AN INVESTIGATION ON THE TEMPERATURE DISTRIBUTION IN THINWALLED CYLINDERS DURING WELDING UP OF WEAR-PROOF LAYERS

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Abstract

This paper describes an attempt to find the space and timewise temperature-distribution in a thinwalled cylinder during building up by welding several layers on top of each other. From the exact one-dimensional solution (thin ring) we found an approximate solution for the cylinder. The approximation used is made possible by the fact that in practice the angular velocity (which is related to the welding speed) is great with respect to the relaxationtime of heat in the material. A number of experiments are given to verify the theoretical results.

Nomenclature

a	coefficient of heat diffusivity ($\lambda/\rho c$)	$\text{m}^2 \text{s}^{-1}$
c	specific heat at constant pressure	$\text{J kg}^{-1} \text{K}^{-1}$
d	thickness of the deposited layer	m
D	wallthickness cylinder	m
λ	coefficient of heat conduction	$\text{W m}^{-1} \text{K}^{-1}$
ρ	specific mass	kg m^{-3}
q	power absorbed by the specimen	W
Q	power per unit of area	W m^{-2}
R	radius of the cylinder	m
S	variable of transformation to the one sided Laplace transform	s^{-1}
t	time	s
T_1	temperature cylinder	K
T_2	temperature reservoir	K
T_0	temperature at $t = 0$	K

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ΔT	$Q/\rho c \omega R$	K
ω	angular velocity	rad.s ⁻¹
ϕ	co-ordinate fixed frame	rad.
ϕ'	co-ordinate $\phi' = \phi - \omega t$	rad.

§ 1. Introduction

In the manufacturing of valvehouses it is common practice to hard-surface with wearresistant alloys, by welding in several layers. A problem here is that whereas the requirements for this layers can be formulated the final mechanical properties as hardness are not known, because the time history of the temperature distribution during the welding process is not known. During the welding process steel suffers temperature changes from about 1800 K to room temperature. The rate of change of temperature is crucial for the eventual structure and its properties. To ensure the quality of the wear-resistant layers it is therefore necessary to find out in an empiric manner for each type of construction in what way this welding should be carried out. This procedure is expensive and tedious and would be much more simpler when an estimate of the space and timewise temperature distribution during welding could be made. This paper describes an attempt to do this. As a typical geometry we choose a semi-infinite thinwalled cylinder. This approximates practical geometry rather well.

A heatsource of strength q , which represents the arc, rotates along the cylinderwall with angular velocity ω (see Fig. 1).

In sections 2 and 3 an approximate calculation of the temperature in the cylinder will be given.

The approximation used is made possible by the fact that in practice ω^{-1} is small with respect to the relaxationtime R^2/a of heat in the material. Here R is the radius of the cylinder and a is the heat diffusivity in steel. In section 2 the problem of a thin ring is considered, which permits an exact solution. In section 3 we make use of the results for the thin ring to develop an asymptotic theory for large values of $\omega R^2/a$ for a semi-infinite thin cylinder. Section 4 gives the solution for an infinite cylinder. In section 5 a number of experiments are described, carried out in the Laboratory for Fluid Mechanics and Heat Transfer of our institute, to verify the theoretical results. Since the present report is a first attempt to give a theoretical account of the temperature distribution in valvehouses

Fig. 1. Semi-infinite thinwalled cylinder.

Fig. 2. Thin steel ring, height h , thickness D .

during welding, we restricted ourselves to considering the heat-conductivity λ as independent of the temperature. The consequences of this approximation will be discussed when later in the paper the experimental results are compared with theory.

§ 2. Thin ring

As a first step, we consider as shown in Fig. 2, a thin steel ring of radius R , height h and thickness D ($h \ll R$, $D \ll R$, h/D of unit order). A heat source of strength q rotates with angular velocity ω in clockwise direction. The heat supply starts at time $t = 0$.

