An Electrostatic Lower Stator Axial-Gap Polysilicon Wobble Motor Part I: Design and Modeling

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Abstract—This paper presents design issues and a theoretical model of electrostatically driven axial-gap polysilicon wobble motors. The motor design benefits from large axial rotor-to-stator overlap and large gear ratios, and motor designs with rotor radii of 50 and 100 μm are capable of generating torques in the nanoNewtonmeter range at high electrostatic fields. Because of the large gear ratio, smaller angular steps and lower rotational speed are obtained, compared to radial-gap motor designs. Aspects like gear ratio, torque generation, excitation schemes and torque coverage, normal forces, friction, rotor kinetics, and dynamical behavior are addressed. The motor design is compliant to the integration of gear linkages with respect to mechanical power transmission. [180]

Index Terms—Electrostatic, micromotor, model.

I. INTRODUCTION

INITIAL micromechanical motor design and fabrication attempts have been axial-gap architectures because of large estimated driving torque [1]–[4]. However, these designs suffered from instabilities in tilting, and to a lesser extent in vertical perturbations, and fabrication complexity that finally led to the development of radial-gap or side-drive micromotors [5]–[8]. Salient-pole and wobble side-driven micromotors have been successfully fabricated.

Although the performance of these micromotors is still improving, they exhibit some drawbacks. The rotor to stator overlap is small, resulting in small driving torques. Typically, a few picoNewtonmeters for salient pole micromotors and a few tens of picoNewtonmeters for wobble motors. This is merely the result of the rotor and stator thickness that is generally only a few microns. Furthermore, it is difficult to implement the radial gap design into a system with respect to mechanical power transmission. In outer stator designs the stator completely surrounds the rotor making it difficult to transfer the mechanical energy to another structure when using planar fabrication techniques, while for inner stator designs it is difficult to make the electrical stator connections [9], [10].

A number of different side-driven wobble motors have been investigated [9]–[13]. Wobble motors are able to generate a larger driving torque compared to salient pole micromotors because of their gear ratio. The gear ratio increases the driving torque at the cost of a decreasing angular speed. Also, in

Fig. 1. Sketch of the operation principle.

wobble motors friction is expected to be lower because of a rolling—instead of a sliding—motion.

A lower stator axial-gap wobble motor design can solve some of the limitations of the radial-gap or side-drive design. This design was first presented by Paratte [14]–[16]. The tilting, vertical, and radial rotor instabilities of this lower stator axial-gap design are constrained by the bearing and stator geometry. The larger rotor-to-stator overlap results in a larger torque generation and successful drive of a gear train has already been realized by a hybrid design based on electroplating and assembling techniques [17].

This paper forms Part I of a set of two papers. Here, the design and modeling of electrostatically driven axial-gap lower-stator polysilicon micromotors is presented, and expressions for the gear ratio, torque generation, normal forces, rotor kinetics, and dynamics are given. Part II deals with fabrication issues and performance characteristics [18].

II. OPERATION PRINCIPLE

The motor is sketched in Fig. 1. The rotor is resting in its center at a pin or ball bearing. When a voltage is applied between the rotor and one of the stator poles, the rotor will be pulled down toward a contact point at the angular center of the excited stator pole. Switching to other stator poles will move this contact point around, and the rotor is forced to roll at its outer radius resulting in a rocking motion. Because of a difference in radius between the rotor and the resulting contact point circle, the rotor will be rotated by a small angle after one sequential activation of all stator poles. By proper commutation of the charge distribution on the stator electrodes and the rotor, continuous motion of the rotor can be achieved. The ratio between the angular speed of the stator poles and the rotor is called the gear ratio n .

