Vorticity dynamics of a dipole colliding with a no-slip wall

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The active role of vorticity in the collision of a Lamb-like dipole with a no-slip wall is studied for Re values ranging between 625 and 20000. The initial approach of the dipole does not differ from the stress-free case or from a point-vortex model that incorporates the diffusive growth of the dipole core. When closer to the wall, the detachment and subsequent roll-up of the boundary layer leads to a viscous rebound, as was observed by Orlandi [Phys. Fluids A **2**, 1429 (1990)] in numerical simulations with Re up to 3200. The net translation of the vortex core along the wall is strongly reduced due to the cycloid-like trajectory. For $Re \leq 2500$ wall-generated vorticity is wrapped around the separate dipole halves, which hence become (partially) shielded monopoles. For $Re \geq \mathcal{O}(10^4)$, however, a shear instability causes the roll-up of the boundary layer before it is detached from the wall. This leads to the formation of a number of small-scale vortices, between which intensive, narrow eruptions of boundary-generated vorticity occur. Quantitative measures are given for the influx of vorticity at the wall and the consequent increase of boundary layer vorticity and enstrophy. © 2007 American Institute of Physics. [DOI: 10.1063/1.2814345]

I. INTRODUCTION

One of the key phenomena in two-dimensional bounded turbulence is the interaction between the vorticity structures inside the domain and the viscous boundary layers at the no-slip walls. For example, in freely evolving turbulence it is often observed that these dipoles collide with the lateral no-slip walls.^{1,2} Typically, under the influence of large-scale vorticity structures the boundary layers are detached from the wall and injected into the domain in the form of small-scale vortices and filaments.^{1–6} This mechanism is nicely illustrated by Wells⁷ in laboratory experiments of a quasi-2D flow in a square container where the background rotation is being modulated. Boundary layers are created and detached from the wall during the stages of wall acceleration and deceleration. Subsequently, the detached vorticity filaments roll-up into vortex structures and enter the interior of the domain. In freely evolving 2D turbulence interior vortices may form dipole pairs, which can translate throughout the flow domain.

To study the vortex-wall interactions in more detail we investigate the collision of a single dipolar vortex with a no-slip wall. Numerical simulations of the dipole-wall collision were first performed by Orlandi,⁸ who revealed that the creation of vorticity at the wall drives a vigorous rebound of the dipolar vortex. Each dipole half forms a new pair with a secondary vortex consisting of boundary layer vorticity. The new dipoles travel along circular trajectories away from the wall. The production of the secondary vorticity at the wall during the dipole-wall collision was later studied by Coutias and Lynov⁹ and by Clercx and van Heijst.¹⁰ The Reynolds number in the reported simulations is limited to $Re=5000$ as

accurate representations of the small-scale structures near the wall requires a high resolution. Walker¹¹ and Peridier *et al.*^{12,13} investigated the behavior of the boundary layer at a no-slip wall affected by a nearby single (point) vortex. This setup permits the use of boundary-layer equations, i.e., a reduced set of the Navier–Stokes equations, on a specially tailored grid to obtain results for large Re values. More recently Obabko and Cassel¹⁴ reported results on this problem based on the full Navier–Stokes equations. Their findings will be discussed in more detail later on.

The present paper aims at giving a clear insight in the dipole-wall collision while concentrating on the vorticity dynamics close to the no-slip wall. We focus mainly on the features of the dipole-wall collisions that are relevant to bounded two-dimensional turbulence. Simulations are performed for several values of the Reynolds number Re , based on the size and translation speed of the dipole. One of the goals is to provide accurate results for the dipole-wall collision for Reynolds numbers up to $Re=20000$, and the dynamics of the small-scale vortices and vorticity filaments which emerge during this process. Furthermore, attention is given to the vorticity flux at the wall during the approach and the rebound of the dipole halves and the consequent increase of boundary layer vorticity and enstrophy. Finally, for different Re values we investigate the formation of the small-scale vortices that originate from the (detached) boundary layers.

II. PROBLEM DESCRIPTION AND NUMERICAL SETUP

The two-dimensional flow of an incompressible fluid in the x, y plane is described by the vorticity equation

$$\frac{\partial \omega}{\partial t} + \mathbf{u} \cdot \nabla \omega = \nu \nabla^2 \omega, \quad (1)$$

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FIG. 1. The evolution of the initial dipole consisting of two isolated vortices: (a) The initial vorticity distribution; (b) shedding of the surrounding vorticity shields; and (c) formation of a downward-travelling Lamb-type dipole and a weaker upward-travelling dipole. The contour levels are drawn for $\dots, -150, -90, -30, 30, 90, 150, \dots$

with ν the kinematic viscosity and the vorticity ω defined as the curl of the velocity field $\mathbf{u}$,

$$\omega = \mathbf{e}_z \cdot \nabla \times \mathbf{u}, \quad (2)$$

where $\mathbf{e}_z$ is the unit vector perpendicular to the plane of flow. In this paper we study the flow in a periodic channel domain $[0, 2) \times [-1, 1]$. The periodic direction of the channel is aligned along the x -axis. On the lateral walls of the channel at $y = \pm 1$ no-slip boundary conditions are applied. The flow is initialized in the form of two shielded Gaussian monopolar vortices, their centers placed at a distance $d=0.2$ apart. The vorticity distribution in each monopole is given by

$$\omega(0, \mathbf{x}) = \omega_e (1 - r^2/r_0^2) \exp(-r^2/r_0^2), \quad (3)$$

where r_0 is the core radius, and $\mathbf{r} = \mathbf{x} - \mathbf{x}_0$ with $\mathbf{x}_0$ the position of the vortex center. The two isolated monopoles are located at $\mathbf{x}_0 = (1.1, 0)$ and $\mathbf{x}_0 = (0.9, 0)$, respectively. Demanding that the root mean square velocity (rms) velocity is initially equal to unity yields the amplitude of the isolated monopole, $\omega_e \approx 300$. The core radius of the shielded monopoles is set to $r_0 = 0.1$. The vorticity amplitude in the surrounding rings decreases exponentially with r . As a result, the circulation of one isolated monopole calculated over a circular contour around the vortex origin decreases exponentially towards zero for increasing contour radius. Hence, no boundary layers are required at the no-slip walls when constructing the initial flow field.

