BOOK REVIEW

Lawrence Dresner, *Similarity Solutions of Nonlinear Partial Differential Equations*, Pitman, London, 1983, 124 pp., \$18.95

This short book is intended to teach the method of similarity solutions as a practical technique for solving nonlinear partial differential equations. It is well written, and I think the author accomplishes his task in a remarkably elegant way.

The basic idea to exploit the algebraic symmetry of a partial differential equation is especially due to the work of Birkhoff (1950), although its roots can be traced back to Boltzmann (1894), who studied the diffusion of particles with a diffusion coefficient proportional to their concentration.

As a preliminary, Chapter 2 is devoted to a discussion of Lie's formula for the integrating factor of an ordinary differential equation, and to the use of the direction field to find the global behavior of the solutions. In Chapter 3 Birkhoff's idea is applied to linear diffusion, especially to a problem that arises when water is desalted by passing it under pressure through a semipermeable membrane. This simple application illustrates the important point that in order to apply the method of similarity solutions to a problem the boundary conditions must have the same invariance as the equation itself. Moreover, as the method reduces a partial differential equation to an ordinary one, some of the original boundary conditions must "collapse" to the same condition so that the solution of the ordinary differential equation is not overdetermined.

In Chapter 4 Dresner illustrates the method for nonlinear partial differential equations by applying it to several variations of Boltzmann's problem quoted before, to the thermal expulsion of fluid from a heated tube, to transient heat transfer in superfluid helium, and to the percolation of a turbulent fluid into a porous half-space. This very practical didactic approach is continued in Chapter 5 with the study of various boundary-layer problems: The Prandtl-Blasius problem of a flat plate, thermal boundary layers, and free convection boundary layers.

Chapter 6 is entirely devoted to nonlinear wave phenomena, for which the symmetry group is either a stretching group or a translation group. As an example of the first Dresner studies longitudinal waves in a nonlinear elastic medium. The method leads naturally to characteristics, Riemann invariants, and shock waves. Next, transverse waves are considered, using the shock-loaded membrane as a guinea pig. In this case the equation of motion is a nonlinear integropartial differential equation; nevertheless the method can still be applied nicely. Another application is to the problem of determining the water profile

behind a breaking dam. Finally the case in which the symmetry group is a translation group is considered by studying the traveling wave with which a nonsuperconducting zone spreads in a superconducting wire.

In the final chapter Dresner shows how for some problems that do not have a similarity solution the limiting behavior can still be described in terms of a similarity solution. It is here that the subject of this monograph makes contact with the method of matched asymptotic expansions, another important tool of modern fluid dynamics.

I greatly enjoyed reading this book. The basic ideas are clearly explained and applied to most interesting examples from the natural sciences. It would make an excellent textbook for any course on this subject, in part because of the collection of 20 additional problems (with outlines of their solutions) at the end. It should also be of general use to any researcher who runs into a nonlinear differential equation and wonders what to do next. Highly recommended.

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