III. THEORETICAL MODEL

A. Gear Ratio

For small rocking angles, in the absence of rotor slip and mechanical deformations, the harmonic reduction ratio of the angular velocity between the stator and rotor is given by the

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Fig. 2. Schematic cross-sectional view of the rotor.

nominal gear ratio. The nominal gear ratio is dependent on motor dimensions and can be found from the ratio of the rotor radius and the difference between the rotor radius and the contact point radius. The radius of the resulting contact point circle R_c can be found from

$$R_c = \sqrt{R^2 + d^2} + (h_b - d) \sin \theta \quad (1)$$

where R is the rotor radius, d is the axial-gap distance at the rotor center, h_b is the height of the bearing pin, and θ is the angle between the rotor and the stator surface, which is approximately equal to d/R (see Fig. 2). This yields the following expression for the nominal gear ratio n_0 :

$$n_0 = \frac{R}{R - R_c} \approx -\frac{2R^2}{2dh_b - d^2}. \quad (2)$$

Note that the gear ratio is negative when R_c is larger than R , which means that the rotor rotates in a direction opposite to the excitation of the stator poles.

B. Torque Generation

The tilt angle θ of the rotor is very small. Therefore the electrostatic field can be assumed to be vertical. Furthermore, fringing fields have been neglected, and the rotor is assumed to be a rigid disk. An analytical model of the torque generation based on these assumptions has been shown to be in good agreement with finite-element-method (FEM) simulations for other axial-gap motors [19]. The axial-gap spacing g between the substrate and the rotor is equal to (see Fig. 2)

$$g = [R - r \cos(\varphi - \alpha)] \sin \theta \approx d \left[1 - \frac{r}{R} \cos(\varphi - \alpha) \right]. \quad (3)$$

The electrostatic coenergy E'_{el} of an excited stator pole can be found by integration over the excited stator region from angle φ_1 to φ_2 and radius R_i to R_o (see Fig. 3) yielding

$$E'_{el} = -\frac{1}{2} CV^2 = -\frac{1}{2} \int_{\varphi_1}^{\varphi_2} \int_{R_i}^{R_o} \frac{\epsilon_0 r dr d\varphi}{g + \frac{d_{ins}}{\epsilon_r}} V^2 \quad (4)$$

where r is the radius, φ is a variable for the angle, α is the contact point angle of the rotor, ϵ_0 is the dielectric constant in air, d_{ins} is the thickness of the dielectric insulator between the stator and rotor, ϵ_r is the relative dielectric constant of this layer, and V is the applied voltage. In the absence of rotor slip, the torque generated by the motor τ_{motor} can be found from the negative derivative of the electrostatic coenergy with respect to the rotor angle α

$$\tau_{motor} = \frac{1}{2} \frac{dC}{d\alpha} V^2. \quad (5)$$

Fig. 3. Illustration of parameters α , φ , φ_1 , φ_2 , R_o , and R_i .

Fig. 4. Single-phase torque of a 3-, 4-, 6-, and 12-stator-pole design as function of the rotor angle. The driving voltage is equal to 100 V. The rotor radius R is 100 μm , the axial-gap distance d is 2 μm , and the thickness of the dielectric layer d_{ins} between the stator and rotor is 0.2 μm and has a relative dielectric constant of 7.5. The stator outer radius is equal to the rotor radius ($R_o = R$), and the inner stator radius R_i is equal to half the rotor radius (50 μm).

An example of the single-phase torque as function of the rotor angle α is shown in Fig. 4 for several stator-pole designs. Dimensional values are given in the figure caption. The torque generation can be in the range of nanoNewtonmeters at electrostatic fields of 10^8 V/m. The maximum torque is generated when the rotor contact point is located at one of the edges of the excited stator pole. The driving torque can be increased by increasing the angular width of the excited stator poles up to π rad by reducing the air-gap spacing and the thickness of the dielectric layer between the rotor and the stator by increasing the relative dielectric constants and by increasing the rotor diameter. The torque scales proportional to the third power with motor radius when the gear ratio and electric field strength are kept constant.