To solve Eq. (1) numerically with a pseudospectral method, both the velocity and vorticity are expanded in a truncated series of Fourier polynomials for the x -direction and in a truncated series of Chebyshev polynomials for the nonperiodic y -direction. A semi-implicit time discretization scheme is applied to the vorticity equation. In most of the cases the nonlinear term in the vorticity Eq. (1) is treated by the explicit second-order Adams–Bashforth (AB) scheme and the diffusion term on the right-hand side of Eq. (1) is treated by the implicit Crank–Nicolson scheme (CN). In some specific cases a third-order accurate semi-implicit scheme (BFD3) is applied, which is based on the backwards differentiating formula. Applying the spatial and time discretizations yields a system of equations for the spectral coefficients for the velocity and vorticity. This system can be resolved using a specially tailored Gaussian elimination technique. The vorticity values at the walls are not known *a priori*. Enforcing the vorticity definition (2) with an influence matrix yields the correct boundary values for the vorticity at

the walls.¹⁵ A more detailed description of the numerical method is given in Ref. 16.

After releasing the initial set of shielded monopolar vortices [Fig. 1(a)], one observes how the cores combine into a dipole, while the surrounding shields are substantially deformed [Fig. 1(b)]. Somewhat later, the dipolar vortex has travelled away in the negative y -direction, and the shields left behind combine into another, much weaker dipolar structure that translates in the opposite direction [Fig. 1(c)]. This weak dipole has no considerable impact on the primary dipole and is not discussed further. Previous studies have revealed that the primary dipole vortex closely resembles the Lamb dipole,¹⁷ i.e., a compact dipolar vorticity structure in a circular area with a linear vorticity-stream-function relationship.^{18,19} For more details on the formation of a dipole from two interacting monopoles the reader is referred to Beckers *et al.*²⁰ and Schmidt *et al.*²¹

The time evolution of the dipole is calculated by the spectral code for several values of the viscosity parameter. The integral-scale Reynolds number for the initial field is given by

$$\text{Re} = \frac{U_{\text{rms}} H}{\nu}, \quad (4)$$

where the characteristic length scale is set to the half-height of the domain $H=1$ and the characteristic velocity to the initial rms velocity $U_{\text{rms}}=1$. This integral Reynolds number differs slightly from the Reynolds number $\text{Re}_d \approx 0.8\text{Re}$ based on the dipole translation speed U_d and dipole radius R . The required number of Fourier polynomials (N) and Chebyshev polynomials (M) depends on the Reynolds number; the resolution is increased for higher Re values in order to cope with

TABLE I. The number of Fourier polynomials N , the number of Chebyshev polynomials M , and the time step Δt used to solve the dipole-wall collision for a number of initial Reynolds numbers. (See Table II for the simulations with $\text{Re}=20000$.)

ν	Re	N	M	Δt
$(\times 10^{-4})$				$(\times 10^{-5})$
16.0	625	512	512	2.5
8.0	1250	512	512	2.5
4.0	2500	1024	768	1.25
2.0	5000	1536	1024	0.625
1.0	10000	1536	1024	0.625

TABLE II. The number of Fourier polynomials N , the number of Chebyshev polynomials M , and time step Δt used to solve the dipole-wall collision for an initial Reynolds number of $Re=20000$. The spatial resolution in the x -direction is increased after $t=0.25$ and the applied time discretization scheme is changed from AB/CN to BDF3 at $t=0.31$.

t	ν	N	M	Δt	Scheme
	($\times 10^{-5}$)			($\times 10^{-6}$)	
0.00–0.25	5.0	2048	1024	6.25	AB/CN
0.25–0.31	5.0	4096	1024	6.25	AB/CN
0.31–0.54	5.0	4096	1024	6.25	BDF3

the smaller scales that appear throughout the domain (Table I). Special effort is required to obtain reliable results for the dipole-wall collision with an initial value of $Re=20000$. The same resolution of 1536×1024 as for $Re=10000$ was used to simulate the approach of the dipole to the no-slip wall. At $t=0.25$ the resolution is increased to 4096×1024 to capture the small scales in the boundary layer. The Chebyshev resolution (normal to the boundary) is already quite good in the boundary layer region, but the Fourier resolution along the boundary layer needed to be enlarged to obtain accurate results. At $t \approx 0.32$ the AB/CN scheme became unstable for the used time step. Instead of decreasing the time step a BDF3 time discretization scheme was used, which has relaxed sta-

bility conditions for large Reynolds numbers.^{16,22} The specific settings used in the simulation are given in Table II.

To verify the accuracy of the numerical method the results are compared to the benchmark results reported by Clercx and Bruneau.¹⁷ For this purpose we have calculated the position of the maximum vorticity, $\mathbf{x}_d$, in the positive dipole half. The position is determined using a second order polynomial fit of the vorticity around its maximum. In Table III the location of the maximum vorticity, $\mathbf{x}_d$, in the positive dipole half is compared with the benchmark results of Clercx and Bruneau¹⁷ obtained with two different numerical methods. One set of numerical simulations was performed with a finite differences code while the other set concerns simula-

TABLE III. Numerical results for the maximum vorticity in the positive dipole half ω_d and its location $\mathbf{x}_d = (x_d, y_d)$. The location is given relative to the point where the dipole axis crosses the no-slip wall. The x_d -coordinate measures the distance along the wall and the y_d -coordinate is the distance normal to the wall. The results are compared to the available benchmark results for a square domain enclosed by no-slip walls as reported in Clercx and Bruneau (Ref. 17).

Re	t	Current results			Clercx and Bruneau		
		x_d	y_d	ω_d	x_d	y_d	ω_d
...	0.0	0.1	1.0	316.84	0.1	1.0	316.84
625	0.6	0.1656	0.1827	158.70	0.165	0.182	158.9
625	0.625	0.1655	0.1674	154.22	0.166	0.168	154.2
625	1.0	0.2543	0.1949	102.64	0.254	0.195	102.6
625	1.4	0.3068	0.2314	71.03	0.307	0.231	71.0
1250	0.6	0.1506	0.1260	219.29	0.151	0.126	219.4
1250	0.625	0.1725	0.1157	216.15	0.174	0.115	216.1
1250	1.0	0.2568	0.1515	170.30	0.257	0.152	170.3
1250	1.4	0.2914	0.1908	132.73	0.292	0.191	132.7
2500	0.6	0.1654	0.1042	261.83	0.165	0.104	261.9
2500	0.625	0.1982	0.1040	259.59	0.199	0.104	260.0
2500	1.0	0.2195	0.1738	231.40	0.219	0.174	231.4
2500	1.4	0.1946	0.2017	201.59	0.195	0.202	201.6
5000	0.6	0.2438	0.0948	286.93	0.244	0.097	286.9
5000	0.625	0.2780	0.1178	285.89	0.275	0.116	285.9
5000	1.0	0.3614	0.1948	269.01	0.366	0.189	269.1
5000	1.4	0.5141	0.2052	252.08	...	...	...
10000	0.6	0.2562	0.0961	301.57	...	...	...
10000	0.625	0.3032	0.1036	300.59	...	...	...
10000	1.0	0.6439	0.2738	...	...	...	...
10000	1.4	0.6865	0.3568	...	...	...	...