In a frame in which the heat source is stationary, at $\phi = 0$, the temperature T in the ring is given by:

$$-\frac{\partial T}{\partial t} + \omega \frac{\partial T}{\partial \phi} = \frac{a}{R^2} \frac{\partial^2 T}{\partial \phi^2}. \quad (2.1)$$

Here a is the heat diffusivity in steel, regarded as constant during the process, and connected with the density ρ , the specific heat c and the heat conductivity λ by:

$$a = \frac{\lambda}{\rho c}. \quad (2.2)$$

The boundary and initial conditions for (2.1) are:

$$T(0, \phi) = 0, \quad (2.3)$$

$$T(t, \phi) = T(t, \phi + 2\pi), \quad (2.4)$$

$$-\frac{\lambda}{R} \left(\frac{\partial T}{\partial \phi} \right)_{\phi=0} + \frac{\lambda}{R} \left(\frac{\partial T}{\partial \phi} \right)_{\phi=2\pi} = QH(t). \quad (2.5)$$

In (2.5) $H(t)$ is Heaviside's unit stepfunction whereas Q is the source density

$$Q = \frac{q}{hD}. \quad (2.6)$$

(2.1) subject to the conditions (2.3)–(2.5) is conveniently solved by defining the Laplace transform of T by:

$$\Theta = \int_0^\infty e^{-st} T(t, \phi) dt. \quad (2.7)$$

We obtain in a straight forward way for Θ the expression:

$$\Theta = \frac{QR e^{p\phi}}{2\lambda sk} \left\{ \frac{e^{-2\pi p} \sinh k\phi + \sinh k(2\pi - \phi)}{\cosh 2\pi k - \cosh 2\pi p} \right\}. \quad (2.8)$$

where

$$p = \frac{\omega R^2}{2a}, \quad (2.9)$$

$$k = \left(p^2 + \frac{2sp}{\omega} \right)^{\frac{1}{2}}. \quad (2.10)$$

By making use of the inverse Laplace transform and defining:

$$l = \frac{n^2}{2p}. \quad (2.11)$$

we obtain by known methods (see e.g. Mc.Lachlan [1])

$$T = \frac{QR\omega t}{4\pi\lambda p} + \sum_{n=1}^{\infty} \frac{QR}{2\pi p\lambda(l^2 + n^2)} [e^{-\omega t l} \{n \sin n(\omega t - \phi) - l \cos n(\omega t - \phi)\} + n \sin n\phi + l \cos n\phi]. \quad (2.12)$$

Under practical circumstances the parameter p which represents the ratio of the angular velocity of the source and the heat diffusion velocity in the material is of order 10 and larger. For example when welding with a welding speed ωR of $7 \cdot 10^{-3} \text{ m s}^{-1}$, we obtain for $R = 10^{-1} \text{ m}$, with $a = 14 \cdot 10^{-6} \text{ m}^2 \text{ s}^{-1}$, the value $p = 50$.

In Fig. 3 the temperature for points on the ring being initially in $\phi = -\pi/4, -2\pi/4$ etc. is drawn, as numerically obtained from (2.12). From this figure we see that the temperature in a given point along the ring rises sharply to $Q/\rho c \omega R$ when the heat source passes this point. Due to the relatively slow diffusion of heat the temperature remains at a constant level until the heat source passes again. Heatconduction is therefore of importance only in the immediate vicinity of the point where the source is located and

Fig. 3. Thin ring solution, $p = 10$, $\omega = (2\pi/100) \text{ s}^{-1}$.

where the temperature rises steeply. The form of the temperature curve here may also be obtained by considering the asymptotic solution of (2.1) for large p :

introducing

$$t = \frac{\tau}{\omega}, \quad \phi = \frac{\psi}{p}, \quad (2.13)$$

(2.1) reads:

$$\frac{1}{p} \frac{\partial T}{\partial \tau} + \frac{\partial T}{\partial \psi} = \frac{1}{2} \frac{\partial^2 T}{\partial \psi^2}. \quad (2.14)$$