C. Excitation Schemes and Torque Coverage

The stator needs a minimum of three poles in order to generate a driving torque. A stator with two poles with an angle of π rad will only result in a rocking motion, but does not turn the rotor. To ensure a one-directional rotation,

Fig. 5. Torque coverage in the case of open-loop, single-pole, and double-pole excitation for a four-pole-stator design with dimensions and variables as given in Fig. 4. Frictional forces are neglected. The average torque in the case of single- and double-pole excitation is, respectively, 305 and 204 pNm.

a power supply with at least three independent phases is required. In this paper, the independent phases are represented by alphabetical symbols, given in bold capitals when activated, and normal format when deactivated. The total amount of alphabetic symbols equals the number of stator poles. For example, the excitation scheme for a motor with three stator poles, which are excited one by one is represented by **A***bc*, *a***B***c*, and *ab***C**.

The overall torque generation τ_{coverage} for different driving schemes or excitation patterns can be found from the enveloping curve of excited stator poles (see Figs. 5 and 6). The average torque τ_{av} can be found from

$$\tau_{av} = \frac{1}{2\pi} \int_0^{2\pi} \tau_{\text{coverage}} d\alpha \quad (6)$$

and the normalized torque ripple τ_{ripple} is

$$\tau_{\text{ripple}} = \frac{\tau_{\text{max}} - \tau_{\text{min}}}{\tau_{av}}. \quad (7)$$

The output power of the motor can be found from

$$P_{\text{motor}} = \tau_{av} \omega_{av} \quad (8)$$

where ω_{av} is the average angular speed of the rotor.

1) *Open Loop*: In open-loop drive, the simplest case for motor operation is a sequential excitation of the different stator poles, which is called “single-pole open-loop excitation” (e.g., **A***bcd*, *a***B***cd*, *ab***C***d*, *abc***D**, etc. for a four-stator-pole design). In this case, the rotor rotates until it reaches the zero-torque position at the angular center of the excited stator pole (when friction is neglected). Switching to the adjacent stator pole results in an inefficient position with respect to torque generation. This can be improved when using a “double-pole open-loop excitation.” In this case, two adjacent stator poles are excited simultaneously, and in the next step, this couple is

Fig. 6. Torque coverage in the case of closed-loop, single-, and double-pole excitation for a four-pole-stator design with dimensions and variables as given in Fig. 3. The average torque for the single-pole excitation is 313 pNm, and the average torque for the double-pole excitation is 350 pNm.

rotated by one stator pole (e.g., **A***B**cd*, *a***B***C**d*, *ab***C***D*, *A**b**c***D**, etc.). Now, the starting position of the rotor is always at the edge of an excited stator couple, where the torque generation is highest. The torque coverage of a four-stator-pole design in the case of single-pole, open-loop, and double-pole open-loop excitation is shown in Fig. 5.

2) *Closed Loop*: Closed-loop control requires a feedback of the rotor position in order to switch to the next stator pole at the right moment. Feedback of the rotor position is not implemented at this point. In the case of closed-loop control, the torque ripple decreases with an increasing number of stator poles. However, the maximum torque also decreases because the capacitor area of one stator pole decreases with an increasing number of poles. By simultaneously exciting a group of adjacent stator poles, the driving torque can be increased again. The torque generation in the case of “single-pole, closed-loop,” and “double-pole closed-loop excitation” is shown in Fig. 6 for a four-stator-pole design.

These single- and grouped-pole excitation schemes require a power supply with a number of independent phases that equals the number of stator poles. This may be a problem in the case of a large number of stator poles, which can be overcome by a parallel connection of stator poles. However, the simultaneous excitation of multiple stator groups will somewhat reduce the torque generation again.