FIG. 2. Sequence of vorticity contour plots showing the flow evolution of a dipole colliding with a no-slip wall for integral-scale Reynolds number $Re = 2500$. The contour levels are drawn for $\dots, -100, -60, -20, 20, 60, 100, \dots$

tions that were conducted with a Chebyshev pseudospectral code. These benchmark simulations were performed for a square domain $[-1, 1] \times [-1, 1]$, enclosed by no-slip walls at all sides. There is a good agreement for the position of the positive dipole half $\mathbf{x}_d$ (absolute error smaller than 10^{-3}) as well as the maximum vorticity ω_d (better than 0.2% accuracy), confirming the accuracy of the simulations.

The influence of the periodic boundaries at $x=0$ and $x=2$ is small. A validation run has been performed for $Re = 1250$ for a domain with a doubled channel length. The difference in measured positions of the vorticity maximum, $\mathbf{x}_d$, does not exceed 1.5×10^{-3} . This indicates that these boundaries are located sufficiently far from the dipole and thus do not have a substantial impact on the evolution of the dipole for the time interval considered.

III. GENERAL OVERVIEW

The collision of a dipole with a no-slip wall and the subsequent rebound was first simulated by Orlandi,⁸ for Re values up to 3200. In order to be able to properly compare with the behavior for higher Re values, let us briefly summarize the sequence of events for $Re = 2500$ as is plotted in Fig. 2. In Fig. 3 a schematic representation is given and the different features are labeled using terminology for the right half of the domain. As the dipole approaches the wall, the vorticity in the boundary layers increases in amplitude. As the dipole reaches the wall, the dipole halves separate from each other and subsequently move along the wall in opposite directions. Boundary layer vorticity is advected by the dipole halves and subsequently detached from the wall forming secondary vortices ($v-$). The newly formed asymmetric dipoles travel along circular trajectories, leading to another collision with the wall at $t \approx 0.65$. For the special case of $Re = 2500$ the secondary vortices of the left and right dipole match up to

form a new dipole at the center line of the domain $x=1$. It is orientated in such a way that it translates away from the boundary along a straight path.

For any two-dimensional viscous flow the total energy $E(t)$ decays according to

$$\frac{dE}{dt} = -\nu \int_{\mathcal{D}} \omega^2 dA = -2\nu\Omega(t). \quad (5)$$

Note that the decay rate is proportional to the total enstrophy, $\Omega(t)$, which is a measure of the (squared) vorticity in the domain. Understanding the evolution of the total enstrophy is therefore of crucial importance for explaining the energy decay. For a domain with no-slip boundaries the change in total enstrophy is governed by

$$\frac{d\Omega}{dt} = -\nu \int_{\mathcal{D}} |\nabla \omega|^2 dA + \nu \oint_{\partial \mathcal{D}} \omega (\mathbf{n} \cdot \nabla \omega) ds. \quad (6)$$

The first term on the right-hand side simply states that the enstrophy decays due to vorticity gradients that are present in

FIG. 3. Schematic representation of the dipole-wall collision for the right half of the domain for two subsequent stages. The primary vortex (positive dipole half) and the later formed secondary vortex are labeled $V+$ and $v-$, respectively. The primary and secondary boundary layers are tagged by $B-$ and $b+$, respectively. Note that the primary features are labeled with an uppercase character, while the secondary features have a lowercase character. The plus and minus sign in the label denote the sign of the vorticity.

FIG. 4. Time evolution of the total energy $E(t)$ and the total enstrophy $\Omega(t)$ for the dipole-wall collision at $Re=2500$. The first maximum in the enstrophy, $\Omega_{\max}$, appears during the first collision.

the flow. The second term represents the vorticity production at the no-slip boundaries. Note that the vorticity influx at the no-slip boundaries is equal to $\nu(\mathbf{n} \cdot \nabla)\omega$.²³ In the case of a square domain with stress-free or periodic boundary conditions the second term on the right-hand side of Eq. (6) vanishes. As a result, the total enstrophy cannot increase for a domain with stress-free or periodic boundary conditions and is thus always bounded by its initial value.²⁴

In Fig. 4 the evolution of the total kinetic energy and total enstrophy are plotted for $Re=2500$. Note that the energy steadily decreases from its normalized initial value of $E=2$ towards $E \approx 1.0$ for $t=2$. At $t \approx 0.32$ the kinetic energy decays faster, which is due to the increased enstrophy on the domain. The peak in the enstrophy coincides with the first collision of the dipole with the wall. During this first collision the boundary layers build up a large amount of vorticity. The enstrophy in the boundary layers is then the main contribution to the total enstrophy. At $t=0.65$ another smaller peak is visible, which is due to the second collision. In Sec. V we will investigate the enstrophy change due to the vorticity flux at the no-slip boundaries, governed by the term $\nu \oint \omega(\mathbf{n} \cdot \nabla \omega) ds$ in Eq. (6), in more detail.

For Reynolds numbers larger than $Re=2500$, the dynamics is more interesting, in particular after the first collision of the dipole with the no-slip wall. The first differences are visible immediately after the first collision: The vorticity filaments that are detached from the wall are thinner but stron-

ger in amplitude (Fig. 5). The filament rolls up, forming a vortex ($v-$), and together with one of the primary vortices ($V+$) it forms the secondary dipole. This boundary-layer induced vortex has a higher vorticity amplitude than the primary vortex. As a result of the high amplitude, some outer vorticity of the primary vortex is advected around the secondary vortex. After the secondary vortex is created, even more vorticity is detached from the wall, which leads to the creation of multiple small vortices. The halves of the dipole ($V+$) separate faster from each other for higher Reynolds number. During the second collision both the primary ($V+$) and the secondary vortex ($v-$) advect boundary layer vorticity away from the wall, resulting in the production of a number of both positive and negative vortices. This is in contrast to the case $Re \leq 2500$, for which the detached boundary-layer vorticity is wrapped around the primary vortex.

