near-bed fluxes would require diffusivities that go negative during periods of flow deceleration. An observed general trend for concentration variability to increase with elevation close to the bed is also irreconcilable with diffusion models driven by a bottom boundary condition. © 2002 Published by Elsevier Science B.V.

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1. Introduction

In recent years, a number of detailed measurements have been made of sediment concentrations and particle velocities in oscillatory sheet flow. The sheet flow regime is the high-energy regime where vortex ripples are absent and sediment is in motion down to several grain diameters below the undisturbed bed level. Several experimental studies focussed on typical beach sand (specific gravity $s \approx 2.65$, median diameter $d_{50} \approx 0.2$ mm), starting with Horikawa et al. (1982) and followed by Ribberink et al. (2000) in the nineties. In addition, Dohmen-Janssen et al. (1998) made measurements with finer ($d_{50} = 0.13$ mm) quartz sand. Concentrations of these fine materials were measured via the electrical conductivity of
the fluid-grain mixture. Dick and Sleath (1991, 1992) and Li and Sawamoto (1995a,b) made measurements with coarser, lightweight particles. Both lightweight materials and sand were studied by Flores and Sleath (1998).

These measurements form a good platform for starting to develop a new generation of models for sediment transport in oscillatory flows of water. They do in fact also demonstrate the need for a new generation of models. In particular, they show strong disagreement with all classical bed concentration \( C_b \) models. This disagreement is clear from published concentration time series but we shall discuss it briefly in Section 2.

Most sediment transport models are formulated in terms of vertical sediment fluxes, e.g., an upward flux due to gradient diffusion or convection of sand trapped in travelling vortices more or less in balance with a downward settling flux. Nevertheless, the vertical sediment fluxes, which can be derived from measured concentration time series, have so far not been extracted and explicitly compared with the available models. This will be done in Section 3.

Based on these vertical fluxes, we draw conclusions related to existing pickup function models, the effective settling velocity, \( w_s \), in the high concentration area near the undisturbed bed, and the relevance of the gradient diffusion concept close to the undisturbed bed level.

### 2. Bed-level concentrations in sheet flow

In relation to existing \( C_b \)-models, the most remarkable feature of measured sediment concentrations near \( z=0 \) in oscillatory sheet flow is that they vary very little with time compared to the existing \( C_b \)-predictors. A very good example is \( C(0.2 \, \text{mm}, t) \) in Fig. 7, Ribberink et al. (2000). In Fig. 1 below, we show a measure \( R \) of the concentration variability with \( z \) for a number of experiments.

![Graph showing concentration variability](image)

**Fig. 1.** Vertical distribution of the coefficient of variation \( R = \sqrt{\text{Var}(C(z,t))} / \bar{C}(z) \) for measured sediment concentrations in oscillatory sheet flow and for the predicted \( C_b(t) \) based on Engelund and Fredsoe (1976) with the Shields parameter varying as \( \cos^2 \theta \) and with \( \theta_c = 0.05 \).

![Graph showing free stream velocity and bed level](image)

**Fig. 2.** Free stream velocity [m/s], —; Instantaneous bed level \( Z_b(t) \) [mm] from the undisturbed bed level, \( \circ \); and \( C(Z_b(t) + 2d_{50}, t) \) [vol/vol], +, determined by interpolation between measured concentrations. Data from Ribberink et al. (2000), 0.245-mm quartz sand.
We see that that the measured concentrations vary very little near \( z = 0 \), as quantified by the coefficient of variation:

\[
R = \sqrt{\text{Var}\{ C(z, t) \} / \bar{C}(z)}
\]  \hspace{1cm} (2.1)

For all the measurements, \( R \) is less than 0.1 in the elevation range \( |z| < 2d_{50} \), while typical \( C_b \) predictors, which typically predict zero concentration for \( \theta \rightarrow 0 \) and approach \( C_b = 0.4 \) for large \( \theta \), give \( R \)-values greater than 0.5.

We further see that \( R \) increases rapidly with \( z \) for \( 0 < z < 1 \) mm for all the measurements. This is in contrast with gradient diffusion models which will (unless the diffusivity is purely imaginary), in general, have oscillating components of \( C(z, t) \), decaying more rapidly with \( z \) than \( \bar{C}(z) \) (cf. Nielsen, 1992, p. 244), and hence have \( R \) decreasing with increasing \( z \).

One reviewer suggested that the \( C_b \)-predictors should, to be fair, be applied to a level \( 2d_{50} \) above the instantaneous bed level—not the undisturbed bed level. However, this only makes things worse for the \( C_b \)-predictors. That is, the measured (= interpolated) concentrations at \( 2d_{50} \) above an “instantaneous bed level”, determined as the level where the measured \( C(z, t) \)-profile hits \( C_{\text{max}} \), show a dip at the time when \( u_{\infty}(t) \), see Fig. 2. This is the opposite of what the \( C_b \)-predictors predict.