D. Normal Forces and Friction

If the sliding frictional torque balances the electrostatic torque τ_{motor} generated by the motor, rotor slip is avoided, resulting in a pure rolling motion. In Fig. 7, the forces and torques are shown when a small part dA of the stator is excited. In the z direction, perpendicular to the stator surface, all forces are in equilibrium. The total axial electrostatic force F_{NeI} can be found by integration over the surface of the excited stator pole from angle φ_1 – φ_2 and radius R_i – R_o . This axial electrostatic force is balanced by normal reaction forces at the rotor contact point $F_{N\text{contact}}$ and the center bearing $F_{N\text{bearing}}$.

Fig. 7. Sketch of the forces and torques acting on the rotor.

This results in the following expression:

$$-\frac{1}{2}\epsilon_0 V^2 \int_{\varphi_1}^{\varphi_2} \int_{R_i}^{R_o} \frac{r}{\left(g + \frac{d_o}{\epsilon_r}\right)^2} dr d\varphi + F_{N\text{contact}} + F_{N\text{bearing}} = 0, \quad (9)$$

As a result of the axial electrostatic force, a torque $d\tau = dF_{Nel} \times r$ is acting on the rotor (see Fig. 7). This torque can be decomposed in a component $d\tau_{ang}$ in the angular direction and a component $d\tau_{rad}$ in the radial direction. The radial component is responsible for the rocking motion and provides the driving torque of the motor. The rotor is not rotating in the angular direction. Therefore, all the torques acting in the angular direction are in equilibrium. The angular electrostatic torque component can be found by multiplying the electrostatic force dF_{Nel} by the arm $r \cos(\varphi - \alpha)$ over which it is acting. The total angular component of the electrostatic torque is again found by integration over the excited region. The torque balance around the center of the rotor gives

$$F_{N\text{contact}} R - \frac{1}{2}\epsilon_0 V^2 \int_{\varphi_1}^{\varphi_2} \int_{R_i}^{R_o} \frac{r^2 \cos(\varphi - \alpha)}{\left(g + \frac{d_o}{\epsilon_r}\right)^2} dr d\varphi = 0. \quad (10)$$

The normal bearing force $F_{N\text{bearing}}$ and normal contact point force $F_{N\text{contact}}$ can be found from the force balance in the normal z direction (9) and the angular torque balance (10). This results in the following expressions for the normal force at the contact point and the normal force at the center bearing:

$$F_{N\text{contact}} = \frac{1}{2}\epsilon_0 V^2 \int_{\varphi_1}^{\varphi_2} \int_{R_i}^{R_o} \frac{r^2 \cos(\varphi - \alpha)}{R \left(g + \frac{d_o}{\epsilon_r}\right)^2} dr d\varphi \quad (11)$$

$$F_{N\text{bearing}} = \frac{1}{2}\epsilon_0 V^2 \int_{\varphi_1}^{\varphi_2} \int_{R_i}^{R_o} \frac{r [R - r \cos(\varphi - \alpha)]}{R \left(g + \frac{d_o}{\epsilon_r}\right)^2} dr d\varphi. \quad (12)$$

Fig. 8. Decomposition of radial torque into motor torque and rocking torque.

Fig. 9. The normal force at the contact point $F_{N\text{contact}}$ and the normal force at the centerpoint $F_{N\text{bearing}}$ as a function of the rotor contact angle α for a four-pole-stator design using the same parameter values as in Fig. 3.

The radial electrostatic torque component can be found by multiplying the electrostatic force dF_{Nel} by the arm $r \sin(\varphi - \alpha)$ over which it is acting. As the rotor is tilted, the radial electrostatic torque τ_{rad} can be divided in a component that provides the rocking motion ($\tau_{rad} \cos \theta$) and a component in the z direction ($\tau_{rad} \sin \theta$) that provides the driving torque of the motor (see Figs. 7 and 8)

$$\tau_{\text{motor}} = -\frac{1}{2}\epsilon_0 \sin \theta V^2 \int_{\varphi_1}^{\varphi_2} \int_{R_i}^{R_o} \frac{r^2 \sin(\varphi - \alpha)}{\left(g + \frac{d_o}{\epsilon_r}\right)^2} dr d\varphi. \quad (13)$$

This expression is equal to (5) that was derived earlier. The normal force at the contact point $F_{N\text{contact}}$ and the normal force at the centerpoint $F_{N\text{bearing}}$ are shown in Fig. 9 for a four-stator-pole design as function of the contact point angle.