IV. TRAJECTORY OF THE DIPOLE

The trajectory of a dipole approaching a free-slip wall perpendicularly is usually modeled using a set of two point vortices of opposite circulation and two mirror point vortices.¹⁸ Each point vortex moves along a hyperbolic path given by

$$1/x^2 + 1/y^2 = 1/a^2, \quad (7)$$

FIG. 5. Sequence of vorticity contour plots showing the flow evolution of a dipole colliding with the wall for $Re=5000$ and 10000 . Only the right-hand side symmetrical part of the domain is shown, $[1.0, 1.5] \times [-1, -0.5]$. The contour levels are drawn for $\dots, -100, -60, -20, 20, 60, 100, \dots$

FIG. 6. A dipole collision with a stress-free wall is studied considering the equivalent problem of two dipoles colliding frontally. At the symmetry line $x=2$ the stress-free boundary condition is satisfied. Vorticity contour plots are given for $t=0$ (a) and $t=0.4$ (b).

where (x, y) gives the position with the origin at the intersection of the wall and the symmetry axis. When the point-vortex pair approaches the free-slip wall the vortex separation first slowly increases, but close to the wall the separation speeds up dramatically under the influence of the mirror vortices. Saffman²⁵ investigated the effect of a finite core size using elliptically shaped vortex patches in an inviscid fluid. He concluded that the finite size itself does not cause a rigorous departure from the point vortex model, and that deformations of the vortex cores when colliding with a wall cannot explain the rebound.

Following Saffman's conclusion²⁵ it can safely be conjectured that the trajectories of the Lamb-dipole halves for a normal collision with a free-slip wall (i.e., inviscid flow) are properly described by Eq. (7). Here, the point (x, y) represents the location of either the maximum or the minimum vorticity in the Lamb dipole. The presence of finite viscosity will cause the gradual growth of the vortices. We will now briefly describe how the effects of viscosity can be included in Eq. (7). Nielsen and Rasmussen²⁶ found that a freely moving Lamb dipole keeps its shape in a viscous fluid, but it gradually expands due to diffusion of vorticity. For the change in radius, R , they found

$$R(t) = R(t_0) \sqrt{(\nu \gamma_{11}^2 / R_0^2)(t - t_0) + 1} \quad (8)$$

with $R(t_0)$, the radius at t_0 , and $\gamma_{11} \approx 3.8317$, the first zero of the Bessel function, J_1 . The distance between the vorticity extrema changes accordingly; thus for the point-vortex model we set

$$a(t) = a(t_0) \sqrt{(\nu \gamma_{11}^2 / R_0^2)(t - t_0) + 1}. \quad (9)$$

This means that $a=a(t)$ is now a function of time, i.e., the viscous growth of the Lamb dipole is modeled by a gradual increase of the vortex separation in the point-vortex model. Choosing $t_0=0.15$, i.e., when the shape of the dipole in the simulations is the Lamb-type, we can calculate the value for $a(t_0)$ by inserting the position of maximum vorticity $\mathbf{x}_d(t=0.15)$ in Eq. (7). If one coordinate of the vortex center, e.g., $y_d(t)$, is specified, the model provides the other coordinate, $x_m(t)$, according to

$$\frac{1}{x_m^2(t)} = \frac{1}{a^2(t)} - \frac{1}{y_d^2(t)}. \quad (10)$$

To investigate this model for the trajectory of a Lamb dipole approaching a stress-free wall, we performed a simulation for an equivalent problem of two dipoles colliding head-on for $\text{Re}=1250$. On an expanded domain $[0, 4) \times [-1, 1]$ a right-traveling dipole is initially located at $(1, 0)$ and a left-traveling dipole is initially located at $(3, 0)$ (Fig. 6). The two dipoles will collide at the line $x=2$, which can be considered to be a stress-free boundary. At this line both the vorticity and the velocity component u are by definition equal to zero, thus satisfying the stress-free boundary condition. The trajectory of the center of one of the dipole halves is shown in Fig. 7. For convenience the axis is shifted and rotated, so the wall is located at $y=0$ and the symmetry line at $x=0$. The hyperbolic trajectory found for the inviscid point vortex model [i.e., with $a(t)$ constant] already describes the

FIG. 7. The trajectory of a dipole approaching a stress-free or no-slip wall for $\text{Re}=1250$. (a) For the stress-free case, the position of the maximum vorticity in the core (drawn) is compared to the path of two point vortices (dotted) and the point-vortex path corrected for viscous diffusion (dashed). (b) The path of a dipole colliding with a no-slip wall (black) compared to the path in the stress-free case (gray). Both figures also give the position of the maximum core vorticity at specific times ($+\circ$), where the time is denoted by the labels.

FIG. 8. Trajectories followed by the positive half of the dipole when colliding with the no-slip boundary for $Re = 1250$ (a), 2500 (b), and 5000 (c). The origin of the axes is shifted to coincide with the wall and symmetry axis.

path of the Lamb dipole quite well. If the viscous correction for the increase of the vortex cores is used to correct the hyperbola, we obtain a good prediction of the path of a Lamb dipole approaching a stress-free wall. When the dipole halves change orientation in proximity of the wall, the shape does not resemble a Lamb dipole. The viscous correction is thus not valid from $t=0.3$ to $t=0.35$. This does not have a large effect as for $Re=1250$ viscous effects seems to be relatively small [see inset Fig. 7(a)].

In Fig. 7(b) the path of a dipole colliding with a no-slip wall is compared to the stress-free case. In the initial stages both trajectories overlap and the translation speed is equal, which indicates that the effect of the boundary layers is still rather weak. From $t=0.3$ the trajectory of the dipole colliding with the no-slip wall departs from the stress-free case. The dipole comes less close to the no-slip wall than to the stress-free wall at $t=0.34$ as it is hindered by (detached) boundary layer vorticity. Thereafter an asymmetric dipole is formed when the primary vortices ($V+$) pair with secondary vortices ($v-$). Without any influence of other vortex structures, the asymmetric dipole would follow a circular trajectory. However, mirror vortices (mimicking the influence of the wall) induce a substantial velocity at both the primary and secondary vortex directed along the wall away from the symmetry line. Consequently, the trajectory of the primary vortex core is cycloid-like.

