A Systematic Approach to Circuit Design and Analysis: Classification of Two-VCCS Circuits

Eric A. M. Klumperink

Abstract—**This paper discusses a systematic approach to the design and analysis of circuits, using a transconductor or voltage controlled current source (VCCS) as a building block. It is shown that two independent Kirchhoff relations among the VCCS voltages and currents play a crucial role in establishing a unique transfer function in two-port circuits with two VCCS's. A class of two-VCCS circuits is defined, which can be subdivided into three main classes and 14 subclasses, based on different imposeable sets of two Kirchhoff relations. The classification is useful for circuit synthesis and analysis, as it reveals all the basically different ways to exploit two VCCS's, and allows for a unified analysis of classes of circuits. To exemplify this, all complementary metal–oxide–semiconducter (CMOS) $V - I$ converter kernels, based on two matched MOS transistor (MOST)-VCCS's, are generated and analyzed with respect to distortion. It is shown that dozens of published transconductor circuits can be classified in only four classes, with essentially different distortion behavior.**

Index Terms—**Circuit analysis, circuit synthesis, CMOS analog circuits, transconductors.**

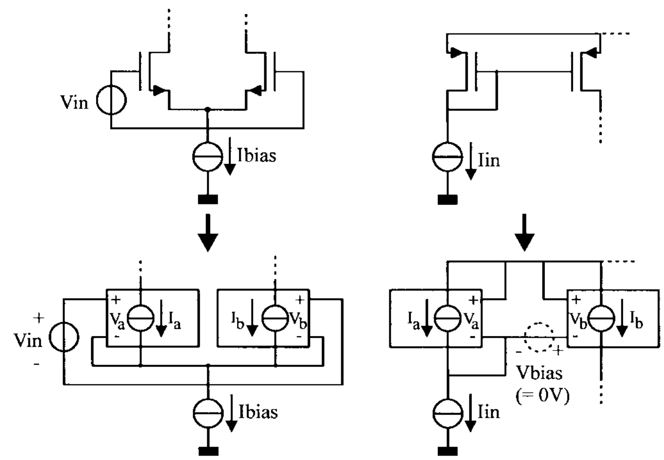

Fig. 1. Well-known examples of circuits with a transfer function determined by the transconductances of MOS transistors: the differential pair and current mirror.

I. Introduction

LINEAR circuits are indispensable in electronic systems, e.g., for amplification and filtering. Such circuits are usually designed by experienced analog designers, using a largely intuitive design approach. Based on past experience, one or a few known circuit topologies are often reused and adapted to fit a specific application [1]. However, the intuitive design method also has the following disadvantages.

1) Known circuit topologies are favored, although there may be alternative topologies that are more suitable. Innovations in circuit topology largely depend on fortuitous insights of designers.
2) It takes a long time for a designer to become experienced, and it is not clear how an intuitive design approach should be taught to new designers. Furthermore, the intuitive approach is not very suitable for implementation in a CAD system.

These observations argue for the development of design methodologies that explore the design space in a more systematic way. A recent Ph.D. thesis [2] addresses this subject for circuits in which the transconductance of metal–oxide–semiconducter (MOS) transistors is exploited (hence transconductance-based circuits). Well-known simple examples are the differential pair and current mirror (Fig. 1). However, many other complementary MOS (CMOS) circuits can be considered as transconductance-based circuits (e.g., [3]–[36]).[1]

Most publications on transconductance-based CMOS circuits focus on specific aspects of one particular circuit. To the knowledge of the author only a few papers try to classify and compare different approaches [20], [21], [27]. Moreover, if a classification is made, it is often on a rather *ad hoc* basis. The present paper aims at a systematic generalized approach. It is shown that this is possible by focusing on the functional kernel of circuits, using a voltage controlled current source (VCCS) as an abstraction. Looking at circuits from this abstract point of view, only a few essentially different operating principles remain. The present paper proposes a formal classification system that can be used to clearly identify this operating principle. It covers all circuits with a functional kernel that can be modeled by two VCCS's and interconnections. Although this may seem a serious restriction, it is shown in [2] that most commonly required linear two ports with a $V-I$, $I-I$, $I-V$, and $V-V$ transfer function can be approximated with only two VCCS's (see also Section II) and that the classification system covers hundreds of MOST circuit topologies. The classification that will be proposed renders an overview of all possibilities of exploiting two VCCS's and allows for a unified analysis

Manuscript received July 7, 1997; revised September 22, 1998. This paper was recommended by Associate Editor D. Zhou.

The author is with the MESA Research Institute, University of Twente, 7500AE, Enschede, The Netherlands.

Publisher Item Identifier S 1057-7122(99)05580-4.

[1] Although this paper mainly discusses MOS circuits, many circuits exploiting bipolar transistors and/or resistors can also be considered as transconductance-based circuits.

of classes of circuits. It will be shown that this provides a powerful means for systematic circuit synthesis and analysis.

The paper is organized as follows. The design philosophy behind transconductance-based CMOS circuits will be reviewed in Section II. In Section III, the role of Kirchhoff relations in establishing a unique transfer function in circuits with two VCCS's is examined. Based on these Kirchhoff relations a classification system with three main classes and 14 subclasses of circuits is proposed in Section IV. Section V illustrates the usefulness of the classification in some elementary circuit synthesis and analysis examples. A more elaborate example is presented in Section VI: all transconductor VI kernels with two matched MOST-VCCS's are generated, in search of kernels with essentially different distortion behavior. Finally, in Section VII conclusions are drawn.

II. A Design Philosophy: Transconductance-Based CMOS Circuits

A. Motivation

The following observation was taken as a starting point for a design philosophy for MOST circuit design. Analog circuits are commonly designed using very simple circuits as building blocks (e.g., differential pairs and current mirrors). Apart from the elegance of simple solutions, there are some other good reasons for this design practice. Adding components tends to limit the high-frequency potential of circuits (few or no internal nodes, e.g., [15]) and tends to increase the noise level and power consumption. Hence, squeezing maximal functionality out of a minimal set of components seems to be a viable design philosophy.

In CMOS circuits, transistors are the main components. The primary transistor property that is exploited in many circuits, is the transconductance of a saturated MOS transistor. Consequently, the principle of operation of many CMOS circuits can be understood by considering them as transconductance-based circuits [2], i.e., circuits with a transfer function that is mainly determined by the transconductance values. Important reasons for the frequent use of transconductance-based CMOS circuits are the following.

1) The fact that a VCCS is a good model for a MOS transistor in a broad-frequency band. This is a major reason for the use of transconductance-C filters at high frequencies [38]. Moreover, the simplicity of transconductance-based circuits often comes with good high-frequency performance.
2) The transconductance of a MOST depends, in general, on its biasing point. This electronic variability allows for on-chip self-correction for spread in IC-processing and temperature variations (e.g., f-tuning in Gm-C filters [37], [38]). Moreover, it enables adaptive signal processing (e.g., AGC [39]).
3) The matching of the transconductance values of equally biased MOST's can be good (better than 0.5% current matching is possible [41]). This allows for accurate current gains (e.g., current mirror).