3. Vertical sediment fluxes in sheet flow

Steady, uniform sheet flow, e.g., see Sumer et al. (1996), can be modelled without consideration of vertical sediment fluxes—they are simply always zero in steady, uniform flows. In oscillatory flows, however, the vertical fluxes vary with time and essential parts of the sheet flow process can be studied by observing their vertical distribution and time variation. It is a fairly straightforward matter to derive vertical sediment fluxes from measured concentration time series. So far, the fluxes have, however, only been considered qualitatively, e.g., Flores and Sleath (1998) noted that the settling out happened much faster for sand than for the lightweight materials in their experiments. The strange, rapid settling by sand is highlighted by the following quantitative analysis and contrasted with previous modelling expectations.

3.1. Vertical sediment fluxes derived from measured concentration time series

Based on measured time series of \( C(z, t) \), the corresponding time series of vertical sediment fluxes \( Q(z, t) \) can be calculated for any level in the measuring domain. There are two ways of doing this: considering the time variation of the total amount of sediment above \( z \) alternatively leads to:

\[
Q_{+}^z(z, t) = \frac{\partial}{\partial t} \int_{z}^{\infty} C(z', t)dz',
\]  \hspace{1cm} (3.1)

considering the total amount of sediment below \( z \) gives:

\[
Q_{-}^z(z, t) = \frac{\partial}{\partial t} \int_{-\infty}^{z} C(z', t)dz'.
\]  \hspace{1cm} (3.2)

Both expressions rely on the assumption of uniformity in a horizontal plane, which seems reasonable for the sheet flow. When this assumption is satisfied, and if the probes are perfect, the two expressions are equivalent.

The upper part of the integral in Eq. (3.1), above the highest measured concentration time series \( C(Z^+, t) \), are estimated in accordance with the observation of Nielsen (1986) and Dick and Sleath (1991, 1992) that the shape of the upper part of the concentration profiles varies roughly as \( \exp[ -z/(30d_{50}) ] \) in sheet flow, i.e.:

\[
\int_{-\infty}^{\infty} C(z, t)dz \approx -\frac{\partial}{\partial t} \int_{-\infty}^{Z^+} C(z, t)dz
+ 30d_{50}C(Z^+, t)
\]  \hspace{1cm} (3.3)

We see that the vertical fluxes are greatest around the undisturbed bed level \( (Z=0.2 \) mm) where the
concentration varies very little with time, cf. Ribberink et al. (2000), Fig. 7.

The data show strong upward and strong downward fluxes as the free stream velocity, \( u_\infty(t) \), accelerates, while \( u_\infty(t) \) is decelerating (\( \mathrm{d}\left[ u_\infty \right]/\mathrm{d}t < 0 \)). As we shall see below, the upward fluxes can reasonably be well predicted by existing pickup functions. However, the concentrated downward fluxes cannot. These strangely strong downward fluxes are possibly peculiar to sand. That is, Flores and Sleath (1998) noted strangely strong downward fluxes are possibly peculiar to sand. That is, Flores and Sleath (1998) noted

3.2. Predictive models of vertical fluxes near \( z=0 \)

As an alternative to using a prescribed \( C_b \) as the bottom boundary condition, Nielsen et al. (1978) and Van Rijn (1984) suggested the use of pickup functions, i.e., a prescribed vertical flux \( p(t) \) of sediment through the bottom of the modelling domain (usually the undisturbed bed level). The pickup function is then a non-negative function of the Shields parameter \( \theta'(t) \), e.g.:

\[
p(t) = \begin{cases} 
0.017w_0 \left[ \theta'(t) - 0.05 \right]^{1.5} & \text{for } \theta'(t) > 0.05 \\
0 & \text{for } \theta'(t) < 0.05 
\end{cases}
\]

(3.5)

where \( w_o \) is the clear water settling velocity of the sediment, cf. Nielsen (1992), Eq. (5.3.9).

This still presents some problems, however, because little is known about the actual behaviour of the bed shear stress \( \tau(t) \), and hence of \( \theta'(t) \) on a movable sand bed in an unsteady flow. Nielsen (1992) suggested to use a Shields parameter based on a roughness of \( 2.5d_{50} \) for estimating skin friction, and the following expression (his Eq. (2.4.15)) for use with an arbitrary time series of the free stream velocity \( u_\infty(t) \) with time step \( \delta_t \):

\[
\theta_{2.5}(t) = \frac{\frac{1}{2}f_{2.5}A_{rms}}{(s-1)gd_{50}} \left[ \cos \varphi_t \omega_\varphi u_\infty(t) + \sin \varphi_t \frac{u_\infty(t + \delta_t) - u_\infty(t - \delta_t)}{2\delta_t} \right],
\]

(3.6)

where \( f_{2.5} \) is the wave friction factor corresponding to a bed roughness of \( 2.5d_{50} \), \( \omega_\varphi \) is the peak angular frequency, and \( \varphi_t \) is the phase shift between free stream velocity and bed shear stress (45° for laminar flow). \( A_{rms} \) is the near-bed semi excursion of a sine wave motion with the same velocity variance as \( u_\infty(t) \), i.e., \( A_{rms} = \frac{\sqrt{2}}{\omega_\varphi} \times \sqrt{\text{Var}\{u_\infty(t)\}} \).