For $Re=1250$ one observes two rebounds [Fig. 8(a)], thereafter the primary vortex core moves slowly along and away from the boundary. Then the primary vortex is surrounded by a ring of opposite vorticity. This shielding causes the very slow translation along the wall. The movement away from the boundary could be caused by the growth in size of the vortex by diffusion, but it is more likely due to the asymmetry of the shielding. When Re is increased to 2500 the primary vortex core follows a cycloidal trajectory with a larger radius after the first collision [Fig. 8(b)]. This implies that the ratio, $\Gamma_r = |\Gamma_{v-}| / |\Gamma_{V+}|$, between the circulation in the secondary ($v-$) and primary vortex ($V+$) has increased. In our view the increased relative strength of the secondary vortex stems from the larger amount of vorticity that is created at the no-slip wall for increased Reynolds numbers. In this particular case, the primary vortices remain close to each other near the symmetry line $x=1$. Recall that for $Re=2500$ the secondary vortices form a small dipole at the symmetry line $x=1$, which subsequently translates away from the wall (Fig. 2). The secondary vortex does not re-enter the region

between the dipole and the wall, leading to a slower drift of the primary vortex off the symmetry axis. In the later stages, $t > 1.0$, the primary vortices are surrounded by a ring of opposite vorticity like for $Re=1250$. For $Re=5000$ the radius of the trajectory of the primary vortex after the first collision becomes even larger than in the previous two examples [Fig. 8(c)]. However, after it reaches its largest distance from the wall, it makes a tighter turn than for the low Reynolds number cases. The secondary vortex is now so strong that primary vorticity is advected around it (Fig. 5), partially shielding the secondary vortex. This shielding causes the dipole to make a tight turn. As a result, the second collision takes place at a larger distance from the symmetry line. Thereafter the trajectory becomes really irregular, due to the presence of a large number of small vortices. An interesting observation is that the mean translation of the primary vortex along the wall is much faster than for the low Reynolds number cases (Fig. 8).

V. BOUNDARY LAYER FORMATION AND DETACHMENT

Lighthill²³ discussed the existence of a diffusive flux of vorticity at a no-slip wall into the flow domain. For a wall moving with velocity V in its own plane we define the x, y -coordinate system with the origin at the wall, x along the wall and y normal to it. For the vorticity flux at the wall, $-\nu \partial \omega / \partial y|_{y=0}$, the x -component of the Navier-Stokes equation yields

$$-\nu \frac{\partial \omega}{\partial y} \bigg|_{y=0} = \rho^{-1} \frac{\partial p}{\partial x} \bigg|_{y=0} + \frac{dV}{dt}. \quad (11)$$

This illustrates that the vorticity flux from the boundaries depends on the tangential pressure gradient at the wall and the boundary acceleration. For a detailed discussion on both these mechanisms, the reader is referred to Morton.²⁷

In the early stages the evolution of the dipole did not differ strongly from the stress-free solution. In the interior domain viscous effects are negligible and thus at the edge of the boundary layer the velocity (free-stream) is assumed to be equal to the velocity u_{sf} at a stress-free wall. Before the impact of the dipole the vorticity at the wall can be approximated by $\omega|_{y=0} \sim -u_{sf} / \delta$ with δ the local thickness of the boundary layer. In Fig. 9 both the vorticity profile and the

FIG. 9. Distribution of vorticity $\omega|_{y=0}$ (black) and vorticity flux, $-\nu\partial\omega/\partial y|_{y=0}$ (gray), along the wall ($y=0$) for $\text{Re}=2500$ at $t=0.2, 0.3$. For $t=0.2$ the dashed line gives the velocity at the wall corresponding to the stress-free case.

vorticity flux Eq. (11) at the wall are shown for the right half of the domain. The profile of the stress-free velocity is similar to the vorticity at the wall. The exact vorticity profile at the wall is due to the cumulative effect of vorticity production at the wall and of inward diffusion of vorticity. For $t=0.3$, the nearby dipole half ($V+$) causes the production of opposite-signed vorticity at the wall for $x \geq 0.1$, as is illustrated by the gray line in Fig. 9. The vorticity at the wall is still of single sign.

Points on the wall where the vorticity changes sign ($\omega|_{y=0}=0$) are dynamically very important. In these points streamlines start or end, indicating that the flow separates from or attaches to the wall. Thus a small region of opposite

boundary layer vorticity (for example, $b+$ in Fig. 3) at the wall indicates the presence of a recirculation cell between the two points on the boundary where $\omega|_{y=0}=0$. To understand the formation of a recirculation region we investigate the vorticity fields at the time $t_{\min}$ when the primary vortex is closest to the wall (Fig. 10). For $\text{Re}=20000$ a detailed view of the boundary region is given in Fig. 11 for times around $t_{\min}$.

At $t=t_{\min}$ the boundary layer ($B-$) is already separating from the wall, although the dipole half ($V+$) hardly moved along the boundary. For all Re values the point of detachment is found about one dipole radius R from the symmetry line $x=1$. The translation speed of the dipole is smaller than

FIG. 10. (Color) Graphs showing the vorticity field for the time $t=t_{\min}$ at which the dipole halves are closest to the no-slip wall for different Reynolds numbers. The recirculation cell is represented by the zero contour of the stream function.

FIG. 11. (Color) Sequence of graphs showing the occurrence of narrow eruptions of vorticity in the boundary layer for $Re=20000$ in the region $[1.13, 1.23] \times [-1, -0.95]$.

the velocity at its edge where it touches the boundary layer. This causes a fast advection of boundary layer vorticity from the rear of the primary vortex to the front. The amount of vorticity in the boundary layer ($B-$) at the front of the dipole is then larger than required to satisfy the no-slip boundary constraint. Oppositely signed vorticity, thus now with the same sign as the vorticity of the primary vortex ($V+$), is produced to balance the situation. Eventually, a secondary boundary layer ($b+$) is formed in front of the primary vortex. Between the points where $\omega|_{y=0}=0$, is connected by a streamline, a recirculation cell is formed which decreases in size for larger Re values.