Hence, for $p \rightarrow \infty$

$$T = A_1(\tau) e^{2\psi} + A_2(\tau). \quad (2.15)$$

The solution that satisfies the condition:

$$-\frac{\lambda}{R} \left(\frac{\partial T}{\partial \phi} \right)_{\phi=0^+} + \frac{\lambda}{R} \left(\frac{\partial T}{\partial \phi} \right)_{\phi=0^-} = QH(t),$$

is for $t > 0$:

$$\left. \begin{aligned} \phi > 0: \quad T &= \frac{Q}{\rho c \omega R}, \\ \phi < 0: \quad T &= \frac{Q}{\rho c \omega R} \exp \frac{\omega R^2}{a} \phi. \end{aligned} \right\} \quad (2.16)$$

By writing: $\phi = \phi' + \omega t$, (2.16) becomes:

$$\left. \begin{aligned} t < -\frac{\phi'}{\omega}: \quad T &= \frac{Q}{\rho c \omega R} \exp \frac{\omega R^2}{a} (\phi' + \omega t), \\ t > -\frac{\phi'}{\omega}: \quad T &= \frac{Q}{\rho c \omega R}. \end{aligned} \right\} \quad (2.17)$$

This solution corresponds with the exact solution in Fig. 3 for the first revolution of the source. For the second and following revolutions similar expressions hold.

§ 3. Welding in several layers on a thinwalled cylinder

The character of the solution (2.17) provides the key for an approximate solution of the semi-infinite cylinder case. If heat would not diffuse in axial direction, which we indicate with z , the temperature

Fig. 4. Theoretical model for the real situation.

distribution would be as in Fig. 3, that is to say constant between subsequent passages of the arc. Due to heat diffusion in z -direction the temperature will decrease in time between these passages. During this process diffusion in ϕ -direction can be left out of account, because the timescale $a/\omega^2 R^2$ is much smaller than the period of the heat source. With reference to Fig. 4 we therefore approximate the real situation with the following model: The temperature of a thin layer of thickness D and height h is brought instantaneously to a value $Q/\rho c \omega R$ above roomtemperature T_0 . The remainder of the cylinder, $0 < z < \infty$, is initially at $T = T_0$ and receives for $t > 0$ heat by diffusion from the layer between $z = 0$ and $z = -h$.

In the real situation the arc puts heat into the area $0 < z < \infty$ through a melting bath. In the model the height h represents the effective height of the burner i.e. the total heatcapacity of the molten steel is represented in the model by an area hD with temperature $Q/\rho c \omega R$.

We may consider $z = -h$ as insulated because heat losses from the ring by processes other than diffusion can be safely neglected.*

Denoting the temperature in the region $-h \leq z \leq 0$ with T_2 and in $0 \leq z \leq \infty$ with T_1 , we have:

$$\frac{\partial T_{1,2}}{\partial t} = a \frac{\partial^2 T_{1,2}}{\partial z^2}, \quad (3.1)$$

* An estimate of heat losses by radiation and free convection from the cylinder leads to the conclusion that this is of the order of 2% of q .

with the conditions:

$$\left. \begin{aligned} T_1(0, z) &= T_0, \\ \frac{\partial T_1}{\partial z} &= \frac{\partial T_2}{\partial z} \quad \text{at } z = 0, \\ T_1 &= T_2 \quad \text{at } z = 0, \\ T_2(0, z) &= T_0 + \Delta T_1 = T_0 + \frac{Q_1}{\rho c \omega R}, \\ \frac{\partial T_2}{\partial z} &= 0, \quad \text{at } z = -h. \end{aligned} \right\} \quad (3.2)$$

This holds during the first revolution $0 < t < 2\pi/\omega$, during which the supplied heat is Q_1 .

The eqs. (3.1) with the conditions (3.2) are solved again by means of Laplace transformation. We obtain:

$$T_1 - T_0 = \frac{\Delta T_1}{2} \left\{ \operatorname{erfc} \left(\frac{z}{2\sqrt{at}} \right) - \operatorname{erfc} \left(\frac{z + 2h}{2\sqrt{at}} \right) \right\}. \quad (3.3)$$

$$T_2 - T_0 = \Delta T_1 \left[1 - \frac{1}{2} \left\{ \operatorname{erfc} \left(\frac{z + 2h}{2\sqrt{at}} \right) + \operatorname{erfc} \left(\frac{-z}{2\sqrt{at}} \right) \right\} \right]. \quad (3.4)$$

For the value $h = 6.5 \times 10^{-3}$ m, (3.3) and (3.4) are graphically drawn in Fig. 5 and Fig. 6 which represent temperature as a function of time for various values of z .

