Analysis of Flatness Measurement and Form Stability of a Granite Surface Plate

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It is rather difficult to compare quantitatively flatness measuring results of one and the same object (e.g. in case of a long-term stability investigation) or the results of a series of products (e.g. from the same machinetool in order to indicate machinetool-influences). An extension of a computerprogram applied to the evaluation of flatness measurement (as described in the CIRP-Annals, vol.26 (1977), no.1), separates the form deviations of a surface into a geometrical part and a random part. The geometric part consists of a best-fitting non-flat, reference plane, composed of the single curvature of a sphere and the double curvature of a real twisted plane. Three parameters (the dimensionless curvature parameters of sphericity and torsion, together with the direction of the torsionvector) are able to characterize the geometrical part of the surface; a fourth one is a measure for the random part. Advantages of the method are demonstrated with the aid of a long-term investigation into the form stability of a granite surface plate. Another extension of the program makes it now possible to measure surfaces with non-rectangular contours in the same way as it is being done in case of a rectangular surface plate.

1. INTRODUCTION

Since 1974 experience has been gained with methods of flatness measurement and evaluation of the measuring results by means of a specially developed computerprogram. The merits of the applied method are described in more detail in [1;2].

In short: every straightness measuring method, applied on a gridpattern of intersecting straight lines on the surface, may be used. In a fair competition two measuring methods have been investigated further: a member of the "direct" method, making use of well calibrated and lapped straightedges in combination with a specially developed contact-indicator and an item of the "indirect" method making use of an electronic level-indicator in combination with an electronic displacement transducer, measuring height differences of pairs of gridpoints.

Both methods have their specific pros and cons. The systematic errors, especially of the straightedge method, have been reduced so that at the moment the results of both computing methods are comparable within the regions of uncertainty. Nevertheless the preference is going to the second method because it is easier to handle, faster and more accurate.

The results of straightness deviations on the lines of a gridpattern on the surface are used in the computerprogram, resulting in a measuring report mentioning:

- height deviations in the measured gridpoints with respect to a regression plane,
- the standard deviation σ of the flatness deviations which can be used to calculate the confidence limits,
- a map of the surface with contourlines of certain heightsteps, found by spline-interpolation of heights within a finer gridpattern.

2. THE NEED OF QUANTITATIVE COMPARISON OF SHAPE ALTERATIONS

The long-term investigation of the formstability of a granite surface plate just after its "birth", which has already been announced in [2] was only possible in the beginning by comparison of the contourmaps and the plotting of changes of heightvalues of (certain) gridpoints. Rather big changes could be detected in this way, certainly in the beginning of lifetime of the surface plate (fig.7 and 8 in [2]).

Now we believe to have found a solution for this problem by separating the shape of the surface into a geometrical part* and a random part, so that characteristic parameters to the surface shape can be adjudged.

Fig.1 Height difference due to the shape of a sphere S

* A hint was given in correspondence by Mr. Kilpi, former CIRP-member of Finland.

3. CURVATURE DUE TO SPHERICITY

The height difference s_{ij} of a point P_{ij} of a real sphere S with respect to a reference plane Q (fig.1) can be expressed as:

$$s_{ij} = -\frac{1}{2} k (x_i^2 + y_j^2) = k \cdot B_{ij}$$

in which k is the curvature in rad/m (k is thus the reciprocal value of the radius of curvature in m/rad). B_{ij} is a factor which is only dependent on the co-ordinates of the point P_{ij} . In fig.1 the impression may be raised that the centre of curvature must be on the Z-axis underneath O ($i=0; j=0$). However by shifting the reference plane Q any position with respect to the measured surface S is possible.

4. CURVATURE DUE TO REAL TORSION

Torsion can be expressed by means of a free vector τ which is supposed to lie in Q and which has been placed in fig.2 in O for convenience sake. Not only the magnitude of τ , but also its direction ϕ with respect to the Y-axis is of importance in the next. Although τ may also have a component along the Z-axis this cannot give rise to height differences with respect to the XOY-plane and it can be ignored furthermore.