This picture is confirmed by investigating the vorticity and vorticity flux at the wall for $t=t_{\min}$ (Fig. 12). Recall that $-\nu\partial\omega/\partial y = \rho^{-1}\partial p/\partial x$ at the wall in the absence of boundary acceleration Eq. (11). The pressure on the wall is in first approximation induced by the primary vortex. In the rear part of the vortex the pressure gradient is negative, and thus the flow is accelerated. In the front part the pressure gradient is positive, and the flow decelerates. This is in agreement with an accelerated advection of boundary-layer vorticity ($B-$) below the rear part of the dipole half and the pile-up of this vorticity below the front half. The part of the boundary

layer that is already detached from the wall also induces a pressure gradient on the wall. This causes the two smaller peaks in the pressure gradient for $Re=2500$, visible in Fig. 12 located near $x=0.2$.

The head of the detached boundary layer ($B-$) tends to roll up, which leads to the formation of the secondary vortex ($v-$). For larger Re values this roll-up is much faster as the vorticity in the boundary layer is of substantially larger amplitude. If $Re > \mathcal{O}(10^4)$ the roll-up does not only occur at the head of the detached boundary layer, but also in the boundary layer upstream of the point where the detachment occurs. This leads to the formation of multiple small-scale vortices, which are situated between the wall and the primary vortex.

VI. SHEAR INSTABILITY IN THE BOUNDARY LAYER

For $Re=20000$, and to a lesser extent $Re=10000$, we observe narrow regions of strong eruptions of vorticity (vorticity spikes) that originate at the wall (Fig. 10). A shear instability causes the boundary layer ($B-$) to roll-up even

FIG. 12. Distribution of vorticity ω (black) and vorticity production, $-\nu\partial\omega/\partial y$ (gray), along the wall ($y=0$) for $Re=2500$ and 20000 at $t=t_{\min}$. The primary vortex and the boundary layer vorticity visualized by the light gray contours serve as a reference.

before it detaches from the wall (Fig. 11). A necessary condition for the instability to occur is that there is an inflection point in the velocity profile (Rayleigh criterion), which is the case for a boundary layer in an adverse pressure gradient (see e.g., Ref. 28). Furthermore, for the instability to appear the Reynolds number, $Re^* = u^* \delta_d / \nu$, based on the displacement thickness of the boundary layer (δ_d) and the free-stream velocity (u^*), has to reach a critical value. The critical Reynolds number for the instability to occur is about $Re^* = 600$ for a boundary layer in zero pressure gradient.²⁹ For our setup the critical Reynolds number is hard to determine as the instability occurs at a certain time and place. For the run with domain-based Reynolds number $Re = 20000$ the velocity induced by the vortex core at the point where the instability occurs measures about $u^* = 8$ and the boundary layer thickness is about 1.0×10^{-2} (measured as the distance from the wall where the vorticity changes sign). If the displacement thickness is estimated to be one-third of the boundary layer thickness (see, for example, Ref. 28), we obtain $Re^* \approx 530$ (where we used $\nu = 5 \times 10^{-5}$) thus supporting the possibility of shear instabilities.

Note that the vortex core induces an adverse pressure gradient along the wall for all investigated Reynolds numbers. Shear instabilities are suppressed by viscosity for $Re \lesssim 5000$, but for $Re \gtrsim 10000$ the stabilizing effect of viscosity is too small.

In the region where the roll-up of the primary boundary layer ($B-$) occurs a thin secondary boundary layer of opposite-signed vorticity ($b+$) is present between the wall and the primary boundary layer ($B-$) (Fig. 11). The roll-up in the primary vorticity boundary layer and subsequent formation of small vortices causes a deformation of the secondary boundary layer. A little bulge emerges between two vortices. Upstream of the first vorticity spike the primary boundary layer again rolls up leading to a new eruption of vorticity. In this way a series of small-scale vortices is created. As these vortices are formed in the boundary layer itself, a complicated distribution of vorticity is required to keep the velocity zero at the wall. The vorticity distribution and the vorticity production at the wall becomes highly peaked as can be seen in Fig. 12. A number of these small vortices merge to form the secondary vortex, which pairs with the primary vortex.

The occurrence of vorticity spikes at the boundary was first observed by van Dommelen³⁰ and by van Dommelen and Shen^{31,32} in recirculation cells in the wake of a cylinder placed in a uniform flow. Their simulations were based on a subset of the Navier–Stokes equation that describes the boundary-layer dynamics. Peridier *et al.*^{12,13} used the same subset of equations to study the problem of a point vortex located at a small distance from a no-slip wall. Their situation closely resembles our problem, the same vigorous eruption of vorticity at the wall was observed. More recently, Brinckman and Walker³³ and Obabko and Cassel¹⁴ reported results that reveal the narrow regions with vorticity eruptions. Their results are based on the full Navier–Stokes calculations of the boundary-layer flow induced by a thick-core vortex. A thick-core vortex is essentially nothing more than one of the halves of a Lamb dipole. The thick-core vortex is

used as an initial field and is directly placed next to the no-slip wall at $t=0$. This means that in order to satisfy the no-slip condition, vorticity is created instantaneously at $t=0$ at the wall. Thereafter, the vorticity production follows from the induced pressure gradients at the wall. This setup is thus basically different from our approach, as no vorticity is instantaneously produced at $t=0$ in our case, and the vortices approach the wall gradually. The advantage of placing the thick vortex core near the wall is the possibility to use specially tailored grids, and consequently, reach higher Reynolds numbers.

Obabko and Cassel¹⁴ distinguish between three different Reynolds number regimes. Two are related to the detachment of the boundary layer (large-scale event) and the formation of vortices in the boundary layer due to the vorticity spikes (small-scale event). The Reynolds number is based on the radius of the thick vortex core and the translation speed along the wall. For low Reynolds numbers, $Re < \mathcal{O}(10^4)$, only the large-scale interaction takes place; the formation of a recirculation zone and consequent detachment of the boundary layer. For $Re > \mathcal{O}(10^4)$ both the large-scale interaction and the small-scale eruption of boundary vorticity occur. Another regime in which $Re \rightarrow \infty$ was distinguished by one of these authors a few years earlier.³⁴ Here the vorticity spikes occur, causing the formation of the small vortices, but the large-scale ejection of boundary layer vorticity in the domain is not observed.

VII. BOUNDARY LAYER SHEAR AND SECONDARY VORTICES

The size and strength of the vortices formed from wall-generated vorticity depend on the thickness of the boundary layer and the vorticity amplitude. If the shear inside the boundary layer is strong, vorticity eruptions occur, creating small vortices with a large vorticity amplitude. For weaker boundary layers, a vortex is formed from the detached boundary layer. The strength of these secondary vortices depends on whether they originate from the small-scale vorticity eruptions or from the large-scale boundary-layer detachment and thus on the strength of the primary boundary layer.

