Letter

Results of Jenike's (1987) radial stress field theory for the flow of granular materials in conical hoppers: flow regimes and flow factors

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Introduction

An important aspect of the design of silos for the storage of bulk solids is the correct prediction of the occurring flow regime. Two flow regimes are usually distinguished when material is withdrawn from the outlet: mass flow where all bulk material is in motion, and funnel flow where stagnant zones form in the hopper. Funnel flow is desirable for abrasive materials, while mass flow is desirable for materials that deteriorate in time. Usually mass flow is preferred because of the smoother operation it provides, contrary to funnel flow where flow is often erratic. The flow regime also has a large influence on the occurring wall pressures.

The flow regime is mainly determined by the geometry of the hopper (hopper half-angle α), the bulk material properties (effective internal friction angle δ) and the interaction between hopper wall and bulk material (wall friction angle ϕ_w).

Jenike's theory [1] is almost exclusively used to predict the flow regime for a given hopper angle α, wall friction angle ϕ_w and effective internal friction angle δ. Jenike [2] noted that this theory is not satisfactory, since the predicted flow regime is regularly incorrect. For this reason Jenike [2] modified his original theory by replacing the hexagonal Mohr–Coulomb yield criterion by a conical yield surface. For wedge-shaped hoppers the results of both theories are identical.

For the case of conical hoppers, predictions by the new theory for the mass and funnel flow regions in an (α, ϕ_w) diagram are presented here for various values of the effective internal friction angle δ.

Another result of radial stress field theories is the flow factor [1]. This flow factor determines, together with the flow function [1] which characterizes the material strength, the minimum outlet diameter that is required to prevent arching in hoppers of silos. Results for the flow factors according to Jenike's (1987) theory are presented.

Jenike's new (1987) theory

Here Jenike's (1987) theory [2] will be briefly outlined. Like Jenike's old theory [1], this new theory is based on the quasi-static equilibrium conditions. The material is considered as a rigid-plastic frictional continuum. The employed yield criterion is a conical yield criterion:

$$\mathrm{tr}(\mathrm{dev}(T)\cdot\mathrm{dev}(T)) = 2(\sin\,\delta)^2\sigma^2 \tag{1}$$

where T is the stress tensor, $\mathrm{tr}(T)$ and $\mathrm{dev}(T)$ respectively denote the trace and the deviator of tensor T and σ is the mean stress. Stress tensor T and strain rate tensor D are related by the (coaxial) Levi flow-rule:

$$\mathrm{dev}(T) = \lambda\mathrm{dev}(D) \tag{2}$$

where λ is a positive scalar.

The spherical coordinate system used (r, θ, χ) is shown in Fig. 1. The associated velocity vector is $(u_r, u_\theta, u_\chi)^\mathrm{T}$. Let σ be the mean stress and ψ the inclination of major principal stress direction to the r-coordinate. For conical hoppers the assumption of a radial stress and velocity field means:

$$\sigma(r, \theta) = \rho g r s(\theta) \tag{3}$$

$$\psi(r, \theta) = \psi(\theta) \tag{4}$$

$$u_r(r, \theta) = \frac{f(\theta)}{r^2} \tag{5}$$

$$u_\theta = 0 \tag{6}$$

$$u_\chi = 0 \tag{7}$$

where ρ is the bulk density of the material and g is the acceleration due to gravity. This assumption of a radial stress and velocity field is only valid near the hopper outlet.
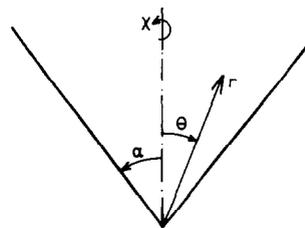

Fig. 1. Definition of the coordinate system.

With the additional assumption of a constant bulk density ρ, the following relation between $f(\theta)$ and $\psi(\theta)$ was derived in [2]:

$$\frac{f(\theta)}{f(0)} = \exp\left(-3\int_0^\theta \tan(2\psi(\theta))\,d\theta\right) \tag{8}$$

Jenike's two theories differ in the employed yield criterion and flow-rule. In the old theory the material satisfies the Mohr–Coulomb yield criterion, and the circumferential stress is determined from the Haar–von Karman hypothesis. In contrast, the new theory is based on the conical yield criterion and the Levi flow-rule.