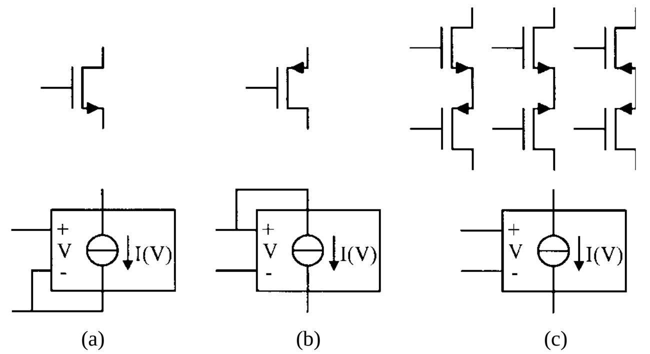

Fig. 2. (a) A single NMOST and (b) PMOST can be represented as a VCCS with a connection between a voltage and current terminal. By combining two of them (c) a VCCS with floating input and output is created (bias sources have been omitted).

4) The large range of transconductance values that the transconductance of a MOST can take (e.g., $1nS \cdots 1S$ [2]), which is much larger than for integrated resistors.
5) The fact that transconductor circuits often render minimum complexity implementations of a certain function, since a single transistor can often implement a transconductor function. This is, for instance, important in massively parallel analog neural networks [40].

B. Formal Modeling Using Two VCCS's

Formally, a MOST can now be modeled as a VCCS, as is shown in Fig. 2(a) and (b) for an NMOST and PMOST.[2] Unfortunately, in both cases there is a connection between one of the voltage and one of the current terminals, which limits the flexibility of use. However, by using two MOST's a more flexible VCCS with a floating input and output port can be implemented, as shown in Fig. 2(c). This VCCS is used as a building block for circuit synthesis. The use of this abstraction introduces hierarchy in the design. Many different MOS circuit topologies (low abstraction level) can be considered as different implementations of a single (higher abstraction level) VCCS-circuit topology. For instance, a VCCS with one connection between a voltage and current terminal can be implemented by a single NMOST [Fig. 2(a)], a single PMOST [Fig. 2(b)], or by a pair of MOST's with one connection added [Fig. 2(c)]. Hence, many different circuits can be viewed as variations on a theme.

A VCCS model fits well to the function of a MOST in many circuits, and in many cases it also fits well to a bipolar transistor. Moreover, it can also be used to model conductances which may be used as well (e.g., passive resistors or triode MOST's). Finally, more sophisticated transconductor circuits can be used if required, e.g., if tight requirements on the linearity are posed.

Two VCCS's are at least needed to implement nonunity V–V and I–I transfer functions (determined by a ratio of two transconductance values) apart from V–I and I–V relations. With two VCCS's as generating elements a graph-based exhaustive circuit topology generation is performed in [2],

[2]A generalized treatment of transconductance-based circuits is pursued, grasping their main features. The body effect of a MOS transistor is considered as a second-order effect and is not taken into account in the first-order model.

leading to 145 graphs of linear[3] two ports with two VCCS's. It appears that two ports with infinite- or finite-valued port impedances can be implemented directly, while two ports with zero port impedances can be approximated for large transconductance values [2]. The set of topologies includes most commonly required linear two ports. Each of these graphs can be implemented in several different ways, using combinations of transistors and resistors so that many hundreds of different circuits are covered.

Summarizing, it has been argued that a VCCS is a very useful model for MOST circuit synthesis and that even with two VCCS's a large class of circuits is covered.

III. The Role of Kirchhoff Relations in 2VCCS Circuits

As discussed in the previous section, many two-port circuits with two VCCS's can be constructed. Since VCCS's are the only components, the two-port parameters of these circuits are determined by transconductance values. Moreover, of course, the interconnections between the VCCS's are important. It will now be shown that two Kirchhoff relations play a crucial role, constituting a suitable basis for a classification system. To clarify the discussion, first some basic assumptions and conventions will be stated and explained in Section III-A. Then the role of Kirchhoff relations is considered in Section III-B, while the different possibilities of imposing Kirchhoff relations are discussed in Section III-C.

A. Basic Assumptions and Conventions

1) Two-Port Circuits with Two Interconnected VCCS's: As was discussed in Section II, circuits with two interconnected VCCS's, each having a floating input and output port, are the subject of this paper.

2) $I(V)$ Characteristic and VCCS Variables: A VCCS has an $I(V)$ characteristic that is, in general, nonlinear. Used in a linear two-port, the small signal transfer in a quiescence point (V_0, I_0) is exploited, which can be approximated as

$$I(V) = I(V_0+v) = I_0(V_0)+i(v) = I_0(V_0)+g(V_0)\cdot v \qquad (1)$$

where v and i are the small-signal voltage and current excursions from the biasing point and $g(V_0)$ is the transconductance dI/dV, which depends, in general, on the biasing point (V_0, I_0).

Only one of the variables V and I of the VCCS is independently controllable, the other is dependent according to (1). Of course, only the input voltage of a VCCS can directly be controlled while imposing a current involves feedback to the voltage terminals.

Two equivalent VCCS's are assumed that will be indicated as VCCS$_a$ and VCCS$_b$ and (1) will be used with indexes a and b (the name assignment is arbitrary, but will be standardized later). Hence, the following VCCS variables will be used: V_a, I_a, V_b, and I_b.

[3]Two ports with nonlinear V–V, V–I, I–V, and I–I characteristics can also be generated [2].

3) Voltage- or Current-Driven Input-, Open-, or Short-Circuited Output: It will be assumed that the circuits have one signal input and output (two port). Independent voltage or current sources will be used to supply the input signal and, if necessary, bias the circuit (for nonlinear VCCS's). The signal output will be presumed to be either open or short circuited. Thus, either zero or infinite source and load impedances are assumed, and biasing by voltage or current sources. This simplifying assumption is often legitimate, as designers often aim at voltage driving or current driving. Moreover, of course, linear two ports driven by a source with finite impedance and/or loaded by a finite impedance can be analyzed, using the results for the cases with zero and infinite impedance.

Since the biasing affects the transconductance of the VCCS's, and thus controls the transfer function of a circuit, the biasing variables will be called control variables from now on. Sometimes it will be convenient to talk about input, control, and output signals, regardless of whether voltage or current variables are concerned. For this purpose, the variables $S_{\rm in}$ (input), S_c (control), and $S_{\rm out}$ (output) will be used.

4) Unique Solution for VCCS Variables: Linear signal-processing functions are often designed, using circuits behaving like two ports for which $S_{\rm out}$ is a continuous function of $S_{\rm in}$ (for every value of $S_{\rm in}$ there exists one and only one value of $S_{\rm out}$). As a result, a unique transfer function $\delta S_{\rm out}/\delta S_{\rm in}$ exists. As VCCS's are presumed to be the only components available to implement a unique transfer function, it will be required that a unique solution exists for V_a, V_b, I_a, and I_b, which will be referred to as VCCS variables. Only the VCCS variables are relevant for implementing a transfer function. The input current of a voltage sense branch of a VCCS is zero (by definition) and, hence, does not contribute to $S_{\rm out}$, while the voltage across a controlled current source can take arbitrary values and, hence, cannot aid in establishing a unique transfer function.