We have determined the circulation of the primary boundary layer, Γ_{B-} , and the secondary boundary layer, Γ_{b+} , by numerically integrating the vorticity in these regions. Separate contiguous regions can be distinguished where the absolute vorticity is larger than a certain level ($\omega = \omega_d/50$). A factor 2 reduction of the selection level leads only to a small increase of the circulation by 0.5% or less. The circulation of the different regions are given in Table IV. Note that when the dipole reaches the wall, it is considerably weaker for lower Reynolds numbers, e.g., the circulation Γ_{V+} of a dipole half is 2.76 for $Re=625$, while it is 3.71 for $Re=20000$. To partially counteract this problem we also give the ratio between the circulation in the primary vortex ($V+$) and the circulation related to the opposite vorticity that is created at the wall $|\Gamma_{B-} + \Gamma_{v-}|$, i.e., a combination of the circulation of the secondary vortex ($v-$) and the circulation of the (partially) detached boundary layer ($B-$). For $Re \gtrsim 10000$ the circulation in the boundary layer seems to become virtually

TABLE IV. Specification of the circulation Γ_{V+} of the positive dipole half, circulations Γ_{B-} , and Γ_{b+} in the primary and secondary boundary layers, respectively. At $t=0.5$ the circulation Γ_{v-} of the secondary vortex is also given. In Table V the values for $t_{\min}$ are specified.

Re	t	Γ_{V+}	Γ_{v-}	Γ_{B-}	Γ_{b+}	$\frac{ \Gamma_{B-}+\Gamma_{v-} }{ \Gamma_{V+} }$	$\frac{ \Gamma_{v-} }{ \Gamma_{V+} }$
625	$t_{\min}$	+2.76	...	-1.67	+0.041	0.605	...
625	0.50	+2.18	-0.86	-0.74	+0.003	0.732	0.394
1250	$t_{\min}$	+3.13	...	-2.00	+0.085	0.641	...
1250	0.50	+2.49	-1.17	-0.72	+0.047	0.762	0.472
2500	$t_{\min}$	+3.38	...	-2.31	+0.143	0.681	...
2500	0.50	+2.72	-1.42	-0.76	+0.079	0.803	0.524
5000	$t_{\min}$	+3.55	...	-2.55	+0.221	0.719	...
5000	0.50	+2.85	-1.52	-0.92	+0.119	0.857	0.535
10000	$t_{\min}$	+3.64	...	-2.80	+0.339	0.769	...
10000	0.50	+3.02	-1.70	-0.90	+0.112	0.863	0.565
20000	$t_{\min}$	+3.71	...	-2.92	+0.389	0.787	...

independent of the Reynolds number, as it only changes from $\Gamma_{B-}=-2.80$ to -2.92 when we increase from $\text{Re}=10000$ to 20000 . This can be expected, as the pressure gradient along the wall Eq. (11) determines the vorticity production.¹⁰ Note that the ratio $|\Gamma_{B-}+\Gamma_{v-}|/|\Gamma_{V+}|$ increases even if the primary vortex core moves away from the wall. Thus vorticity is still being produced during this stage of the evolution. A part of the vorticity that is created at the wall will end up in the secondary vortex. The strength of this vortex is highly dependent on the strength of the boundary layers, and thus increases for higher Reynolds numbers.

From $t=t_{\min}$ to $t=0.5$ the circulation in the primary vortex decreases due to cross-diffusion with boundary-generated vorticity. An equal amount of positive and negative vorticity is lost, thus the circulation related to boundary-produced vorticity has changed by an amount $\Gamma_{V+}(t=0.5)-\Gamma_{V+}(t_{\min})$. If we

add this number to $\Gamma_{B-}+\Gamma_{v-}$, we can correct for the loss due to cross-diffusion. The resulting circulation related to boundary-produced vorticity is -3.22 for $\text{Re}=10000$. Compare this to the circulation in the primary vortex at $t_{\min}$, $\Gamma_{V+}=+3.64$. Thus a nearly equal amount of circulation is generated at the wall, as is present in the primary vortex. For lower Reynolds numbers the difference is larger, e.g., -2.18 generated against $+2.76$ in the primary vortex for $\text{Re}=625$. After the first collision multiple small vortices are generated if $\text{Re}=20000$, and the vorticity distribution becomes too complex to measure the circulation of the various structures. Clercx and van Heijst¹⁰ found that the circulation in the boundary layer becomes independent of the Reynolds number for $\text{Re} \gtrsim 20000$. It is thus likely that the amount of vorticity generated during the first collision does not exceed the circulation of the primary vortex.

The secondary vortices are stronger for larger Reynolds numbers. This also leads to an increased ratio between the strength of the secondary and primary vortex, $|\Gamma_{v-}|/|\Gamma_{V+}|$, if the Reynolds number is increased to 5000 . For $\text{Re}=10000$ secondary vortices are also created by the shear instability in the boundary layer. Some of the smaller vortices merge to form one secondary vortex. The relative strength of this vortex is higher than for $\text{Re}=5000$, as reflected by the higher ratio $|\Gamma_{v-}|/|\Gamma_{V+}|$.

It is interesting to see how the enstrophy changes due to the production of vorticity at the wall. This change is governed by Eq. (6), where the term

$$\nu \oint_{\partial \mathcal{P}} \omega(\mathbf{n} \cdot \nabla \omega) ds \quad (12)$$

represents the enstrophy increase or decrease by vorticity flux at a straight wall. For the dipole-wall collision, the enstrophy is always increased by the production of vorticity at the wall (Fig. 13). The integrand in Eq. (12) is not necessarily positive for all points on the wall, e.g., in Fig. 9 for $t=0.3$ there is a region where the vorticity ω at the wall is positive while there is an influx of negative vorticity. The peaks in the enstrophy production coincide with the time the primary vortices are close to the wall. If they move away from the wall the enstrophy production drops to zero, reflecting that there is minimal vorticity production at the wall.

FIG. 13. Graphs showing the change in total enstrophy due to vorticity production at the wall for $\text{Re}=2500$ and 10000 .