The assumption of radial stress and velocity fields leads to two ordinary nonlinear differential equations in σ and ψ. Fully developed wall friction at the hopper wall and the symmetry condition at the centre line determine the boundary conditions.

Here these two differential equations, together with the boundary conditions, are solved numerically by a Runge–Kutta method using adaptive step-size control [3] in combination with a shooting method.

Concerning the yield criterion used in the new theory, it should be noted that true-triaxial test data on dense sand [4] seem to indicate that the yield surface is best described by the hexagonal Mohr–Coulomb yield surface, as used in Jenike's original theory.

Mass and funnel flow regions

According to Jenike's new theory mass flow occurs when the radial velocity is always positive. This is the case if $90° < \psi(\theta) < 135°$.

Using this criterion and the results of the numerical calculations, the mass flow regions have been determined. These regions are plotted in Figs. 2–4, for $\delta = 30°$, $40°$ and $50°$. Also depicted are the mass flow regions according to Jenike's old theory.

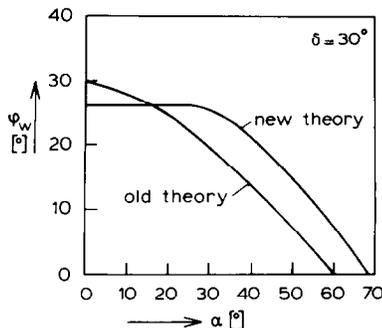Comparison of the mass flow regions of the old and the new theory shows that the mass flow region according

Fig. 2. Boundary line between mass and funnel flow for Jenike's old and new theory; effective internal friction angle $\delta = 30$ °.
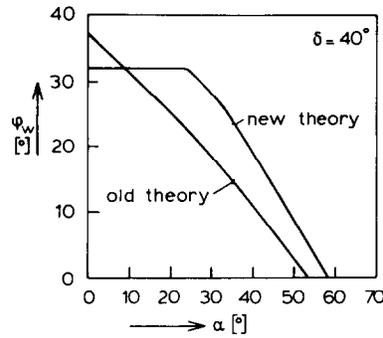

Fig. 3. Boundary line between mass and funnel flow for Jenike's old and new theory; effective internal friction angle $\delta = 40$ °.
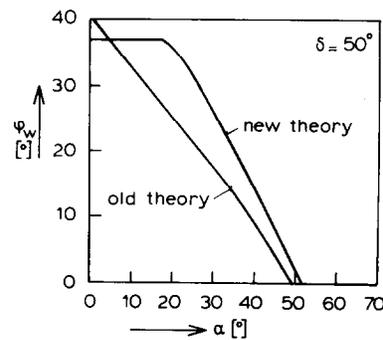

Fig. 4. Boundary line between mass and funnel flow for Jenike's old and new theory; effective internal friction angle $\delta = 50$ °.

to the new theory is roughly 8° 'larger' than that according to the old theory.

Comprehensive experiments of the occurring flow regimes have been reported in [5]. It was found that Jenike's old theory does not predict the flow regime accurately, while Jenike's new theory gives a better prediction of the flow regime. Another interesting aspect of these experiments is the dependence of the flow regime on the material level in the bin: below a critical height mass flow changes to funnel flow. It is not possible to explain this phenomenon with a radial stress field theory.

Flow factors

The flow factors, as defined by Jenike [1], are widely used in the design of silos to determine the critical outlet diameter D_c. This is the minimum outlet diameter that is required to prevent arching in the hopper of the silo.

The flow factor ff is determined from the stress factor at the wall $s(\alpha)$:

$$ff = s(\alpha)\,\frac{1+\sin(\delta)}{2\sin(\alpha)}\,H(\alpha) \tag{9}$$

where $H(\alpha)$ is a geometrical factor given in [6]. The critical outlet diameter D_c depends on the unconfined

yield strength σ_p of the bulk material. This unconfined yield strength σ_p is assumed to depend on the major principal consolidation stress σ_1 by:

$$\sigma_p(\sigma_1) = X + Y\sigma_1 \qquad (10)$$

The critical outlet diameter is determined from:

$$D_c = \frac{X}{1 - ffY} \frac{H(\alpha)}{\rho g} \qquad (11)$$

The flow factors ff were determined using the numerically computed radial stress field functions. They are plotted in Figs. 5–7, for $\delta = 30°$, $40°$ and $50°$.