B. Two Crucial Kirchhoff Relations

In general, two types of relations determine the voltages and currents in a network: 1) element equations and 2) Kirchhoff relations. It will now be shown that two Kirchhoff relations play a crucial role in establishing a unique solution for the VCCS variables which are the key to $S_{\rm out}$. For simplicity, the case of linear VCCS's will first be discussed. If linear VCCS's are assumed, biasing sources can be omitted (S_c put to zero), as the transfer function does not depend on biasing. This leaves us with the circuit shown in Fig. 3 which is a linear circuit comprised of six ports: a one port at the input (independent voltage source or current source), a one port at the output (short circuit or open) and two two ports (two VCCS's). The solution for all the 12 port variables (six port voltages and six port currents) requires 12 independent linear relations. As six element equations are available: 1) $I_a(V_a)$, 2) $I_b(V_b)$, 3) either an independent input voltage or current; 4) either $V_{\rm out} = 0$ or $I_{\rm out} = 0$; 5) input current is zero for VCCS$_a$; 6) idem for VCCS$_b$), six Kirchhoff relations are required to solve all variables. However, it is possible to calculate the solution for

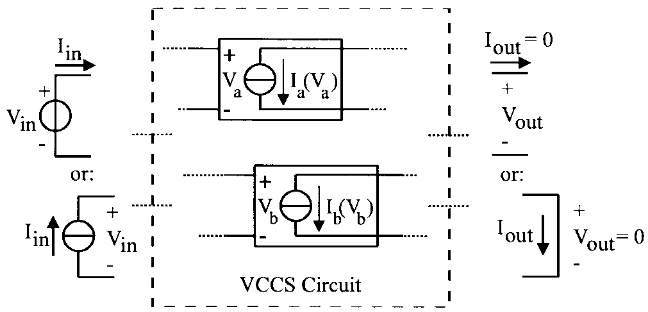

Fig. 3. Schematic representation of a two port consisting of two VCCS's with a short-circuited or open output and driven by a voltage or current source.

the VCCS variables from a smaller set of equations, because of the following.

1) As discussed in the previous section, the voltages across the controlled current sources can take arbitrary values and, hence, do not put constraints on the VCCS variables (i.e., skip two relations).
2) The voltage across an independent current source or open output and the current through an independent voltage source or short circuit can take arbitrary values and, hence, do not put constraints on other variables (i.e., skip two relations).
3) The current in the voltage-sense branches of the two VCCS's is zero (i.e., eliminate two variables).
4) Either $V_{\rm out}$ or $I_{\rm out}$ is zero (open or short output) (i.e., eliminate one variable).
5) The independent source value can be substituted for either $V_{\rm in}$ (voltage source) or $I_{\rm in}$ (current source) (i.e., eliminate one variable).

Together, this leaves us with only four equations, $I_a(V)_a)$, $I_b(V_b)$ and two Kirchhoff relations among the four VCCS variables (four unknowns) and independent source variables. If these two Kirchhoff relations are independent, four independent linear relations in four unknowns are available, which is necessary and sufficient for a solution. The other relations mentioned above either do not impose constraints on the VCCS variables or force them to zero or to a (known) independent source value.

If the $I(V)$ relations are nonlinear, two independent Kirchhoff relations can also be sufficient for a solution, but alternatively no or multiple solutions may exist. In [2] it is shown that a unique solution exists in almost all cases[4] with a square-law (half-parabolic) and exponential $I(V)$ characteristic. Hence, the above derivation is also useful for these nonlinear cases (and probably more). The main difference is that additional independent sources are introduced, appearing in the Kirchhoff relations. These sources bias the VCCS's, and act as control variables (change transconductance and, hence, transfer parameters). Fig. 4 illustrates the role of Kirchhoff relations. Two Kirchhoff relations establish a unique function from $S_{\rm in}$ to the VCCS variables. A further Kirchhoff relation assigns a linear combination of VCCS variables and the input variable

[4]The V_Δ, I_Δ, and V_Σ, I_Σ class sometimes render no solution or two solutions.

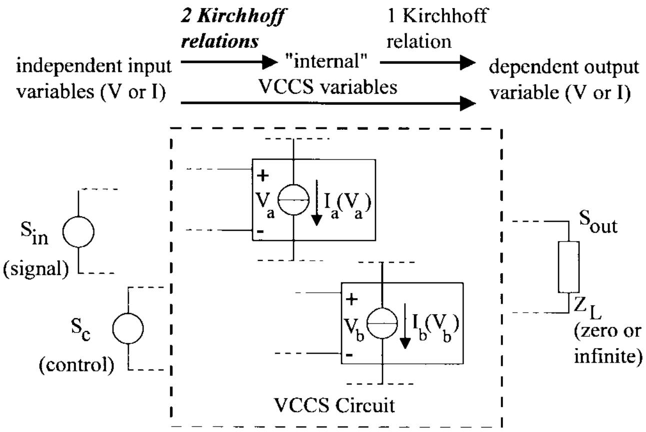

Fig. 4. Two Kirchhoff relations establish a unique relation between the independent variables $S_{\rm in}$ and S_c (S denotes a voltage or current) and the VCCS variables. A third relation assigns a linear combination of independent and VCCS variables to output $S_{\rm out}$.

to the output variable. However, the latter relation does not affect the VCCS variables.

Since two independent Kirchhoff relations play such a crucial role, they constitute a suitable classification criterion. The Kirchhoff relation that assigns the output variable ($V_{\rm out}$ or $I_{\rm out}$) will not be taken into account in the classification, as the core circuit essentially works the same, independent of the output variable that is used [loosely speaking, this Kirchhoff relation can be viewed as a selector of (a linear combination of) variables that are ready for use].

Summarizing, it has been shown that two Kirchhoff relations among the VCCS variables and independent input variables (Fig. 4) play a crucial role in establishing a transfer function for a two port with two VCCS's and, hence, constitute a suitable basis for a classification system.

C. Different Kirchhoff Relations

Before dealing with the actual classification, different types of possible Kirchhoff relation need to be considered. Considering first KVL, the relation between the VCCS voltage variables V_a and V_b and an independent voltage (or sum of voltages) $V_{ind.}$ can be written in the following general form:

$$\alpha_a V_a + \alpha_b V_b = V_{\rm ind.} \tag{2}$$

$$\alpha_a,\, \alpha_b, \quad \in \quad \{\{-1,\,0,\,1\} | \alpha_a \neq 0 \vee \alpha_b \neq 0\} \tag{3}$$

where α_a and α_b indicate how the corresponding VCCS voltages are connected in the voltage loop. The values -1 and 1 allow for different orientations of the positive and negative voltage terminals of the VCCS's. The value zero indicates that the voltage terminals of a VCCS do not occur in the voltage loop. This leads to eight possible equations. Fortunately, this number can be reduced because of the following.

1) Many relations differ only in sign. Such a change of sign corresponds to exchanging the terminals of the independent source. Obviously, this does not change the transfer characteristics of the VCCS.
2) Some relations are equivalent. Forcing VCCS voltage V_a is equivalent to forcing V_b since the VCCS's are equivalent. The name assignment is arbitrary, yet necessary,

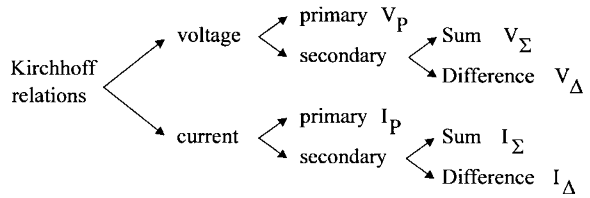

Fig. 5. Overview of possible KVL and KCL relations amongst VCCS variables.

to relate mathematical analysis results in a unique way to a circuit topology.