For $h \ll z$ (3.3) may be written as:

$$T_1 - T_0 = \frac{\Delta T_1 h}{(\pi at)^{\frac{1}{2}}} + 0 \left\{ \left(\frac{h}{z} \right)^2 \right\}, \quad (3.5)$$

which agrees with the wellknown source solution of the heat equation. Indeed at distances $z \gg h$, the temperature should behave as if at $t = 0$ an amount of heat $\rho ch \Delta T_1$ per m^2 is applied.

It is also useful to consider the asymptotic behaviour of $T_1 - T_0$ and $T_2 - T_0$ for large times. For $h \ll 2\sqrt{at}$ and $z \ll 2\sqrt{at}$, both the right hand sides of (3.3) and (3.4) reduce to:

$$\frac{h \Delta T_1}{\sqrt{\pi at}} + 0 \left\{ \left(\frac{z}{2\sqrt{at}} \right)^3 \right\}.$$

Fig. 5. Thinwalled cylinder solution T_1 for $h = 6,5 \cdot 10^{-3}$ m; $a = 1,4 \cdot 10^{-5}$ m² s⁻¹; $\Delta T = 2000$ K; $T_0 = 300$ K.

Fig. 6. Thinwalled cylinder solution T_2 for $h = 6,5 \cdot 10^{-3}$ m; $a = 1,4 \cdot 10^{-5}$ m² s⁻¹; $\Delta T = 2000$ K; $T_0 = 300$ K.

Therefore we may write:

$$T_1 - T_0 = \frac{\Delta T_1 h}{(\pi a t)^{\frac{1}{2}}}, \quad z \ll \sqrt{a t}. \quad (3.6)$$

For $t = 100$ s and $a = 14 \cdot 10^{-6} \text{ m}^2 \text{ s}^{-1}$, $(a t)^{\frac{1}{2}} \approx 4 \cdot 10^{-2} \text{ m}$.

This shows that when the period of the burner has a typical value of 100 s, the temperature distribution in the first centimeter of the material depends only on time at the end of the first revolution. This is illustrated also in Figs. 5 and 6.

At large z the temperature is given by (3.5). Now we consider what happens during the next revolution of the welding head. A new layer of thickness d_1 is deposited and the process starts again, however the initial conditions in the material are given now by (3.3) and (3.4) for $t = 2\pi/\omega$.

Since these conditions are for values of $z \ll 2\sqrt{a t}$ independent of z we may shift the origin to $z = -d_1$, as shown in Fig. 7.

We may now repeat the analysis of the first revolution, replacing however, z and t by $z' = z + d_1$, $t' = t - 2\pi/\omega$, respectively. For $t' = 0$, (3.3) gives the initial condition in $z' > 0$. The introduced error is in the far field, because the near field is at $t' = 0$ independent of z . Replacing in (3.5) z by z' gives:

$$\exp\left[-\frac{(z' - d_1)^2}{4at}\right] = \exp\left[-\frac{z'^2}{4at} - \frac{d_1^2}{4at} + \frac{z'd_1}{2at}\right] \approx \exp\left[-\frac{z'^2}{4at}\right]$$

in the region $z' \gg d_1$.

Fig. 7. Shifting the origin the to $z' = z + d_1$.