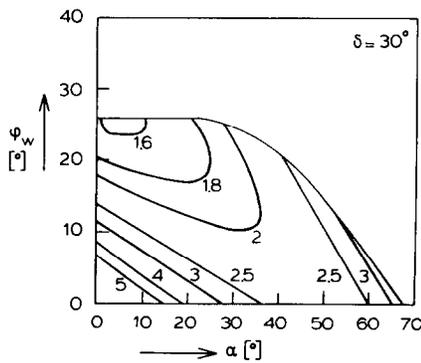Comparison of these flow factors with those given in [1] shows that the new flow factors are higher. This

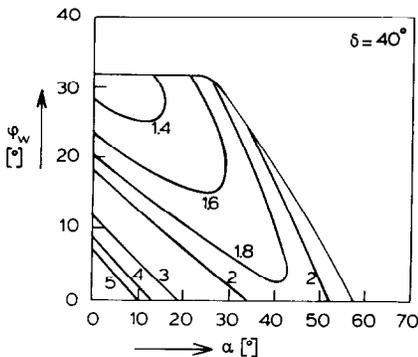Fig. 5. Flow factors; effective internal friction angle $\delta = 30$ °.

Fig. 6. Flow factors; effective internal friction angle $\delta = 40$ °.
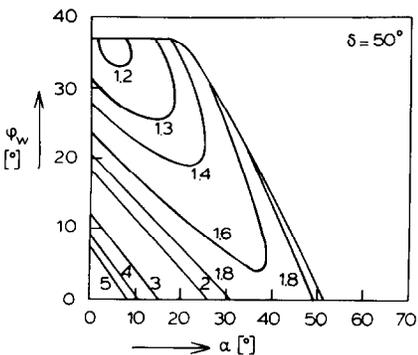

Fig. 7. Flow factors; effective internal friction angle $\delta = 50$ °.

implies that the new critical outlet diameter D_c is larger than that predicted by the old theory. Experiments reported in [7] show that even the old theory over-estimates the critical outlet diameter D_c. So in this aspect the new theory is not an improvement.

Discussion

Jenike's old theory does not give accurate predictions of mass and funnel flow regions. The mass and funnel flow regions according to Jenike's new theory are presented here. An advantage of Jenike's new theory is that it leads to an improved prediction of mass and funnel flow regions for conical hoppers.

Jenike's old theory gives conservative estimates of the critical outlet diameter. The flow factors according to Jenike's new theory are given here. These flow factors are higher than according to Jenike's old theory. Hence, a disadvantage of Jenike's new theory is that it gives an even more conservative prediction of the critical outlet diameter.

This means that Jenike's new theory does not remedy *both* defects, incorrect prediction of flow regime and overestimation of the critical outlet diameter, of Jenike's old theory.

An interesting possibility to try to remedy both defects within the context of radial stress field theories is the adoption of other yield functions and flow rules, as suggested in [8].

List of symbols

D strain rate tensor, s^{-1}
D_c critical diameter required to prevent arching, m
$f(\theta)$ function of θ defined by eqn. (5), –
ff flow factor, –
g acceleration due to gravity, m s^{-2}
$H(\alpha)$ shape factor, –
r radial coordinate, m
$s(\theta)$ function of θ defined by eqn. (3), –
T stress tensor, N m^{-2}
u_χ azimuthal velocity, m s^{-1}
u_r radial velocity, m s^{-1}
u_θ zenithal velocity, m s^{-1}
X constant in eqn. (10), N m^{-2}
Y constant in eqn. (10), –

Greek letters

α hopper half-angle, °
δ effective internal friction angle, °
ϕ_w wall friction angle, °
χ azimuthal coordinate, °
λ positive scalar in the flow-rule eqn. (2), N m^{-2} s

ψ inclination of the major principal stress direction to the r-direction, °

ρ bulk density, kg m^{-3}

σ mean stress, N m^{-2}

σ_1 major principal stress, N m^{-2}

σ_p unconfined yield stress, N m^{-2}

θ zenithal coordinate, °

References

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