Two high-frequency mutual inductance bridges with high resolution

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Abstract Two mutual inductance bridges are described for operation up to about 100 kHz. Special attention is paid to the sensitivity and resolution of the bridges. Both bridges can be used to measure variations of about 10 pH in the mutual inductance. The first bridge consists of passive elements only whereas the second bridge is equipped with active circuits.

1 Introduction

The dynamic susceptibility of magnetic materials is usually measured by means of a mutual inductance bridge (Van de Klundert *et al* (1975) and references therein). In case one is interested in the isothermal susceptibility only (e.g. in measuring the Curie law in thermometry experiments), the bridge may operate at a fixed frequency which is often chosen in the range from 10 to 1000 Hz in order to avoid impurity effects of the various bridge elements. Recently two examples of these fixed-frequency bridges were published by Brodbeck *et al* (1978) and Whitmore *et al* (1978). In our laboratory we perform spin-lattice relaxation experiments and therefore investigate the susceptibility as a function of the frequency. The frequency range of the mutual inductance bridge determines the range of the time constant to be measured so that it is necessary to have a bridge with a broad bandwidth.

In this paper we describe two mutual inductance bridges for operation up to about 100 kHz. Special attention has been paid to the sensitivity and resolution of the bridges. Both bridges can be used to measure variations in the mutual inductance of about 10 pH.

The first bridge consists of passive elements only. Using a toroidal system in order to minimise stray fields and furthermore three continuously variable variometers, the unknown mutual inductance is directly available from the meter readings. We have already used this bridge successfully for several years in our relaxation experiments.

The second bridge is an electronic mutual inductance bridge. In this bridge some of the principles used by Brodbeck *et al* (1978) are combined with the principles of the passive bridge. The result is a very sensitive high-frequency mutual inductance bridge which has a small size, is easy to make and simple in operation.

2 General principle and cryostat coils

A simplified diagram of the mutual inductance bridges we use for our spin-lattice relaxation experiments is given in figure 1. The magnetic sample is placed in a primary-secondary coil system (cryostat coils) inside a superconducting magnet. The primary system consists of two astatically wound coaxial coils. The secondary coil consists of two sections with an equal

Figure 1 A simplified diagram of the mutual inductance bridge. A and B are connected to the differential input of a phase sensitive detector, whose reference signal is derived from Ref.

number of turns wound in opposite direction. Denoting the coefficients of mutual inductance between the primary coil and the two secondary sections by M_{ps1} and M_{ps2} respectively, the induced voltage can be written as

$$U_{ind} = i2\pi\nu(M_{ps1} - M_{ps2})I_{pr} + i2\pi\nu M_{ps1}f\chi I_{pr}$$

where I_{pr} is the primary alternating current (frequency ν), f the filling factor and χ the susceptibility of the sample placed in the first section. Although M_{ps1} is made as near as possible equal to M_{ps2} there always remains a certain imbalance which has to be compensated. As M_{ps1} and M_{ps2} are not ideal inductances at higher frequencies, the compensating system consists of a variable mutual inductance and a resistive network. The susceptibility is a complex quantity in relaxation experiments, $\chi = \chi' - i\chi''$, where $\chi'(\nu)$ is the dispersion and $\chi''(\nu)$ the absorption. With the aid of a pure variable inductor the phase-detection apparatus is adjusted in such a way that χ' and χ'' are measured simultaneously on two X-Y recorders as a function of the external field.

In order to be able to perform accurate measurements up to high frequencies attention must be paid to:

- (i) the inductive coupling between the primary-secondary coil system and the superconducting magnet,
- (ii) the purity of the bridge elements,
- (iii) the cable capacities.

The coupling between the cryostat coils and the superconducting magnet, which leads to a field-dependent empty-coil effect, is minimised by the astatic design and the small size of the primary and secondary system (figure 2). It is further minimised by optimal adjustment of the cryostat coils inside the magnet.

A primary-secondary coil system with high resonance frequencies can be obtained using coils consisting of a single layer of turns of silk covered wire (in our case with a diameter

Figure 2 The primary-secondary coil system and the superconducting magnet. A, adjusting mechanism; B, superconducting magnet; C, transformer housing; D, transformer.

of 0.07 mm). The primary coil has 763 turns, the anti-primary coil 351 turns and the two secondary sections each 188 turns (resonance frequencies > 3 MHz). This system is used for the range 10 Hz–100 kHz. For the range 1 Hz–10 kHz we use a coil where the secondary coil has sections of ten layers of 188 turns (resonance frequencies > 0.6 MHz). The primary current is maximal 150 mA so that the harmonic magnetic field is about 1.03 kA m^{-1} (13 Oe). We generally use about 10 mA. In §§3 and 4 attention is paid to the purity of the other bridge elements.