The temperature distribution during the second revolution is in the region $z' > 0$, $2\pi/\omega < t < 4\pi/\omega$

$$T_1 - T_0 = \frac{\Delta T_1}{2} \left\{ \operatorname{erfc} \left(\frac{z + d_1}{2\sqrt{at}} \right) - \operatorname{erfc} \left(\frac{z + 2h + d_1}{2\sqrt{at}} \right) \right\} + \\ + \frac{\Delta T_2}{2} \left\{ \operatorname{erfc} \left(\frac{z + d_1}{2\sqrt{a(t - 2\pi/\omega)}} \right) - \operatorname{erfc} \left(\frac{z + d_1 + 2h}{2\sqrt{a(t - 2\pi/\omega)}} \right) \right\}. \quad (3.7)$$

Here $\Delta T_2 = Q_2/\rho c \omega R$. Because in one revolution the supplied heat may be different from the next, we indicate this value with Q_2 for the second revolution. Note that the solution (3.7) is so easily obtained here because the initial temperature distribution for the second revolution is a solution of the differential equation.

Likewise we obtain after two revolutions:

$$T_1 - T_0 = \frac{\Delta T_1}{2} \left\{ \operatorname{erfc} \left(\frac{z + d_1 + d_2}{2\sqrt{at}} \right) - \right. \\ \left. - \operatorname{erfc} \left(\frac{z + 2h + d_1 + d_2}{2\sqrt{at}} \right) \right\} + \\ + \frac{\Delta T_2}{2} \left\{ \operatorname{erfc} \left(\frac{z + d_1 + d_2}{2\sqrt{a(t - 2\pi/\omega)}} \right) - \operatorname{erfc} \left(\frac{z + 2h + d_1 + d_2}{2\sqrt{a(t - 4\pi/\omega)}} \right) \right\} + \\ + \frac{\Delta T_3}{2} \left\{ \operatorname{erfc} \left(\frac{z + d_1 + d_2}{2\sqrt{a(t - 4\pi/\omega)}} \right) - \operatorname{erfc} \left(\frac{z + 2h + d_1 + d_2}{2\sqrt{a(t - 4\pi/\omega)}} \right) \right\}. \quad (3.8)$$

For comparison with experiments the quantities q_1 , q_2 , q_3 , d_1 , d_2 and h are required. The quantities d_1 , d_2 can be obtained from making a cut through the cylinder after the experiment (see for further details section 5). The quantities q_1 , q_2 and q_3 can unfortunately not be measured directly. Of course, in an experiment the total heat output by the arc is known but a considerable fraction of this is radiated away and the portion supplied to the material is not known. For the specific configuration of a flat plate Eichhorn [2] determined experimentally the ratio of power output by the arc to the heat absorbed by the specimen. Since for each configuration this ratio will be a different one and also the power output in

the experiments by Eichhorn was an order of magnitude larger than in our experiments, no reliable data can be deduced pertinent to the present case.

§ 4. Infinite configuration

The equations and conditions are identical with the equations of the semi-infinite cylinder with the exception of the insulating condition, see (3.2)

This becomes: $\partial T_2 / \partial z = 0$ at $z = -h/2$, since the problem now is symmetric. The equations for the infinite configuration can be found by replacing h by $h/2$ in the arguments of the complementary error functions in (3.3), (3.4), (3.7) and (3.8).

§ 5. Experiments

In order to obtain the best possible approximation of the practical situation a MAG* welding apparatus of great capacity was used for the supply of heat. MAG welding was chosen to prevent formation of slug as to make it possible to deposit several layers on top of each other. Rotation was provided by means of a turning table with variable angular speed. Temperature was measured with Chromel-Alumel thermocouples. The millivoltage delivered by these thermocouples was recorded during the measurements. The power delivered by the welding apparatus was recorded continuously.

5.1. Determination of the quantities q_1 , q_2 and q_3

As we have seen before we can write for $(z + 2h)/2\sqrt{at} \ll 1$, (3.3) as:

$$T_1 - T_0 = \frac{\Delta T_1 h}{\sqrt{\pi at}},$$

with

$$\Delta T_1 = \frac{q_1}{\rho c \omega R D h}.$$

Hence

$$q_1 = \rho c \omega R D \sqrt{\pi at} (T_1 - T_0). \quad (5.1)$$

(5.1) contains two unknown parameters: q_1 and T_1 .

* MAG welding is a welding process with an electric arc burning between a consumable wire and the workpiece. The wellpool is protected from the air by a so-called inactive gas, e.g. CO_2 or CO_2 containing gas mixtures.