3) Some relations are not independent relations between the VCCS variables and independent sources. Thus they do not help to establish a solution.

Eliminating these cases, it appears that three different KVL relations can be imposed, that can be divided into primary and secondary ones in the following ways.

1) Forcing a single VCCS control voltage (by convention V_a): $V_P = V_a$. (P stands for *primary* variable, and refers to a input voltage or output current of a VCCS)
2) Forcing the sum of the VCCS control voltages $V_\Sigma = V_a + V_b$ or difference $V_\Delta = V_a - V_b$ (sums or differences of primary variables will be referred to as *secondary* variables).

For currents, a similar reasoning can be followed, leading to forcing I_P, I_Σ, and I_Δ. Fig. 5 schematically shows all resulting possibilities.

IV. Classification Based on Different Sets of Two Kirchhoff Relations

As mentioned in the previous section, the set of two Kirchhoff relations that establishes a unique solution for the VCCS variables constitutes a suitable classification criterion. The different possible sets of two Kirchhoff relations will now be determined and used as a classification criterion. Since two relations are needed, while KVL and KCL relations are available, the following three main classes can be distinguished:

1) sets with two KVL relations: the $\{V, V\}$ *class*;
2) sets with two KCL relations: the $\{I, I\}$ *class*;
3) sets with one KVL and one KCL relation: the $\{V, I\}$ *class*.

Three possibilities for both KVL and KCL can be used: the primary, sum, or difference relation. Since the two relations should be independent, only sets of two different KVL or KCL relations are to be considered. Furthermore, the case with two primary relations need not be considered, since in that case the circuit can be separated in two independent circuits with a single VCCS. Such 1VCCS circuits can be classified in two classes: the $\{V\}$ and $\{I\}$ class, corresponding to forcing the VCCS voltage or current. However, if at least one of the Kirchhoff relations is a secondary one, the circuit can no longer be divided into two one-VCCS circuits and will be designated as a two-VCCS circuit.

Together, three main classes with 14 subclasses are found, as shown in Table I: three subclasses for the $\{V, V\}$ and $\{I, I\}$ class and eight for the $\{V, I\}$ class. The variables of VCCS_a

TABLE I
Overview of all Classes of Two-VCCS Circuits That Can Be Classified in Three Main Classes and 14 Subclasses Based on Different Sets of Two Imposeable KVL and/or KCL Equations

Main classes	**Subclasses**	$\mathbf{V_{ind1}=...}$	$\mathbf{V_{ind2}=...}$
	$\{V_P, V_\Sigma\}$	V_a	V_a+V_b
$\{V,V\}$	$\{V_P, V_\Delta\}$	V_a	V_a-V_b
	$\{V_\Sigma, V_\Delta\}$	V_a+V_b	V_a-V_b
	Subclasses	$\mathbf{I_{ind1}=...}$	$\mathbf{I_{ind2}=...}$
	$\{I_P, I_\Sigma\}$	I_a	I_a+I_b
$\{I,I\}$	$\{I_P, I_\Delta\}$	I_a	I_a-I_b
	$\{I_\Sigma, I_\Delta\}$	I_a+I_b	I_a-I_b
	Subclasses	$\mathbf{V_{ind}=...}$	$\mathbf{I_{ind}=...}$
	$\{V_P, I_\Sigma\}$	V_a	I_a+I_b
	$\{V_P, I_\Delta\}$	V_a	I_a-I_b
	$\{V_\Sigma, I_P\}$	V_a+V_b	I_a
$\{V,I\}$	$\{V_\Delta, I_P\}$	V_a-V_b	I_a
	$\{V_\Sigma, I_\Sigma\}$	V_a+V_b	I_a+I_b
	$\{V_\Sigma, I_\Delta\}$	V_a+V_b	I_a-I_b
	$\{V_\Delta, I_\Sigma\}$	V_a-V_b	I_a+I_b
	$\{V_\Delta, I_\Delta\}$	V_a-V_b	I_a-I_b

have always been assigned a positive sign. As a result, α_a in (2) is always equal to one, which eliminates this variable. A similar argument holds for β_a in KCL relations.

If a circuit is to be classified, the set of KVL and/or KCL equations must be determined by writing equations in terms of VCCS variables and independent source variables. For example, for the differential pair in Fig. 1(a), this leads to

$$V_{\text{in}} = V_a - V_b \quad (4)$$

$$I_{\text{bias}} = I_a + I_b \quad (5)$$

thus, it belongs to the $\{V, I\}$ class and $\{V_\Delta, I_\Sigma\}$ subclass. In a similar fashion it can be verified that the current mirror in Fig. 1(b) belongs also to the $\{V, I\}$ class, yet to the $\{V_\Delta, I_P\}$ subclass.

In summary, it has been shown that two independent Kirchhoff relations among the VCCS variables and independent source variables play a crucial role in establishing a unique solution for the VCCS variables in two-VCCS circuits. Based on different sets of two independent imposeable Kirchhoff relations, two-VCCS circuits can be classified in three main classes and 14 subclasses.

V. Application Example I: Circuit Synthesis and Analysis

The usefulness of the classification will now be illustrated by means of some simple circuit synthesis and analysis examples.