The capacities of the cables connecting the various bridge elements limit to a high extent the maximal usable bridge frequency. The cable lengths should be kept as short as possible.

The mutual inductance between the primary set and one section of the secondary coil amounts to about $2 \times 10^{-4} \text{ H}$ for the high-frequency system and $2 \times 10^{-3} \text{ H}$ for the low-frequency system. It is quite possible to balance the primary-secondary system in such a way that the imbalance is within 1% at liquid-helium temperatures. A bridge with a range of $20 \mu\text{H}$ is required to compensate this off-balance signal. The effect of the sample is usually considerably smaller than this value. For instance, a single crystal of cerium magnesium nitrate with perpendicular susceptibility $\chi_{\perp} \approx 3.5 \times 10^{-3}$ (SI units, $\chi_{\perp}$ is dimensionless) at 4.2 K produces an effect of $0.07 \mu\text{H}$ in the high-frequency set and $0.7 \mu\text{H}$ in the low-frequency set when the filling factor is 10^{-1} . In relaxation experiments one is often interested in small changes of the susceptibility (in the order of $10^{-3}\chi$, Flokstra *et al* (1978)) so it is necessary to have a bridge with a good resolution. If the high-frequency coil set is used a resolution in the order of 10 pH is advisable.

3 The passive mutual inductance bridge

The first bridge we developed for relaxation measurements has a variable mutual inductance consisting of a toroidal system (figure 3) and a variometer system (figure 4). The primary toroidal coil has 180 turns and one turn is wound back parallel to the centre line. This coil, wound on a perspex toroid with a rectangular cross section, is fixed in a toroidal perspex box, on which the secondary turns are brought. There are ten sections of ten turns; each section consisting of 2×5 turns diametrically, so obtaining an astatic coil. Further the toroid is equipped

Figure 3 The toroidal system: A, the single turn; B, the five astatic sections each of two turns; C, one astatic section of ten turns.

Figure 4 The variometer: A, housing; B, secondary ring covered with a layer of gold; C, perspex case; D, connecting spindle between the two secondary parts; E, spacer; F, movable primary system; G, stop; H, sliding contact.

with five sections each of two astatic turns and finally a single turn. The coefficient of mutual inductance for one winding is $0.320 \mu\text{H}$ so that the maximal range of the variable inductor is about $35 \mu\text{H}$.

The variometer system is composed of three variometers covering different ranges. The first one is continuously variable from -320 nH to $+320 \text{ nH}$, the second from -32.0 nH to $+32.0 \text{ nH}$ and the third from -3.20 nH to $+3.20 \text{ nH}$. Each variometer is divided into 200 scale parts so that one scale part on the most sensitive variometer corresponds to 32 pH . The secondary side is the same for all three variometers. The primary windings are movable on to a cylindrical case enabling the three variometers to be adjusted. The most sensitive variometer is achieved by two turns, wound opposite and close to each other so that an almost compensated situation is obtained.

The resistive network of the compensating system is equipped with metal film resistors. The voltage at B in figure 1

Figure 5 The resistive network of metal film resistors. The values of the resistors are given in Ω .

Figure 6 The frequency response of the bridge without cryostate coils when a ten-turn (T10) or a two-turn (T2) section of the toroid has been switched on. $M(\nu)$ is the coefficient of mutual inductance.

is usually written as $I_{pr}R_{eff}$, so $R_{eff} = r_1 r_2 / (r_1 + r_2 + R)$. The R_{eff} range is from -5.5 to $+5.5 \Omega$; the smallest step is about $10^{-6} \Omega$. The negative values of R_{eff} are obtained by a symmetrical configuration of the resistors with regard to the ground (figure 5). Negative R_{eff} values are necessary in order to obtain compensation under all circumstances.

In figure 6 the frequency response of the bridge without the cryostate coils is given when a ten-turn or a two-turn section of the toroid is switched on. The toroidal mutual inductance is very pure up to 100 kHz. This result was also obtained for the three variometers and the resistive network. The resonance at 470 kHz is due to the wiring of the bridge. The length of the wiring is mainly a result of the large spacing between the variometers which is necessary in order to prevent magnetic coupling between them (size of the bridge: $650 \times 650 \times 450$ mm).

The resonance frequency of the bridge decreases considerably when the cryostat coils are connected (to ≈ 140 kHz). The capacity of the cables to the coils plays an important part. Compensation up to 100 kHz is possible, so that measurements can be performed up to this frequency.