By measuring T_1 we can determine q_1 . In the same way q_2 and q_3 can be found by measuring T_1 in the second and third revolution respectively.

$$q_2 = \rho c \omega R D \sqrt{\pi a t} (T_1 - T_0) - q_1, \quad (5.2)$$

$$q_3 = \rho c \omega R D \sqrt{\pi a t} (T_1 - T_0) - q_2 - q_1, \quad (5.3)$$

because the total amount of supplied heat is defined by:

$$Q_{\text{tot}} = \frac{q_{\text{tot}}}{Dh} = \int_0^{\tau} \delta(t) Q(t - \tau) d\tau = Q(t = 0) + \\ + Q\left(t = \frac{2\pi}{\omega}\right) + Q\left(t = \frac{4\pi}{\omega}\right) = \frac{q_1}{Dh} + \frac{q_2}{Dh} + \frac{q_3}{Dh}. \quad (5.4)$$

In the sense described here, our theory is semi-empiric but as explained at the end of section 3 there is no other way to determine independently q with sufficient accuracy.

5.2. Determination of h , d_1 and d_2

The effective height of the burner h cannot be exactly determined. Based upon visual observation we took, in calculating the temperature distribution, h equal to D in (5.4) and similar ones. In the experiments carried out in our laboratory we chose the wallthickness D equal to the effective diameter of the welding torch. The quantities d_1 and d_2 were determined as follows:

Fig. 8. Schematized photograph of a cut through the cylinder.

Fig. 8 schematizes a photograph of a cut through the cylinder after the experiment. From pictures of this type we measured for each revolution a representative value for d_1 and d_2 .

5.3. Experiments carried out in the laboratory

In preparing the theoretical curves shown in the following figures we used for T_1 (3.3), (3.7) and (3.8).

To calculate the temperature during the first revolution in $z = 0$, (3.4) for T_2 has been used.

The figures represent results for the following series of measurements.

Figure 9 Series I: Semi-infinite cylinder, temperature at $z = 0$.

Figure 10 Series II: Semi-infinite cylinder, temperature at $z = 10^{-2}, 2 \cdot 10^{-2}, 3 \cdot 10^{-2}, 4 \cdot 10^{-2}$ m.

Figure 11 Series III: Infinite cylinder, temperature at $z = 10^{-2}$ m.

5.4. Discussion

We used for each experiment 4 thermocouples at the same z -location but at different values of ϕ' . All these thermocouples gave the same

Fig. 9. Semi-infinite cylinder, $p = 25,8$; $\omega = 0,09 \text{ s}^{-1}$; $d_1 = 1,5 \cdot 10^{-3} \text{ m}$; $d_2 = 2 \cdot 10^{-3} \text{ m}$; $h = 6,5 \cdot 10^{-3} \text{ m}$; $z = 0 \text{ m}$; $q_1 = 1648 \text{ W}$; $q_2 = 2269 \text{ W}$; $q_3 = 1996 \text{ W}$.

Fig. 10. Semi-infinite cylinder, $p = 23$; $\omega = 0.09\text{ s}^{-1}$; $d_1 = 1.5 \cdot 10^{-3}\text{ m}$; $d_2 = 2 \cdot 10^{-3}\text{ m}$; $h = 6.5 \cdot 10^{-3}\text{ m}$; $z = 0.01, 0.02, 0.03, 0.04\text{ m}$; $q_1 = 1170\text{ W}$; $q_2 = 1492\text{ W}$; $q_3 = 1409\text{ W}$.

temperature curve as a function of $(t - \phi/\omega)$. This supports the assumption that the diffusion in ϕ -direction was negligible. Apart from the errors made by measuring and by the approximate character of the theory (which is correct to order p^{-1}) we notice the following differences:

Fig. 11. Infinite cylinder, $p = 22$; $\omega = 0,087 \text{ s}^{-1}$; $d_1 = d_2 = 0 \text{ m}$; $h = 6,5 \cdot 10^{-3} \text{ m}$; $z = 0,01 \text{ m}$; $q_1 = 1706 \text{ W}$; $q_2 = 2398 \text{ W}$; $q_3 = 2494 \text{ W}$.