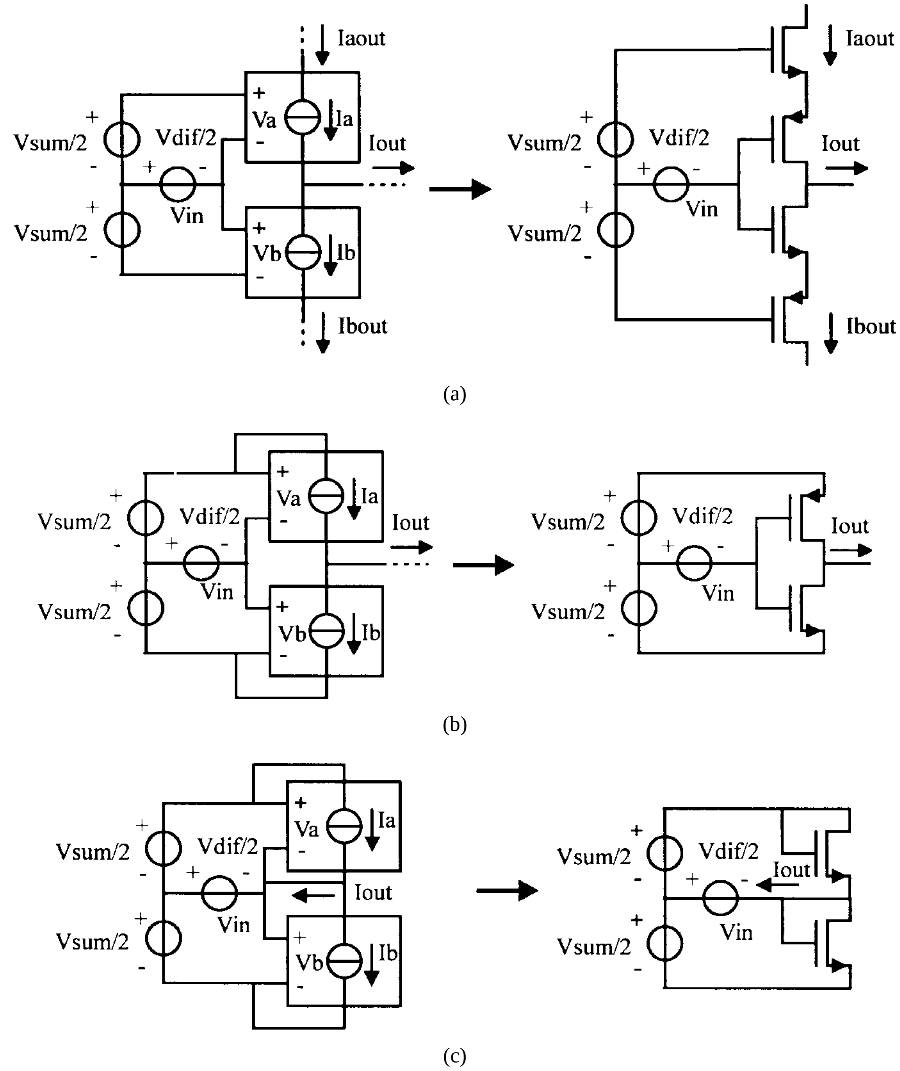

Fig. 6. Different implementations of $\{V_\Sigma, V_\Delta\}$ circuits resulting in: (a) the transconductor proposed in [10]; (b) the same in [15]; and (c) the I–V converter proposed in [12].

A. $\{V_\Sigma, V_\Delta\}$ *Circuit Synthesis*

As an example of circuit synthesis it will be shown how some well-known $\{V_\Sigma, V_\Delta\}$ circuit topologies [10], [15], [12] shown in Fig. 6 can be found by systematically considering different design options. The VCCS schematics at the left in Fig. 6 show three circuits in which the sum $V_a + V_b$ is forced equal to $V_{\rm sum}$ (by two sources $V_{\rm sum}/2$), while the difference $V_a - V_b$ is forced equal to V_{dif} (source $V_{dif}/2$). The three VCCS circuits differ in the available output variables. In Fig. 6(a), the individual VCCS currents I_a and I_b and their difference $I_{\rm out} = I_a - I_b$ are all available as output variables. This requires a VCCS with separated voltage and current terminals, which can be implemented by a complementary pair of MOST's (Fig. 2(c), [10]). If only the difference $I_{\rm out}$ is needed as the output signal, two common-terminal connections for the VCCS's can be allowed. This can also be implemented by using the CMOS pairs but, alternatively, a single NMOST and PMOST can be used, as shown in Fig. 6(b) [15]. Finally, if $I_{\rm out}$ is to be fed back to the input, to implement an I–V converter or electronically variable resistor, the VCCS's have yet an additional common terminal. Again, the previously mentioned VCCS implementations of Fig. 6(a) and (b) can be used. However, alternatively, two MOST's of the same type can be used as shown in Fig. 6(c) and [12]. Thus, it has been shown that three well-known circuits can be viewed as different implementations of a $\{V_\Sigma, V_\Delta\}$-class circuit. Similar relations exist between other circuits (see [2], [39]).

B. Circuit Analysis for One Subclass

Circuits belonging to a subclass share certain properties and can be analyzed in one run, provided that a generalized VCCS model equation is available. For instance, in the above discussed $\{V_\Sigma, V_\Delta\}$ circuit example, the following square-law V–I relation can be used:

$$I = k_{eq}(V - V_{Teq})^2. \tag{6}$$

This model can be used for a single MOST and a CMOS pair [13]. Doing so, it can easily be shown, that the square-law term cancels in the relation between V_{dif} and $I_{\rm out} = I_a - I_b$ if and only if k_{eq} of VCCS$_a$ and VCCS$_b$ is equal. For all three cases in Fig. 6, the output current $I_{\rm out}$ is then given by

$$I_{\rm out} = k_{eq}(V_{\rm sum} - 2V_{Teq})V_{dif} \tag{7}$$

revealing a linear relation between V_{dif} and $I_{\rm out}$ with a transconductance tunable by means of $V_{\rm sum}$.

C. Circuits Analysis for an Entire Class

In the case above, the analysis is restricted to one subclass. In some cases it is even possible to do an analysis for an entire class in one run. Taking a linear analysis for the $\{V, I\}$ class as an example, the following equations hold:

$$v_a + \alpha_b \cdot v_b = v_{\rm ind} \tag{8}$$

$$i_a + \beta_b \cdot i_b = i_{\rm ind} \tag{9}$$

where the coefficient α_b and β_b take values depending on the subclass

$$\{\alpha_b, \beta_b\} \in \left\{ \begin{matrix} \{0, 1\}, \{0, -1\}, \{1, 0\}, \{-1, 0\} \\ \{1, 1\}, \{1, -1\}, \{-1, 1\}, \{-1, -1\} \end{matrix} \right\}. \tag{10}$$

The solution for the primary current variables is

$$v_a = -\frac{\beta_b g_b v_{\rm ind}}{\alpha_b g_a - \beta_b g_b} + \frac{\alpha_b i_{\rm ind}}{\alpha_b g_a - \beta_b g_b} \tag{11}$$

$$v_b = \frac{g_a v_{\rm ind}}{\alpha_b g_a - \beta_b g_b} - \frac{i_{\rm ind}}{\alpha_b g_a - \beta_b g_b} \tag{12}$$

$$i_a = -\frac{\beta_b g_a g_b v_{\rm ind}}{\alpha_b g_a - \beta_b g_b} + \frac{\alpha_b g_a i_{\rm ind}}{\alpha_b g_a - \beta_b g_b} \tag{13}$$

$$i_b = \frac{g_a g_b v_{\rm ind}}{\alpha_b g_a - \beta_b g_b} - \frac{g_b i_{\rm ind}}{\alpha_b g_a - \beta_b g_b}. \tag{14}$$

Looking at the above four equations, we see that singularities can occur since the denominator of the relations can become zero. This happens for the $\{V_\Sigma, I_\Sigma\}$ (coefficients α_b and β_b both equal to one) and $\{V_\Delta, I_\Delta\}$ class (both coefficient equal to -1) for $g_a = g_b$. For these cases, there is no unique solution or no solution.

Since both the differential pair and current mirror belong to the $\{V, I\}$ class, (11)–(14) hold for both of them. It can be verified that the differential pair ($\{V_\Delta, I_\Sigma\}$ class with $\{\alpha_b, \beta_b\} = -1, 1$) has a transconductance from differential input voltage $v_{\rm ind}$ to i_a of $g_a \cdot g_b/(g_a + g_b)$ for a constant tail current ($i_{\rm ind} = 0$). For the current mirror ($\{V_\Delta, I_P\}$ class with $\{\alpha_b, \beta_b\} = \{-1, 0\}$ with $V_\Delta = 0$ and, thus, $v_{\rm ind} = 0$) a current gain from $i_a = i_{\rm ind}$ to i_b is equal to g_b/g_a is found. Thus, one set of equations describes the transfer properties of an entire class of circuits, allowing for a systematic analysis of large groups of circuits [2].