Sliding and rolling friction results from two bodies that are, respectively, sliding or rolling against each other. The frictional forces are generally depending on the normal forces between the two bodies. A pure rolling motion exists when the sliding frictional torque is larger than the electrostatic torque on the rotor. For the motor, this no-slip condition is given by

$$\text{sgn}(\dot{\alpha})(F_{N\text{contact}} R \mu_{\text{contact}} + F_{N\text{bearing}} r_{\text{bearing}} \mu_{\text{bearing}}) - \tau_{\text{motor}} \geq 0 \quad (14)$$

where μ_{contact} and μ_{bearing} are the sliding frictional coefficients at, respectively, the contact point and bearing.

In Fig. 9, it is shown that the normal force at the contact point is very large around the region of the excited stator pole, and it even changes sign when the contact point reaches a

position at the opposite side of the excited stator pole because the rotor wants to flip over. In this region, the frictional torque as a result of the normal force at the contact point will be zero. However, there still is a frictional torque in this region as a result of the normal force at the bearing. Provided that the bearing radius and sliding frictional coefficient at the bearing are large enough (for example, $R_{\text{bearing}} = 10 \mu\text{m}$ and $\mu_{\text{bearing}} = 0.3$), the sliding frictional torque is still in the same range as the motive torque in this region. An appropriate choice of excitation scheme can be used to sustain continuous large normal forces that will prevent rotor slip. For example, an excitation scheme where the step angle is smaller than the totally excited angular region like in the double-pole excitation will always operate at large normal forces. During normal operating conditions our theoretical model indicates that the motor will always work under no-slip conditions.

1) *Kinetic Behavior*: The motion of a point on the rotor is a superposition of three movements: 1) a rotation of the rotor around its center; 2) a circular translation of the center itself; and 3) the rocking motion of the rotor. The position $\vec{p}$ of a fixed point on the rotor can be described by

$$\begin{aligned} \vec{p} = & \left[R_M \cos \alpha + r \cos \left(\frac{\alpha}{n} + \varphi \right) \right] e_x \\ & + \left[R_M \sin \alpha + r \sin \left(\frac{\alpha}{n} + \varphi \right) \right] e_y \\ & + d \left[1 - \cos \left(\alpha \frac{n-1}{n} + \varphi \right) \right] e_z \end{aligned} \quad (15)$$

where the radius of the center point rotation R_M is given by (see Fig. 2)

$$R_M = \frac{h_b d}{R}. \quad (16)$$

The kinetic behavior is complex, and the motion is related to a cycloid. The movement of a point at the outer radius of the motor is shown in Fig. 10. As a result of the circular translation of the center, a point at the outer radius of the motor even moves in the opposite direction during short time intervals of a wobble cycle. This effect, which is also present in side-driven wobble motors, is strongly exaggerated in this figure by using a small gear ratio, but may have consequences for certain applications of wobble motors.

2) *Dynamic Behavior*: The velocity of a point on the rotor can be found from the time derivative of its position $\vec{p}$ (15). From the velocity, the kinetic energy T of the rotor is easily obtained

$$\begin{aligned} T = & \frac{\pi}{2} \rho h R_M^2 R^2 \dot{\alpha}^2 + \frac{\pi}{4} \frac{1}{n^2} \rho h R^4 \dot{\alpha}^2 \\ & + \frac{\pi}{8} \left(\frac{n-1}{n} \right)^2 \rho h d^2 R^2 \dot{\alpha}^2. \end{aligned} \quad (17)$$