TABLE V. For time $t_{\min}$ the dipole halves are closest to the wall, the total enstrophy is subdivided in the enstrophy of the dipole half Ω_{V+} , of the primary boundary layer Ω_{B-} , and of the secondary boundary layer Ω_{B+} . As a comparison, the time $t_{\max}$ is given when the total enstrophy $\Omega_{\max}$ is maximum.

Re	$t_{\min}$	Ω_{V+}	Ω_{B-}	Ω_{B+}	$t_{\max}$	$\Omega_{\max}$
625	0.3744	166.9	280	1.4	0.3710	466.9
1250	0.3424	247.1	671	10.2	0.3414	949.6
2500	0.3278	300.1	1283	50.3	0.3278	1656
5000	0.3208	331.0	2177	219.9	0.3234	2770
10000	0.3182	348.0	3559	747.7	0.3201	4713
20000	0.3160	357.4	5544	1102	0.3129	7385

The enstrophy that is located in the boundary layers for the time the primary vortex is closest to the wall is specified in Table V. Clercx and van Heijst¹⁰ found that the peak enstrophy scales like $\Omega_{\max} \propto \text{Re}^{0.5}$ for $\text{Re} > 20000$. The peak in the enstrophy occurs when the primary vortex is closest to the wall. For lower Reynolds number they found that the peak enstrophy decreases much faster with decreasing Re, consistent with our observation of a decrease faster than $\text{Re}^{1.0}$ between $\text{Re}=1250$ and 625. The scaling relation, $\Omega_{\max} \propto \text{Re}^{0.5}$, is based on the assumption that the circulation in the boundary layer is independent of the Reynolds number. An assumption that seems to be reasonable, in agreement with the minor change of the circulation of the boundary layer as observed in the previous section when Re is changed from 10000 to 20000. The deviation at lower Reynolds numbers is not surprising, as viscous diffusion weakens the primary vortex considerably before it reaches the wall. The approach of the dipole to the wall for $\text{Re} \geq 20000$ depends less on the Reynolds number; thus the interior flow can be considered to be the same. However, the observed scaling behavior at large Reynolds numbers is quite surprising as the boundary layer itself is very active at $\text{Re}=20000$ and does not resemble a laminar single-signed vorticity boundary layer as assumed for the model. We also observe a strong increase of enstrophy contained by the secondary boundary layer, changing from 0.5% to 20%, relative to the enstrophy in the primary boundary layer, when Reynolds is increased from 625 to 20000.

VIII. CONCLUSION

The dipole-wall collision is a model problem to investigate vortex interactions with the wall. If the dipole is still at some distance from the no-slip wall, the dynamics are not different from the free-slip or stress-free case. The thin boundary layer created to satisfy the no-slip constraint, is not very strong and remains attached to the wall. A strong departure from inviscid flow theory occurs when the dipole is close to the wall and the boundary layer detaches from the wall. The detachment of the boundary layer stems from the adverse pressure gradient at the wall induced by the dipole halves. The adverse pressure gradient causes the formation of a secondary boundary layer with vorticity with sign opposite to that of the primary boundary layer. Hence, a circulation cell is formed that advects primary boundary layer vorticity into the interior of the domain.

Secondary vortices are created from the vorticity generated at the wall by two different mechanisms. For Reynolds numbers smaller than $\mathcal{O}(10^4)$, the detached boundary layer rolls up and forms a single vortex. If $\text{Re} > \mathcal{O}(10^4)$ a shear instability occurs in the boundary layer. As a result strong vorticity eruptions occur in the secondary boundary layer that cut off parts of the primary boundary layer. Multiple small-scale vortices are created between a dipole half and the no-slip wall, which later merge to one secondary vortex.

The secondary vortex forms together with the primary vortex and an asymmetric dipole that translates along a cycloidal trajectory. For $\text{Re} \leq 2500$ the dipole halves are shielded by opposite vorticity in the later stages, slowing down the translation along the wall. For larger Reynolds numbers, the separation between the two dipole halves increases much faster, though still not as fast as observed when the stress-free conditions are applied at the wall.

The increased separation speed is of some interest to the problem of airplane trailing vortices that interact with the ground during touchdown. It is known that these vortices can hover (in the absence of a strong cross-wind) for a considerable time over the landing strip, thereby hindering the landing of following airplane. The Re value of the present results is smaller than typical for trailing vortices, i.e., $\text{Re}=5 \times 10^6$. In the results of our simulations we observe that the separation between the two dipole halves increases faster for higher Reynolds number. The path that the dipole half follows becomes irregular for $\text{Re}=5000$. This is due to the multiple interactions with secondary and tertiary wall-generated vortices. Spalart *et al.*³⁵ uses simulations with different turbulence models to obtain results for $\text{Re}=5 \times 10^6$. At this Reynolds number they also observe the viscous rebound of the dipole. It depends on the used turbulence model whether the typical cycloidal trajectory is retrieved. This emphasizes the importance to resolve the fine-scale structures near the wall. The narrow vorticity eruptions that we observe for $\text{Re} \geq 10000$ might have an influence on the trajectory of the separated dipole halves. Three-dimensional instabilities might also have a considerable effect on the dipole-wall collision and in particular on the lifetime of the vortices.

Of interest to two-dimensional turbulence is the detachment of wall-generated vorticity. Clercx and van Heijst¹ argued that this injection of small-scale vorticity acts as a forcing of the interior flow at a forcing scale comparable to the boundary-layer thickness. In their numerical simulations of

decaying two-dimensional turbulence in a bounded domain they observed an inverse energy cascade starting at the boundary-layer scale. This is opposed to numerical simulations on a periodic domain, where in the same range a direct enstrophy cascade was observed. Later, Wells *et al.*⁷ provided more proof with a numerical simulation in which the injection of wall-generated vorticity was the only mechanism of forcing the interior flow. The size and strength of the vortices that interact with the wall in the turbulence simulations correspond to the lower Reynolds number dipole-wall collisions (typically $Re \leq 2500$). For higher Reynolds number it can be stated that the vortices injected into the flow become smaller but have a higher vorticity amplitude. This is of importance, as boundary-generated vortices can destabilize even domain-sized vortices.^{36,37} The question arises if the stronger wall-generated vortices can even prevent the formation of a domain-sized structure. If the Reynolds number is increased even further, e.g., $Re > 10000$, the eruption of vorticity spikes at the boundary leads to the formation of multiple vortices. The small-scale vorticity spikes can bring the forcing effects of the wall to even larger wave numbers.

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