VI. Application Example II: Classification of MOS Transconductors

A. Introduction

Although many papers on MOS transconductor circuits exist (for an overview, see [2], [39], and [42]), very few consistently compare different approaches. Such a comparison is burdened by many differences in circuit implementation. However, using the classification presented above, it can be shown that many circuits are variations on a theme as far as the actual V–I kernel is considered. We will consider the class of V–I kernels, consisting of two matched MOST's operating as VCCS's. The aim is to identify kernels that behave essentially differently with respect to distortion.

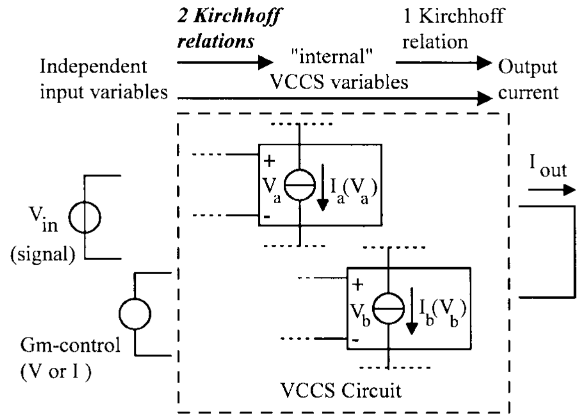

Fig. 7. Schematic representation of V–I kernels with two VCCS's and independent sources.

B. Systematic Generation of All V–I Kernels with Two VCCS's

Although a single MOST can be used as a VCCS, it will appear that there are good reasons to use two of them rather than one. All V–I converter kernels consisting of two VCCS's, as shown schematically in Fig. 7, will now be generated systematically, using the classification system. Only classes for which at least one independent voltage is forced need to be considered (to be used as input voltage $V_{\rm in}$). Table II presents these cases. The first column shows the set of two Kirchhoff relations (class) and the second one lists the independent voltage that is used as input. It appears that three classes with two voltage relations exist ($\{V, V\}$ sets), each subdivided into two cases, while eight classes with a voltage and a current exist ($\{V, I\}$ sets).

To find circuits different from a single VCCS with respect to distortion, the transfer function and nonlinearity of the various classes was considered. To analyze nonlinearity, individual VCCS's have been modeled by a third-order Taylor series (assume mainly HD2 and HD3)

$$I(V_0 + V_{\rm in}) - I(V_0) = g_1 V_{\rm in} + g_2 V_{\rm in}^2 + g_3 V_{\rm in}^3 \tag{15}$$

where g_1, g_2, and g_3 are proportional to the first-, second-, and third-order derivatives of $I(V)$ to V in bias point V_0. The nonlinearity for the V–I kernels has been analyzed symbolically and Table II lists the coefficients of the first-, second-, and third-order terms of I_a and I_b. Taking a single VCCS as reference, we can find the cases with essentially different distortion behavior. For six cases the coefficients appear to be equal to those of the constituting VCCS (g_1, g_2, g_3), which could just as well be achieved by a single VCCS. In two cases ($\{V_\Sigma, I_\Sigma\}$ and $\{V_\Delta, I_\Delta\}$) a unique solution does not always exist [2]. However, the remaining six cases differ essentially

TABLE II
OVERVIEW OF THE 11 IMPOSEABLE SETS OF TWO KIRCHHOFF RELATIONS USEFUL FOR V–I KERNELS, WITH FOUR DIFFERENT TYPES OF BEHAVIOR (IN BOLD) (@ = EQUAL g COEFFICIENTS; $*$ = NOT ALWAYS A UNIQUE SOLUTION)

Class	V_{in}	1st order coeff. I_a,I_b	2nd order coeff. I_a,I_b	3rd order coeff. I_a,I_b
$\{V_P,V_\Sigma\}$	**V_P**	**$g_{1a}, -g_{1b}$**	**g_{2a}, g_{2b}**	**$g_{3a}, -g_{3b}$**
	V_Σ	$0, g_{1b}$	$0, g_{2b}$	$0, g_{3b}$
$\{V_P,V_\Delta\}$	**V_P**	**g_{1a}, g_{1b}**	**g_{2a}, g_{2b}**	**g_{3a}, g_{3b}**
	V_Δ	$0, g_{1b}$	$0, g_{2b}$	$0, g_{3b}$
$\{V_\Sigma,V_\Delta\}$	V_Σ	$g_{1a}/2, g_{1b}/2$	$g_{2a}/4, g_{2b}/4$	$g_{3a}/8, g_{3b}/8$
	V_Δ	$g_{1a}/2, -g_{1b}/2$	$g_{2a}/4, g_{2b}/4$	$g_{3a}/8, -g_{3b}/8$
$\{V_\Sigma,I_P\}$	V_Σ	$0, g_{1b}$	$0, g_{2b}$	$0, g_{3b}$
$\{V_\Delta,I_P\}$	V_Δ	$0, -g_{1b}$	$0, -g_{2b}$	$0, -g_{3b}$
$\{V_P,I_\Sigma\}$	V_P	$g_{1a}, -g_{1a}$	$g_{2a}, -g_{2a}$	$g_{3a}, -g_{3a}$
$\{V_P,I_\Delta\}$	V_P	g_{1a}, g_{1a}	g_{2a}, g_{2a}	g_{3a}, g_{3a}
$\{V_\Sigma,I_\Sigma\}$	V_Σ	*	*	*
$\{V_\Sigma,I_\Delta\}$	**V_Σ**	**$g_1/2, g_1/2$**	**$g_2/4, g_2/4$**	**$g_3/8, g_3/8$**
$\{V_\Delta,I_\Sigma\}$	**V_Δ**	**$g_1/2, -g_1/2$**	**0 , 0**	$\pm\left(\frac{g_3}{8} - \frac{g_2^2}{4g_1}\right)$
$\{V_\Delta,I_\Delta\}$	V_Δ	*	*	*

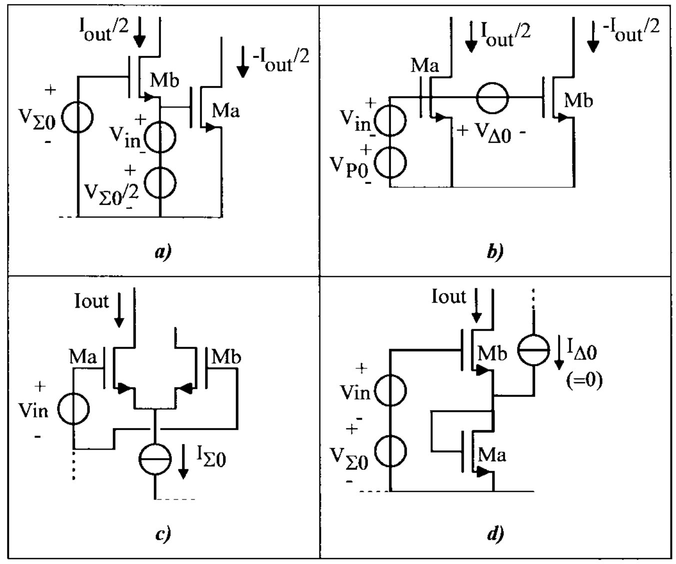

Fig. 8. Example circuits for the four different transconductor classes listed in Table II. (a) $\{V_P, V_\Sigma\}$ with constant V_Σ. (b) $\{V_P, V_\Delta\}$ with constant V_Δ. (c) $\{V_\Delta, I_\Sigma\}$ with constant I_Σ. (d) $\{V_\Sigma, I_\Delta\}$ with constant I_Δ.

from a single VCCS. In fact, four different types of distortion coefficients are found (printed in bold) as follows.