The expression (3.6) corresponds to laminar-like flows where the shear stress magnitude varies as the first power of the velocity magnitude. A modified version more closely mimicking the turbulent \( \tau \sim u^2 \) rather than the laminar \( \tau \sim u^2 \) behaviour is:

\[
\theta_{2.5}(t) = \frac{\frac{1}{2}f_{2.5}}{(s-1)gd_{50}} \left[ \cos \varphi_t u_\infty(t) + \sin \varphi_t \frac{u_\infty(t + \delta_t) - u_\infty(t - \delta_t)}{2\omega_\varphi \delta_t} \right]^2
\]

(3.7)

Fig. 4 shows corresponding time series of \( u_\infty(t) \) and \( p(t) \) based on Eq. (3.7) inserted into Eq. (3.5) together with the measured vertical sediment flux through the undisturbed bed level for Experiment Mh of Ribberink et al. (2000). The justifications for this approach to \( \theta'(t) \) and its applicability to net sediment transport rates in sheet flow and swash zones is discussed by Nielsen (2002).

We see that the pickup function model [(3.5)+(3.7)] is quite capable of modelling the shape of the entrainment peak during the acceleration of \( u_\infty(t) \). However, for the rest of the wave period, the predicted shape of \( p(t) \) does not match that of the measured \( Q_z \).

By definition, the total sediment flux \( Q_z \) is composed of the upward flux represented by \( p(t) \) near \( Z=0 \) and a settling flux proportional to the local sediment concentration and the settling velocity:

\[
Q_z(z,t) = p(t) - w_zC(z,t)
\]

(3.8)

Thus, although not identical, \( Q_z(z,t) \) and \( p(t) \) should only approximately differ by a constant, if
$C(z,t)$ and $w_s$ are constant, as they indeed are. Fig. 1 shows that the concentrations vary very little with time near $Z=0$, and $w_s$ is assumed to be a function of concentration only. With the last term in Eq. (3.8) being almost constant, the graphs of $Q_z(0,t)$ and $p(t)$ should be almost parallel curves. This is definitely not the case: $Q_z$ shows a strong dip just before $t=2$ s, while the predicted $p(t)$ shows only a much weaker dip.

Features like the secondary flux peak, related to velocity reversal, prominent in Fig. 2 near $t=2$ s, is also not predicted by pickup functions like Eq. (3.5) which is based on the Shields parameter only.

### 3.3. Settling flux near $z=0$

The settling velocity applicable in Eq. (3.8) near $z=0$, where the concentrations are of the order $C_{\text{max}}/2$, is expected to be less than the clear water settling velocity, $w_o$, and that is confirmed by the data. The actual magnitude may be estimated by requiring the right-hand side of Eq. (3.8) to average zero. For Experiment Mh and with the values of $p(t)$ shown in Fig. 3, this leads to $w_s \approx 0.31$ mm/s on the average. The concentrated downward flux peak, however, requires $w_s \approx 2.1$ mm/s. Both values are significantly lower than expected on the basis of the formulae suggested by Richardson and Jeronimo (1979), which, for the conditions in Fig. 4 ($d_{50}=0.245$ mm, $w_o=30$ mm/s, $C=\bar{C}(0) \approx 0.38$), give $w_s \approx 0.2w_o \approx 6$ mm/s. These differences are so large that they will persist even if some plausible changes were made to the calculation of $p(t)$. The difference is perhaps not surprising since Richardson and Jeronimo based their formulae on experiments from very different physical settings from oscillatory sheet flow.

### 3.4. Could the observed vertical fluxes be due to gradient diffusion?

Since most of the existing sediment suspension models are based on the diffusion equation, it is of interest to see if the observed vertical sediment fluxes match such a description.

It has previously been noted (Clark and Nielsen, 1996) that a pure convection model seems to be in
better agreement with sheet flow data, for $0 < Z < 100$ mm, than a pure gradient diffusion model, and we have noted, in connection with Fig. 1, that the observed increase of $R$ with elevation, for $0 < Z < 1$ mm, is contrary to trends predicted by diffusion models. In addition, we shall see in the following that the observed vertical sediment fluxes correspond to very “exotic” sediment diffusivities.