If the torsion per unit of length is constant τ (rad/m) a point P_{ij} encounters a height deviation t_{ij} due to torsion only, which obeys:

$$t_{ij} = \tau \cdot l \cdot r = \tau \left[\frac{1}{2} (y_j^2 - x_i^2) \cdot \sin 2\phi - x_i y_j \cos 2\phi \right]$$

$$t_{ij} = \tau \cdot T_{ij}$$

The factor T_{ij} is not only dependent on the co-ordinates of P_{ij} , but also, as could be expected, on the direction of τ .

Fig.2 Height difference due to torsion

5. SUPERPOSITION OF SPHERICITY AND TORSION

In [1 and 2] it has been explained that the measuring procedure results in finding a flat reference plane R so that from that reference the height h_{ij} of a point P_{ij} in the measured surface could be calculated (fig.3).

Next step up to now was to define height difference d_{ij} between a flat regression plane Q and the measured surface in P_{ij} by:

$$d_{ij} = h_{ij} - (a + j \cdot \alpha + i \cdot \beta)$$

By minimising $\sum d_{ij}^2$ it was possible to find the position of that regression plane Q ($a; \alpha; \beta$) and from that the positive and

negative form deviations d_{ij} of the measured surface.

Fig.3 Defining form deviations with respect to a regression plane

From now on the height difference d_{ij} may be defined with respect to a non-flat regression plane Q, which may be the superposition of a flat, a real sphere and a real twisted component:

$$d_{ij} = h_{ij} - [a + j \cdot \alpha + i \cdot \beta + k \cdot B_{ij} + \tau \cdot T_{ij}]$$

In order d_{ij}^2 is minimum: $\frac{\partial d_{ij}^2}{\partial a} = \frac{\partial d_{ij}^2}{\partial \alpha} = \frac{\partial d_{ij}^2}{\partial \beta} = \frac{\partial d_{ij}^2}{\partial \tau} = \frac{\partial d_{ij}^2}{\partial k} = 0$.

These 5 equations make it possible to solve a , α , β , τ and k .

If flatness has been measured in p points of the gridpattern the restnoise R may be defined as:

$$R = \sqrt{\frac{\sum_{i=1}^p d_{ij}^2}{p-6}}$$

One differentiation $\frac{\partial d_{ij}^2}{\partial \phi}$ is missing. This is rather a cumbersome procedure so that in the program it has been replaced by a calculation of $R = f(\phi)$ and finding that ϕ where R is a minimum.

Fig.4 At the definition of the dimensionless bowrise b

Although real sphericity k and real torsion τ are expressed in the unit rad/m so that they are independent of the magnitude of the measured surface, preference is going to dimensionless parameters as a measure for the curvature due to sphericity and torsion.

In fig.4 a dimensionless bowrise b may be defined by $b = \frac{h}{l}$.

The dimensionless bowrise b_s due to true sphericity is thus:

$$b_s = \frac{1}{8} k \cdot l$$

Finding k in the computerprogram in $\mu\text{rad/m}$ the unit of b_s is thus $\mu\text{m/m}$.

Fig.5 A surface with true torsion

True torsion results in a hyperbolic paraboloid or a "saddle"-surface, containing two families of straight lines (fig.5).

The extreme bowrise $t = \frac{t'}{4}$, where $t' = \tau \cdot r^2 = \frac{\tau \cdot l^2}{2}$.

The dimensionless bowrise b_t , due to true torsion is thus:

$$b_t = \frac{t}{l} = \frac{1}{8} \tau \cdot l$$

Here, also, finding τ from the computerprogram in $\mu\text{rad/m}$, the unit of b_t is $\mu\text{m/m}$.

A simple example of a map of a surface with true torsion is shown in fig.6.

The torsion-bowrise $b_{t,\theta}$ in any direction θ with the horizontal axis may be derived from: $b_{t,\theta} = b_t \cdot \sin 2(\theta - \phi)$.

In any real twisted surface two directions of extreme curvature can be indicated:

- maximum curvature (symbol $\bigcirc$) in the direction $\alpha_1 = \phi + 45^\circ$
- minimum curvature (symbol $\bigcirc$) in the direction $\alpha_2 = \phi - 45^\circ$

These so-called main directions are under directions of $\pm 45^\circ$ with the τ -vector.

Moreover there are two directions ψ where straight lines in the surface may be expected ($b_{t,\psi} = 0$).