In this expression, the first term on the right is due to the motion of the center of the rotor, the second term is a result of the rotation of the rotor, and the last term is caused by the rocking motion of the rotor. From Hamilton's principle [20], using the expressions for the electrostatic coenergy and kinetic energy as given in (4) and (17), respectively, the equation of motion for the rotor can be found. When additional terms are added to account for damping mechanisms, this yields

$$J \ddot{\alpha} + C_v \dot{\alpha} + \text{sgn}(\dot{\alpha})(C_{d1} + C_{d2} V^2) = \tau_{\text{motor}(\alpha)} \quad (18)$$

where J is the rotational inertia of the rotor, related to the contact point angle, given by

$$J = \pi \rho h R^2 \left[R_M^2 + \frac{R^2}{2n^2} + \left(\frac{n-1}{n} \right)^2 \frac{d^2}{4} \right]. \quad (19)$$

The rotational inertia of the rotor is mainly determined by its inertia about the rocking axes, which is represented by the last term on the right-hand side of (19). The $\dot{\alpha}$ terms in (18) have been added to include damping mechanisms. As proposed by Tai [21] and Bart [22] in the case of side-driven micromotors, the next damping mechanisms have been used: C_v is the coefficient of viscous drag, C_{d1} is a constant coulombic friction term, which results from constant normal forces like adhesion forces and gravity, C_{d2} is a voltage-dependent coulombic friction term resulting from electrostatic normal forces as shown before, and $\text{sgn}(\dot{\alpha})$ is the sign function of angular velocity, which is $+1$ if $\dot{\alpha} > 0$ and -1 if $\dot{\alpha} < 0$. For small oscillations around an equilibrium position, the differential (18) can be linearized. In this case, the natural oscillating frequency of the rotor ω_N can be found from

$$\omega_N = \sqrt{\left[\frac{d\tau}{d\alpha} \right]_{\alpha=0} / J}. \quad (20)$$

3) *Viscous Damping*: Because of the surrounding gas, a damping torque will develop as a result of viscous drag. The viscous drag torque can be found by solving the Navier–Stokes equations. Because of the complex motion of the rotor this problem cannot be solved easily. Furthermore, realized rotor structures exhibit slots as a result of the fabrication process. Therefore, a three-dimensional (3-D) FEM fluid analysis of the problem is required.

To obtain an indication of the drag torque a strongly simplified approximation has been used. The rotor is assumed to be a thin rigid disk, and the contributions due to the edges are ignored. The gas is treated as a continuous medium and is assumed to behave as a Newtonian fluid under laminar flow. The viscous drag forces are assumed to be mainly due from squeeze film damping in the small gap between the rotor and the stator surface as a result of the rocking motion of the rotor. With these assumptions, the problem simplifies to a thin inclined rigid disk that is rotating over a horizontal surface, where only the drag forces in the gap between the disk and the surface are considered. The rotational axis of the disk is perpendicular to the surface, and the disk is rotating with an angular speed equal to that of the contact point. The shear stress τ_{shear} can be found from Newton's equation

$$\tau_{\text{shear}} = \mu \frac{dv}{dz} \quad (21)$$

where μ is the viscosity of air, dv/dz is the speed gradient of the fluid under the rotor equal to $\omega r/g$, and ω is the angular speed of the disk. The frictional torque τ_{viscous} can be found by integrating $\tau_{\text{shear}} \cdot r$ over the bottom surface of the rotor

$$\tau_{\text{viscous}} = \int_0^R \int_0^{2\pi} \frac{\mu \omega r^3}{d \left(1 - \frac{r}{R} \cos \varphi \right)} dr d\varphi = \frac{4\pi \mu R^4 \omega}{3d}. \quad (22)$$

Fig. 10. (a) Three-dimensional, (b) top, and (c) side view of the movement of a fixed point at the rotor outer radius in arbitrary units. A small gear ratio of about 11 has been used to clearly show the behavior.