Fig. 12. Semi-infinite cylinder with different values of D , $p = 25,8$; $\omega = 0,09 \text{ s}^{-1}$; $d_1 = 1,5 \cdot 10^{-3} \text{ m}$; $d_2 = 2 \cdot 10^{-3} \text{ m}$; $h = 6,5 \cdot 10^{-3} \text{ m}$; $z = 0 \text{ m}$; $D_1 = 6,5 \cdot 10^{-3} \text{ m}$; $D_2 = 5,4 \cdot 10^{-3} \text{ m}$; $D_3 = 4,6 \cdot 10^{-3} \text{ m}$; $q_1 = 1648 \text{ W}$; $q_2 = 1606 \text{ W}$; $q_3 = 930 \text{ W}$.

Fig. 13. Welding up layers on the lower half of a cut valvhouse.
Fig. 14. Welding up layers in a full valvhouse.

- The maximum theoretical temperature (at $t = 0$) will be higher than the highest temperature measured, because the theory does not include a melting bath.
- The theory supposes constant properties of the material resulting in a smooth temperature profile.

However, at about 1400 K the structure suffers a transformation leading to a discontinuity in λ . Indeed our experimental results, especially Fig. 9 and 12, show definitely a discontinuity in the temperature profile. We used the value for λ for temperatures < 1400 K (the ferritic value) and because this temperature range embodies most of the total temperature range, the effect of the different value of λ above 1400 K does not affect the results significantly.

- The theory supposes D and h as a constant; Fig. 8 shows that D varies over the different layers; variation in h is not probable. To improve the theoretical results we calculated also the temperature for series I, however this time using instead of a constant D the pertinent values of the thickness obtained from the photographs as shown in Fig. 8. These results are together with the experimental data represented in Fig. 12. From this figure we see that a major part of the difference between theory and experiment as appearing in Fig. 9 is accounted for by the variation in D .

5.5. *Experiments with actual valvehouses**

During these experiments we recorded the temperature distribution in the practical situation (see also Fig. 13 and 14). The configuration of a valvehouse is not quite exactly a cylinder. The heat supply does not take place over the whole wallthickness. The theory of the foregoing sections is therefore not applicable without modification.

We followed this procedure: First experiments were carried out with deposition of a layer on the lower half of a cut valvehouse (Fig. 13). These measurements were brought to fit in with the theory for a semi-infinite cylinder by looking for a best choice of an effective value for D (Fig. 15).

Subsequently measurements were carried out with a full valvehouse and these measurements were compared with the theoretical

* These experiments were carried out at Dijkers N.V. at Hengelo, Netherlands.

Fig. 15. Semi-infinite cylinder (cut valvehouse), $p = 41,7$; $\omega = 0,09 \text{ s}^{-1}$; $d_1 = 2 \cdot 10^{-3} \text{ m}$; $h = 5 \cdot 10^{-3} \text{ m}$; $z = 0 \text{ m}$; $q_1 = 1172 \text{ W}$; $q_2 = 1405 \text{ W}$.

Fig. 16. Infinite cylinder (full valvehouse), $p = 41,7$; $\omega = 0,09 \text{ s}^{-1}$; $d_1 = 1,5 \cdot 10^{-3} \text{ m}$; $d_2 = 3 \cdot 10^{-3} \text{ m}$; $h = 5 \cdot 10^{-3} \text{ m}$; $z = 5 \cdot 10^{-3} \text{ m}$; $q_1 = 4090 \text{ W}$; $q_2 = 3268 \text{ W}$; $q_3 = 3064 \text{ W}$.

results for an infinite cylinder, using the above mentioned effective value for D . These results are represented in Fig. 16. We conclude from Fig. 16 that it is possible to get a reasonably accurate prediction for the temperature distribution in a practical valvehouse geometry, from calculations on a cylinder, once by one experiment an effective value for D is obtained.

Received 9 January 1975

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