In fig.6 these lines are a and b at $\psi_1 = 0$ and $\psi_2 = 0$. They are indicated with the symbol $\equiv$.

Fig.6 Map of a surface with true torsion

If true sphericity and true torsion are superimposed (fig.7) the total dimensionless bowrise b_r in any direction θ with the horizontal axis equals:

$$b_r = b_s + b_t \cdot \sin 2(\theta - \phi)$$

Fig.7 Map of a surface with true sphericity and torsion

Now the maximum curvature may be expected in the direction $\alpha_1 = \phi + 45^\circ \cdot \text{sign } b_t$ and the minimum curvature in the direction $\alpha_2 = \phi - 45^\circ \cdot \text{sign } b_t$.

Here sign b_t has been added in the formulae, because in the computerprogram the direction θ of the τ -vector is always between 0° and 90° , but due to the kind of torsion in the surface τ may be positive or negative, making b_t positive or negative.

If $|\frac{b_s}{b_t}| < 1$ the surface has two directions of straight lines, because the saddle-shape overrules the sphericity. They fulfill:

$$\psi_1 = \phi + \arcsin(-b_s/b_t)$$

$$\psi_2 = \phi - \arcsin(-b_s/b_t)$$

If $|b_s| = |b_t|$ the surface has a cylindrical shape with one axis $\psi = \eta$

If $|\frac{b_s}{b_t}| > 1$ the surface is a part of

- an ellipsoid ($b_t \neq 0$),
- or a sphere ($b_t = 0$).

So at the end of the calculation by the computer not only the height deviations d_{ij} with respect to a flat regression plane are mentioned, but also:

- the dimensionless bowrise parameters b_s and b_t ,
- the direction ϕ of the torsionvector $\underline{t}$ (if any),
- the restnoise-value R ,
- depending on the values of b_s and b_t an indication whether the surface is mainly a saddle-surface, a cylinder, an ellipsoid, a sphere or random.

In fact the parameters b_s , b_t , ϕ and R are sufficient to characterize the shape of the measured surface. The above-mentioned flat regression plane can be considered as a sphere with curvature $k_s = 0$.

Fig.8 Alterations of the b_s -parameter in course of time.

6. LONG-TERM INVESTIGATION OF A GRANITE SURFACE PLATE

The investigation into the form stability of a "just born" granite surface plate (630x400x80 mm³) started at the end of 1976. It was inspired on some vague doubts about the stability of this kind of material, caused by

- measurements on several plates of different origins of material,
- experience of form changes of optical glass material, which is being processed in about the same way and the chemical composition of which has similarity with that of granite.

6.1 The "childhood"-period

After more than two months of measuring every 3 $\frac{1}{2}$ day (measurements nr.1-15) and once a week (nr.16-18) it was thought on the quasi-stability of the heightvalues and the contourmaps that the most spectacular changes of form were behind us.

Re-calculating the data of measurements on punch cards with the aid of the extended computerprogram it is now possible to plot the b_s -parameter in a sequence of measurement (fig.8), where-as fig.9 shows the b_t -parameter and the angle ϕ of the torsionvector in that same sequence.

Surprising are:

- the decreasing trend of b_s , which was not ended at all after measurement nr.18,
- the more or less "constant" level of b_t -values and ϕ -values in that period.

6.2 Experiments with illumination

Measurements nr.19 and 20 were based on the steady state at illumination of the upper surface with a normal 40 W-filament lamp at 1 m height above the centre of the plate. Between 19 and 20 the bulb was turned over 180° along its vertical axis in order to eliminate the unsymmetrical Lux-profile on the surface.

Fig.9 Alterations of the b_t -parameter and ϕ in course of time

In measurement nr.21 and 23 a 40 W-fluorescent lamp was mounted at 1 m height and parallel to one diagonal NW-SE, whereas measurement nr.22 belongs to the position of that lamp along the other diagonal direction SW-NE.

These measurements only result in a change of sphericity (fig.8). The rise Δb_s of about 2 $\mu\text{m/m}$ can fully be derived from the thickness of the plate (80 mm), the coefficient of linear expansion ($\alpha = 5,6 \cdot 10^{-6} \text{ K}^{-1}$) and the temperature difference between upper and lower surface of only +0,25 K.