In the case of a slider bearing, a comparable approximation was found to be somewhat lower, but accurate within a factor of two with a solution from the Navier–Stokes equations [23]. Therefore, the expression above is considered to be a reasonable estimate of the viscous drag torque on the rotor.

4) *Dynamic Behavior:* A theoretical prediction of the transient response can be obtained from the equation of motion given in (18). The rotational inertia for a rotor, with dimensions as given in Fig. 11, is calculated using (19) from which a value of 8×10^{-22} kgm² is obtained. For an air viscosity of 1.83×10^{-5} kg/ms, the viscous damping constant C_v is estimated by (22) to be about 3×10^{-15} Nms. Measurements on side-driven wobble motors showed that coulombic friction is dominated by the voltage-dependent term (i.e., $C_{d1} \ll C_{d2}V^2$). From the measured dynamic frictional torque of fabricated axial-gap wobble motors with a radius of $100 \mu\text{m}$, the value of C_{d2} is calculated to be 9×10^{-15} Nm/V² [18]. Now, the generated torque can be calculated for a given driving voltage by (13). For ease of numerical computation,

the function $\pi/2 \arctan(\dot{\alpha}/10^{-4})$ has been used in place of $\text{sgn}(\dot{\alpha})$.

The transient response of a four-stator-pole motor in the case of a single-pole excitation is shown in Fig. 11. The response time is strongly dependent on the driving voltage. Because of coulombic friction, the rotor does not reach the zero-torque position of the excited stator pole, but stops at an angle that is dependent on the driving torque. For decreasing driving voltages, this stopping angle approaches about -0.2 rad, which is the angle, where torque coverage is minimal.

IV. CONCLUSION

The design of an electrostatically driven lower-stator axial-gap wobble motor has been presented. In contrast to side-drive motors, the stator poles are located underneath the rotor instead of surrounding the rotor sides. This results in higher torque generations and easily accessible rotor structures that are suited for mechanical power transmission to other structures, fabricated on the same wafer.

Fig. 11. Transient response of a four-stator-pole design with single-pole excitation for a driving voltage of 10, 30, and 100 V. Starting angle is $-\pi/2$ rad, $R = 100 \mu\text{m}$, $R_i = 50 \mu\text{m}$, $R_o = 100 \mu\text{m}$, $d = 2.3 \mu\text{m}$, $d_{\text{ins}} = 0.46 \mu\text{m}$, $\epsilon_r = 7.5$, $\varphi_1 = -\pi/4$ rad, $\varphi_2 = \pi/4$ rad, $J = 2.0 \times 10^{-21} \text{ kgm}^2/\text{s}^2$, $C_v = 4.1 \times 10^{-18} \text{ Nms}$, $C_{d1} = 0 \text{ Nm}$, and $C_{d2} = 9 \times 10^{-15} \text{ Nm/V}^2$.

Based on small tilt angles, a rigid disk rotor and simplified electrostatic fields, a theoretical model describing the static and dynamic behavior of the motor, has been given. The theoretical torque generation is in the range of nanoNewtonmeters at high electrostatic fields. The torque coverage for different excitation schemes in the case of both open- and closed-loop excitation, which requires position feedback, is discussed. Because of large normal forces, generated by axial electrostatic fields, the motors can always be operated under no-slip conditions. The kinetic behavior is complex and related to a cycloid. A fixed point at the rotor even moves opposite to the rotational direction of the rotor during small time intervals of a wobble cycle. This may have consequences for certain applications of wobble motors. Based on side-driven micromotor results, an equation of motion for the rotor has been derived that includes rotational rotor inertia, viscous drag and coulombic frictional terms. The rotational inertia of the rotor is mainly determined by its inertia about the rocking axes. A theoretical prediction of the transient response of the motor has been given.

The presented theoretical model can be used as a framework for further development of axial-gap wobble motors.

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