4 The electronic mutual inductance bridge

A diagram of the electronic mutual inductance bridge is presented in figure 7. Compensation of the off-balance signal of the cryostat coils is obtained by the mutual inductor T_1 and the active circuits I and II. In circuit I an amplifier A_1 is used, which generates a variable current I_m in phase with the primary current I_{pr} . With the aid of the mutual inductor T_2

Figure 7 Diagram of the electronic bridge. A and B are connected to the differential input of a phase sensitive detector, whose reference signal is derived from Ref.

the inductive part of the off-balance signal can then be compensated. In circuit II, A_2 and A_3 generate a voltage in phase and 180° out of phase with I_{pr} respectively, so that the losses can be compensated using the phase detector as null-detector.

The mutual inductor T_1 is a toroidal system similar to that described in §3. The toroid used is a modification of toroid No. 7 of table 1 in the paper of Van de Klundert *et al* (1975). The secondary coil of toroid No. 7 is now used as primary coil whereas the primary windings have been replaced by a secondary system equal to that of figure 3. A single turn represents an inductance of $0.233 \mu\text{H}$ so that the range of T_1 is about $26 \mu\text{H}$.

The range of circuit I for the compensation of inductive signals should cover at least the inductance of the single turn of T_1 . We use a precision high speed op-amp LM 318N which has a small-signal bandwidth of 15 MHz, a slew rate of 70 V/ μ s and an output current of 15 mA. Because I_{pr} does not exceed 150 mA, we need an inductor T_2 with an inductance of at least 2.3 μ H. We chose toroid No. 4 of table I in the paper of Van de Klundert *et al.* (1975), which has a mutual inductance of 4.47 μ H. Thus compensation at even the largest primary currents used is secured whereas the compensation range at the normally used value $I_{pr}=10$ mA covers several toroid turns. The input voltage for the op-amp can be varied by a switch and two ten-turn potentiometers. The switch selects the voltage over an array of 10 Ω resistors. The potentiometers are used for coarse and fine compensation. The linear response of this potentiometer arrangement is slightly disturbed, but it is not the linearity of a ten-turn potentiometer we use but its resolution. For instance, if the potentiometer for coarse compensation is half way along its range we can vary the input voltage of the op-amp 2% with the resolution of a ten-turn potentiometer. We chose a fixed value for the amplification factor ($10\times$) because the broad bandwidth of the op-amp may lead to oscillations caused by the inductive properties of a ten-turn potentiometer in the feedback loop. Even now we had to insert a 10 pF capacitor in the feedback loop. The phase shift between I_{pr} and the

voltage induced by T_2 is not exactly 90° . This is caused by the feedback capacity, the two ten-turn potentiometers and the not pure resistive character of the load of A_1 . The latter causes only a small effect, for the self inductance of the coil is $4.47 \mu\text{H}$ so that the coil impedance at 100 kHz is about 3Ω whereas the resistance is about 1100Ω . The total deviation from the 90° phase shift is about 6° at 100 kHz.

The compensation of signals in phase with I_{pr} (circuit II) can be adequately achieved by one ten-turn potentiometer of 200Ω and a voltage divider for the range from 10^{-6} to 0.5. The small signal resolution is very high. The large signal resolution is limited to that of a ten-turn potentiometer but in experiments large loss signals seldom occur. The amplification factor of A_2 and A_3 (type LM 318N) is +10 and -10 for compensation of the in phase and 180° out of phase signals, respectively. The phase shift between V_B and I_{pr} is about 5° at 100 kHz, mainly caused by the ten-turn potentiometer.

Due to the phase shift the two compensation circuits are not quite independent but in practice compensation was rapidly achieved even in the high-frequency range of the bridge. The actual settings of selectors and potentiometers are not necessary for measuring χ' and χ'' but only one element, pure up to 100 kHz, is required for a correct adjustment of the phase detection quadrature and the calibration of the X - Y recorder deflection. We again took for this element the single turn of the toroid. Recorder calibration for signals less than that of one toroid turn can be achieved with circuit I. The voltage change over the load resistance can be measured when the single turn of the toroid is compensated. A simple relation between this voltage change, $V_{1 \text{ turn}}$, and I_{pr} can be obtained by adjusting the load resistance so that it is easy to calibrate the recorders in parts of the inductance of the single turn. In our case $V_{1 \text{ turn}}$ is equal to $50 I_{\text{pr}}$.

We tested the linearity of the electronic bridge using the variable inductance of the passive bridge for the generation of the off-balance signal. A typical result is given in figure 8 where the response of the electronic bridge is plotted against the variometer setting. We found the linearity of the bridge to be better than 0.2%.

Figure 8 The response of the electronic bridge as a function of the variometer setting of the passive bridge.

The electronic bridge could be constructed much smaller than the passive bridge because the variometers are replaced by high-resolution electronic circuits. The bridge (size $410 \times 130 \times 250 \text{ mm}$) can be easily built in a 482 mm (19 in) rack. We are now working on an electronic bridge where the large toroidal system T_1 is omitted.

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