The same experience as to sign and magnitude of the effect is being confirmed in [3;4].

6.3 Experiments with humidity

For some reason it was decided to increase the relative humidity within the aluminium tent around the surface plate up to 80 & 85%. Although there were changes in the "character" of the contourmaps of those measurements (nr.24-31) the new parameter b_s shows in fig.8 that these phenomena were more on account of the original "trend" than on account of humidity!

Nevertheless it was decided to go to the utmost: putting wet towels on the upper surface during 24 hours and then to measure immediately after removal.

Fig.10 shows the result of this measurement nr.32 as a contourmap. Effects of curvature can better be valued from fig.8 and fig.9.

The temperature of the upper surface was just before the measurement 18,8°C and afterwards 19,4°C: a normal rise due to the presence of the observer in the tent and some indirect illumination. The general temperature in the tent rose from 17,8°C till 18,0°C. Relative humidity was in the beginning 83%, at last 78%. From these data it can be derived that during the measurement the dewpoint was probably at $15 \pm 2^\circ \text{C}$.

Evaporation of a waterfilm on the surface would make the upper surface cooler than the lower surface. This would tend to a concave shape. However water in the micro-cracks of the finishing operation of the upper surface and in between crystal grains at the surface causes internal compressive stresses which decrease at lower levels under the upper surface. This effect (also demonstrable in optical glass material) causes thus a convex upper surface and is to our opinion mainly responsible for the shape of contourmap nr.32. Other investigators have mentioned this effect, all with the same sign of curvature (convex shape), but with various magnitude [3;5;6;7]. The last item seems to be dependent on the chemical and structural composition of the granite material. As to our black granite data are mentioned in an appendix.

6.4 The second drying period

Measurement nr.34-43 belong to another drying period in which the shape alterations have been followed with increasing time intervals. After measurement nr.43 it was believed that the original, more or less characteristic contourmap had been reached again and intervals between the measurements were increased to several weeks.

Now with the new facilities it is clear that better attention should have been paid to questions as:

- at what level and why the b_s -parameter passed a minimum,
- why the b_t -parameter significantly changed its trend,
- why the angle φ of the torsionvector τ significantly changed its nearly stable level of $\varphi = 47^\circ \pm 4^\circ$ into a decreasing trend (correlation coefficient 0,8) ?

6.5 First loadings on the surface

Because it was thought that no further measurements were of interest measurement nr.45 was done in unloaded state of the plate and measurement nr.46 followed immediately afterwards putting a constant "point"-weight of 18,2 kg (a cylindrical square, $\varnothing = 90 \text{ mm}$, height 400 mm, supporting $\varnothing = 90 \rightarrow 70 \text{ mm}$) in the centre of the plate in order to demonstrate the formdeviation due to this normal attribute on a surface plate. This weight was only for one hour on the table. Because this measurement nr.46 is not of interest in the scope of this paper the values of parameters have been omitted in the graphs.

In the beginning of 1979 a constant weight of the same order, but on a larger supportarea was allowed on the plate during a week or two.

In spite of waiting more than 4½ month (the plate strictly in unloaded condition) measurement nr.47 showed a quite strange contourmap notwithstanding the normal measuring conditions. Both curvatures due to real sphericity b_s and due to real torsion b_t give quite unnormal values. The angle φ changed too.

Nevertheless measurement nr.48, after another 35 weeks of unloaded state of the plate, promises a recovering to the original values b_s , b_t and φ .

Some measurements after composing this paper will confirm this supposition or upset it.

7. THE REMAINING PARAMETERS CHARACTERIZING THE FLATNESS MEASUREMENT

Up to now nothing has been mentioned about the last characterizing parameter of a flatness measurement: the restnoise R. This parameter has been plotted as a function of the sequence of measurements in fig.11.

Up to and including measurement nr.37 this value is surprisingly constant at $R = 0,55 \pm 0,03 \mu\text{m}$. To begin with that concave shape (nr.38) there is more scatter in the R-values. The reason of that cannot be explained.

Finally a measure for the accuracy of each flatness measurement is the standard deviation σ based on the mean value of flatness deviations per gridpoint over the whole gridpattern.