1) $\{V_P, V_\Sigma\}$: constant V_Σ with V_P as input. The second-order terms of I_a and I_b have the same sign, while the first- and third-order terms have different signs. Consequently, second-order distortion is cancelled if I_a and I_b are subtracted and $g_{2a} = g_{2b}$ (for matched MOST's $V_P = V_\Sigma/2$) (balancing). Since the first- and third-order term are both doubled due to current subtraction, HD3 remains the same, independent of the VCCS characteristic. The same behavior is found for case $\{V_\Sigma, V_\Delta\}$ with V_Δ as input. The cases are equivalent if $V_P = V_\Sigma/2 + V_\Delta/2$, which is the usual case (HD2 cancelling). An example of a circuit implementation is shown in Fig. 8(a), [12], and [14]. However, many other (sub)circuits are based on the same basic principle [5]–[7], [10], [13], [15]–[17], [24]–[28], [30], [31].

2) $\{V_P, V_\Delta\}$: constant V_Δ with V_P as input. I_a and I_b have different g coefficients if their bias-point is different. If I_Δ is used as output, the differences of corresponding g coefficients of VCCS$_a$ and VCCS$_b$ are found, multiplied by, respectively, $V_{\rm in}$, $V_{\rm in}^2$, and $V_{\rm in}^3$. As a result, the net G_m range is extended at the low end. If $\Delta g_2/\Delta g_1 < g_2/g_1$ or $\Delta g_3/\Delta g_1 < g_3/g_1$ linearity is improved, which depends strongly on device characteristics. The same behavior is found for case $\{V_\Sigma, V_\Delta\}$, although now with V_Σ as signal input, provided that $V_P = V_\Sigma/2 + V_\Delta/2$. Fig. 8(b) shows an example of a $\{V_P, V_\Delta\}$ circuit [18]. Others have been proposed in [14], [19], [22], [23], [29], and [36]. Using differential transconductors as a starting point, a $\{V_P, V_\Delta\}$ configuration is also used in several circuits (sometimes referred to as crosscoupling or "current differencing," e.g., [8], [9], and [35]). Note that there is an essential difference between circuits in this class and in the previously discussed one. For constant V_Σ the VCCS voltages are driven in antiphase, while they are in phase for constant V_Δ. With I_Δ as output, the signal currents are added in the first case and subtracted in the second one (however, the noise is added in both cases).

3) $\{V_\Delta, I_\Sigma\}$: constant I_Σ with V_Δ as input. The second-order terms are zero for equally biased MOST's. The third-order term also now depends on the second-order term (g_2 dependence). The net result depends on the VCCS characteristic, especially on the sign of the third-order term. The omnipresent different pair is the best known example of a circuit belonging to this class [see Fig. 8(c)].

4) $\{V_\Sigma, I_\Delta\}$: constant I_Δ (=0) and V_Σ as input. Since the input voltage is divided over two equal devices, the input voltage can be two-times larger for the same distortion (this holds for both HD2 and HD3). This effect is independent of the device characteristic. Fig. 8(d) shows a circuit example with equal NMOST's [26]. Circuits belonging to the same class are the CMOS pair in Fig. 2(c) used in [10] and [13] and series connections of differential pairs (e.g., [5] and [34]).

In summary, all V–I converter kernels with two MOST-VCCS's have been generated and analyzed with respect to distortion, using the proposed classification. This systematic approach unveils that dozens of published circuits can be classified in only four classes with basically different distortion behavior.

VII. Conclusions

It has been shown that two independent Kirchhoff relations among the VCCS variables and independent variables play a crucial role in establishing a unique transfer function in two-VCCS circuits. Based on different sets of two imposeable Kirchhoff relations, two-VCCS circuits can be classified in three main classes and 14 subclasses.

The classification system is shown to be useful for circuit design and analysis because of the following.

1) It presents designers with an overview of all ways to combine two VCCS's [e.g., all V–I kernels with two VCCS's have been generated (Table II)]. As VCCS's can be implemented in various forms, a two-VCCS circuit can have several transistor level implementations (Fig. 6).
2) The classification system for two-VCCS circuits divides circuits in classes with common properties, i.e., it helps to recognize that many circuits can be considered as variations on a theme (e.g., [5]–[7], [10], [13], [15]–[17], [24]–[28], [30], and [31]).
3) Having recognized a circuit as a two-VCCS circuit, it can be analyzed using hierarchical symbolic macro-models that express the properties of a VCCS circuit in properties of the VCCS implementation (e.g., Table II. The Taylor coefficients of a two-VCCS circuit are expressed in Taylor coefficients of the VCCS). The hierarchy saves a great deal of analysis effort and allows for an easy comparison of alternative VCCS implementations.

Acknowledgment

The author would like to thank C. H. J. Mensink, H. Wallinga, and A. J. M. van Tuijl for useful discussions on the work presented in this paper. Moreover, the comments of reviewers are gratefully acknowledged.