According to the traditional diffusion model, the sand is brought upwards into suspension by gradient diffusion and returned to the bed by settling:

$$Q_z(z, t) = -e_s \frac{\partial C}{\partial z} - w_s C(z, t).$$

(3.9)

The sediment diffusivity, $e_s$, may in general be a function of both elevation and time and the settling velocity may, for a given sand size, be a function of the local concentration $C(z, t)$. If the other quantities have been measured or reliably estimated, this equation can be used to find the required diffusivity for modelling the observed entrainment process:

$$e_s = - \frac{Q_z + w_s C(z, t)}{\frac{\partial C}{\partial z}}.$$  

(3.10)

Based on the measured concentrations at $z = -0.2$ and $+0.2$ mm, the measured $Q_z(0.2 \text{ mm}, t)$-values shown in Fig. 4, and with $w_s = 0.31$ mm/s corresponding to $Q_z(0.2 \text{ mm}, t) = 0$, the required diffusivity behaviour is the same as that in Fig. 5.

The general magnitude and the behaviour of $e_s(0, t)$ during the acceleration phases are sensible enough. However, the strongly negative values of $e_s(0, t)$ during the deceleration after the wave crest would be hard to justify on physical grounds.

While details of the behaviour of $e_s(0, t)$ in Fig. 5 depend on the calculated $p(t)$ and the estimated $w_s(0, t)$, it is not possible to make the non-physical negative $e_s$-values disappear by reasonable changes to the values of $p(t)$ and $w_s(0, t)$. It is therefore fair to conclude that the measurements near the undisturbed bed level lend no support to the gradient diffusion model (3.9).

4. Conclusions

Measured sediment concentration time series from sheet flow conditions under the waves have been compared to the two classical methods of describing sediment entrainment. Both models show qualitative disagreement with the data.

The oldest and probably still the most widely used model type are the $C_b$ models, where the sediment concentration at the bottom of the model domain is prescribed as a function of the bed shear stress. Such models will usually predict quite a strong variation of $C_b(t)$. In contrast, the measured concentration time series close to the undisturbed bed level show almost no variation through the wave period. Perhaps even worse, concentrations at $2d_{50}$ above the instantaneous bed level—as opposed to the undisturbed bed—show a time variation, qualitatively opposite to that of the $C_b$-predicters, Fig. 2. That is, concentrations at $2d_{50}$ above the instantaneous bed dip around the time of maximum $u_\infty(t)$ or maximum $\theta(t)$, while the $C_b$-predicters show a peak. This means that the $C_b$-predicters represent, at best, some nominal quantity—not the physical concentrations.

In contrast to the almost constant concentrations, the vertical sediment fluxes near the undisturbed bed level vary strongly with time. This is in better qualitative agreement with existing pickup function models. However, only part of the variation, namely the upward fluxes associated with $d |u_\infty|/dt > 0$, can be
modelled by existing \( p(t) \)-models. The strong downward fluxes which always occur (with sand) for \( d|\mu_\infty|/dt < 0 \) are not anticipated by existing models. Similarly, the secondary entrainment peaks, which sometimes occur near the velocity reversals (just before \( t = 2 \) s in Fig. 3), are not predicted by pickup functions which only depend on the Shields parameter.

The observed vertical sediment fluxes also yield some insight into the effective settling velocity, \( w_s \), in the high-concentration area \((0.3 < C < 0.4)\) near \( Z = 0\). The data indicates that \( w_s \) is usually only a few percent of the clear water settling velocity, \( w_o \). This is several times smaller than predicted from the commonly used formulae of Richardson and Jeronimo (1979).

With respect to the validity of the commonly used gradient diffusion model, it is noted that it is in disagreement with the qualitative behaviour of the relative variation \( [\text{Var}\{C(t)/C\}] \) close to the undisturbed bed, Fig. 1. The data shows upward increasing time variation while diffusion models necessarily predict decreasing variation. Also, the hypothetical sediment diffusivity, which would be required to deliver the measured concentrations, is not physically plausible. Hence, gradient diffusion is not the appropriate conceptual framework in this layer \((z < 5d_{50})\). The requirement of unrealistic diffusivities displayed by Fig. 5 for one test is a general feature for all of the sand experiments of Ribberink et al. (2000).

Whether any of the existing granular flow models, e.g., Li and Sawamoto (1995a,b), Ono et al. (1996), Katori et al. (1996), Kaczmarek and Ostrowski (1996, 1998) and Dong and Zhang (1999), are able to model the observed fluxes in detail is an interestingly open question. It might be particularly challenging to model the strong downward fluxes which are observed with sand but not with lighter materials (Flores and Sleath, 1998). Some of these models seem to predict fairly constant concentrations near \( z = 0 \), in agreement with measurements.

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