Up to and including measurement nr.42 this σ -value is surprisingly constant at a level $\sigma = 0,12 \pm 0,02 \mu\text{m}$. In between that measurement and the next one 43 for some reason a pair of permanent magnets has been placed beside the Talyvel-unit of the measuring system and only for half an hour. Since then we never have succeeded in getting the accuracy at the original level.

8. ANOTHER EXTENSION OF THE COMPUTERPROGRAM

A simple question of the Instituut voor Kernfysisch Onderzoek (I.K.O.) in Amsterdam to measure for them at the manufacturer's in Germany dipole-sets for an electron spectrometer to be built

Fig.10 Shape of the surface plate just after wetting

Fig.11 Restnoise and standard deviation of flatness measurements in course of time

in that Institute, caused a problem that these surfaces (area about 2 "a 3 m²) had no rectangular or square, but bowshape contours. The entrance side of the first twin started at about 0,4 m length, the exit side had about 0,8 m length. The second set had a more "conventional" shape.

These pairs of magnets of special soft-iron material have to generate a very strong but homogeneous magnetic field within the gap between the parallel surfaces. On the drawing flatness was tolerated within 10 μm per surface. Last finishing operation was surface grinding.

First impulse was to find a suitable gridpattern that best fitted into the circumference of a surface. Especially at the first pair this is nearly impossible without ignoring relatively large areas of the surface, but which are functionally very important (side effects). At last a solution was found to put a larger grid over the surface, that has a suitable number of measuring points in common with the magnet surface and ignoring the rest of the gridpattern. Of course this procedure gives loss of the number of degrees of freedom in the approximation procedure of finding the reference plane R, but this can be suffered to some extent. Fig.12 gives the contourmap of such an item of the first pair of magnets. The machineshop Lorenz (Ettlingen, Germany) succeeded in keeping all pairs of surfaces within that 10 μm flatness tolerance, however comparing all machined products of the same grinding machine these 5 surfaces (one surface was a prototype to show the possibilities of the workshop) have quite a lot of torsion with the vector τ practically always in the same direction. We are convinced that this effect is due to a systematic way of

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Fig.12 Map of a surface with bowshape contours

movement of the machineslide, which can be corrected for in order to get (even) better products.

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10. APPENDIX

The black granite material of the surface plate concerned originates from a quarry in West Sweden. Arriving at the manufacturer, the firm Mytri, at Apeldoorn, the Netherlands, it has been cut, ground and polished to become a surface plate (630x400x80 mm³) in about two weeks. After the last finishing process it has been carefully transported to the university and measured as to flatness.

On account of this measuring report a last hand-finishing and form-improving operation has been carried out in-situ. Just after that last finishing touch the plate has been installed on the same 3 supports of manufacturing in a tent-frame, covered with aluminium foil in an air-conditioned room. Moreover the plate was packed in isolation material on all sides with exception of the functional upper surface. 24 Hours after finishing and installation the first measurement of the series started on 1976-12-24.

10.1 Chemical composition

A spectro-chemical analysis of material of that quarry resulted in 1976 in:

Silicium- oxide	SiO ₂	56,1 %	Titanium- oxide	TiO ₂	0,33 %
Aluminium- "	Al ₂ O ₃	19,1	Mangan- "	MnO	0,19
Calcium- "	CaO	12,0	Vanadium- "	V ₂ O ₅	0,015
Iron- "	Fe ₂ O ₃	6,5	Nickel- "	NiO	0,01
Magnesium- "	MgO	3,0	Gallium- "	Ga ₂ O ₃	0,01
Sodium- "	Na ₂ O ₅	1,5	Copper- "	CuO	0,005
Potassium- "	K ₂ O	1,2	Lead- "	PbO	0,004
			Zirconium- "	ZrO ₂	0,003

10.2 Structural composition

In fig.13 the crystalline structure of the black granite material is shown. A sample of about 20x10x1 mm³ has been cut from a larger piece, ground and polished until a slice with thickness of less than 0,015 mm was left.

Now the crystalline state can be studied in colour-interference microscopy and /or by macro-photography of which the picture is an example. This black kind of granite seems to have a rather big "density" of grains in comparison with other kinds of granite. The nature of the various crystals can better be explained with the aid of a coloured picture in interference contrast.

Fig.13 Crystalline structure of the granite concerned