References

[1] M. Ismail and J. Franca, Eds., *Introduction to Analog VLSI Design Automation.* Boston, MA: Kluwer, 1990.
[2] E. A. M. Klumperink, "Transconductance based CMOS circuits: Circuit generation, classification and analysis," Ph.D. thesis, Univ. of Twente, Enschede, The Netherlands, 1997.
[3] A. Nedungadi and T. R. Viswanathan, "Design of linear CMOS transconductance elements," *IEEE Trans. Circuits Syst.*, vol. 31, pp. 891–894, Oct. 1984.
[4] H. Khorramabadi and P. R. Gray, "High-frequency CMOS continuous-time filters," *IEEE J. Solid-State Circuits*, vol. SC-19, pp. 939–948, Dec. 1984.
[5] R. R. Torrance, T. R. Viswanathan, and J. V. Hanson, "CMOS voltage to current transducers," *IEEE Trans. Circuits Syst.*, vol. CAS-32, pp. 1097–1104, Nov. 1985.
[6] J. L. Pennock, "CMOS triode transconductor for continuous-time active integrated filters," *Electron. Lett.*, vol. 21, pp. 817–818, Aug. 1985.
[7] T. R. Viswanathan, "CMOS transconductance element," *Proc. IEEE*, vol. 74, pp. 222–224, Jan. 1986.
[8] Z. Czarnul and Y. Tsividis, "MOS transconductors and integrators with high linearity," *Electron. Lett.*, vol. 22, pp. 245–246, Feb. 1986.
[9] Z. Czarnul and Y. Tsividis, "MOS tunable transconductor," *Electron. Lett.*, vol. 22, pp. 721–722, June 1986.
[10] C. S. Park and R. Schaumann, "A high-frequency CMOS linear transconductance element," *IEEE Trans. Circuits Syst.*, vol. 33, pp. 1132–1138, Nov. 1986.
[11] S. L. Garverick and C. G. Sodini, "Large signal linearity of scaled MOS transistors," *IEEE J. Solid-State Circuits*, vol. 22, pp. 282–286, Apr. 1987.
[12] K. Bult and H. Wallinga, "A class of analog CMOS circuits based on the square-law characteristic of a MOS transistor in saturation," *IEEE J. Solid-State Circuits*, vol. 22, pp. 357–365, June 1987.
[13] E. Seevinck and R. F. Wassenaar, "A versatile CMOS linear transconductor/square-law function circuit," *IEEE J. Solid-State Circuits*, vol. 22, pp. 366–377, June 1987.
[14] K. Bult, "Analog CMOS square-law circuits," Ph.D. dissertation, Univ. of Twente, Enschede, The Netherlands, 1988.
[15] B. Nauta and E. Seevinck, "Linear CMOS transconductance element for VHF filters," *Electron. Lett.*, vol. 25, no. 7, pp. 448–450, Mar. 1989.
[16] E. A. M. Klumperink and E. Seevinck, "MOS current gain cells with electronically variable gain and constant bandwidth," *IEEE J. Solid-State Circuits*, vol. 24, pp. 1465–1467, Oct. 1989.
[17] S. N. Filho, M. C. Schneider, and R. N. G. Robert, "New CMOS ota for fully integrated continuous-time circuit applications," *Electron. Lett.*, vol. 25, no. 24, pp. 24–26, Nov. 1989.
[18] Z. Wang, "Novel linearization technique for implementing large-signal MOS tunable transconductors," *Electron. Lett.*, vol. 26, no. 2, pp. 138–139, Jan. 1990.
[19] Z. Wang and W. Guggenbühl, "A voltage-controllable linear MOS transconductor using bias offset technique," *IEEE J. Solid-State Circuits*, vol. 25, pp. 135–138, Feb. 1990.
[20] P. M. Van Peteghem, F. G. L. Rice, and S. Lee, "Design of a very linear CMOS transconductance input stage for continous-time filters," *IEEE J. Solid State Circuits*, pp. 497–501, Apr. 1990.
[21] S. T. Dupuie and M. Ismail, "High frequency CMOS transconductors," in *Analogue IC Design: The Current-Mode Approach*, C. Toumazou *et al.*, Eds. London, U.K.: Peregrinus, 1990.
[22] W. J. Adams and J. Ramirez-Angulo, "Extended transconductance adjustment/linearization technique," *Electron. Lett.*, vol. 27, no. 10, pp. 842–844, May 1991.
[23] P. Wu and R. Schaumann, "Tunable operational transconductance amplifier with extremely high linearity over very large input range," *Electron. Lett.*, vol. 27, no. 14, pp. 1254–1255, July 1991.
[24] M. C. H. Cheng and C. Toumazou, "Linear composite MOSFET's (COMFET's)," *Electron. Lett.*, vol. 27, no. 20, pp. 1802–1804, Sept. 1991.
[25] J. Sevenhans and M. Van Paemel, "Novel CMOS linear OTA using feedback control on common source node," *Electron. Lett.*, vol. 27, no. 20, pp. 1873–1875, Sept. 1991.
[26] E. J. van der Zwan, E. A. M. Klumperink, and E. Seevinck, "A CMOS OTA for HF filters with programmable transfer function," *IEEE J. Solid-State Circuits*, vol. 26, pp. 1720–1724, Nov. 1991.
[27] G. Wilson and P. K. Chan, "Comparison of four CMOS transconductors for fully integrated analogue filter applications," *Proc. Inst. Electr. Eng.*, vol. 138, pp. 683–688, Dec. 1991.
[28] S. Szczepanski, J. Jakusz, and A. Czarniak, "Differential pair transconductor linearization via electronically controlled current-mode cells," *Electron. Lett.*, vol. 28, no. 12, pp. 1093–1095, June 1992.
[29] R. Raut, "A novel VCT for analog IC applications," *IEEE Trans. Circuits Syst. II*, vol. 39, pp. 882–883, Dec. 1992.
[30] Z. Wang, "Two CMOS large current-gain cells with linearly variable gain and constant bandwidth," *IEEE Trans. Circuits Syst.*, vol. 39, pp. 1021–1024, Dec. 1992.
[31] M. C. H. Cheng and C. Toumazou, "Linear Composite MOSFET: Theory, Design, and Applications," *IEEE Trans. Circuits Syst. I*, vol. 40, pp. 297–306, May 1993.
[32] S. Szczepanski, A. Wyszynski, and R. Schaumann, "Highly linear voltage-controlled CMOS transconductors," *IEEE Trans. Circuits Syst. I*, vol. 40, Apr. 1993.
[33] E. A. M. Klumperink, "Cascadable CMOS current gain cell with gain insensitive phase shift," *Electron. Lett.*, vol. 29, no. 23, pp. 2027–2028, Nov. 1993.
[34] I. E. Opris and G. T. A. Kovacs, "Large-signal subthreshold CMOS transconductance amplifier," *Electron. Lett.*, vol. 31, no. 9, pp. 718–720, Apr. 1995.
[35] E. A. M. Klumperink, C. T. Klein, B. Rüggeberg, and A. J. M. v. Tuijl, "AM suppression with low AM-PM conversion with the aid of a variable-gain amplifier," *IEEE J. Solid-State Circuits*, vol. 31, pp. 625–633, May 1996.

[36] S. Szczepanski, R. Schaumann, and B. Pankiewicz, "A CMOS transconductance element with improved DC gain and wide bandwidth for VHF applications," *Analog Integr. Circuits Signal Processing*, vol. 10, pp. 143–156, 1996.
[37] Y. P. Tsividis, "Integrated continuous-time filter design—An overview," *IEEE J. Solid-State Circuits*, vol. 29, pp. 166–176, Mar. 1994.
[38] Y. P. Tsividis and J. O. Voorman, *Integrated Continuous-Time Filters*. New York: IEEE, 1993.
[39] M. W. Hauser, E. A. M. Klumperink, R. G. Meyer, and W. D. Mack, "Variable-gain, variable-transconductance, and multiplication techniques: A survey," in *Analog Circuit Design*, W. M. C. Sansen *et al.*, Eds. Dordrecht, The Netherlands: Kluwer, 1996, pp. 291–322.
[40] E. A. Vittoz, "Analog VLSI signal processing: Why, where, and how?," *Analog Integr. Circuits Signal Processing*, vol. 6, pp. 27–44, 1994.
[41] M. J. M. Pelgrom, A. C. J. Duinmaijer, and A. P. G. Welbers, "Matching properties of MOS transistors," *IEEE J. Solid-State Circuits*, vol. 24, pp. 1433–1440, Oct. 1989.
[42] C. H. J. Mensink, "Analogue transconductors for sub-micron CMOS technology," Ph.D. thesis, Univ. of Twente, Enschede, The Netherlands, 1996.

Eric A. M. Klumperink was born on April 4th, 1960, in Lichtenvoorde, The Netherlands. He received the B.Sc. degree from HTS, Enschede, The Netherlands, in 1982 and the Ph.D. degree from Twente University, Enschede, The Netherlands, in 1997.

In 1984 he joined the Faculty of Electrical Engineering of the University of Twente, where he was mainly engaged in analog CMOS circuit design. He is currently an Assistant Professor at the University of Twente. He is involved in teaching and research at the IC-Design Laboratory of the Faculty of Electrical Engineering and the IC-Design Theme of the MESA Research Institute, especially in RF analogue CMOS circuits. His research interests are in the areas of design issues in RF front-end circuits for integrated CMOS transceivers, such as dynamic range of low noise amplifiers, up conversion of $1/f$ noise in oscillators and mixers, and phase noise in frequency synthesizers.