ORIGINAL ARTICLE

# Mental maps and travel behaviour: meanings and models

Els Hannes · Diana Kusumastuti · Maikel León Espinosa · Davy Janssens · Koen Vanhoof · Geert Wets

Received: 30 October 2009/Accepted: 27 October 2010/Published online: 12 November 2010 © Springer-Verlag 2010

**Abstract** In this paper, the "*mental map*" concept is positioned with regard to individual travel behaviour to start with. Based on Ogden and Richards' triangle of meaning (The meaning of meaning: a study of the influence of language upon thought and of the science of symbolism. International library of psychology, philosophy and scientific method. Routledge and Kegan Paul, London, 1966) distinct thoughts, referents and symbols originating from different scientific disciplines are identified and explained in order to clear up the notion's fuzziness. Next, the use of this concept in two major areas of research relevant to travel demand modelling is indicated and discussed in detail: spatial cognition and decision-making. The relevance of these constructs to understand and model individual travel behaviour is explained and current research efforts to implement these concepts in travel demand models are addressed. Furthermore, these mental map notions are specified in two types of computational models, i.e. a Bayesian Inference Network (BIN) and a Fuzzy Cognitive Map (FCM). Both models are explained, and a numerical and a real-life example are provided. Both approaches yield a detailed quantitative representation of the mental map of decision-making problems in travel behaviour.

**Keywords** Mental map · Travel behaviour · Bayesian decision network · Fuzzy cognitive map

e-mail: els.hannes@uhasselt.be

E. Hannes Architecture Research Institute—ArcK, PHL University College, Campus Diepenbeek, Agoralaan—Building E, 3590 Diepenbeek, Belgium

M. L. Espinosa

Center of Studies on Informatics, Central University of Las Villas, Road to Camajuani Km 5 $^{1}\!/_{2},$  54830 Santa Clara, Villa Clara, Cuba

E. Hannes  $(\boxtimes) \cdot D$ . Kusumastuti  $\cdot D$ . Janssens  $\cdot K$ . Vanhoof  $\cdot G$ . Wets Transportation Research Institute—IMOB, Hasselt University, Wetenschapspark 5/6, 3590 Diepenbeek, Belgium

**JEL Classification** D83 Search; Learning; Information and Knowledge; Communication; Belief · R49 Other

# 1 Introduction

The mental map is commonly used to represent the internal knowledge base of a human data processor, i.e. notions and know-how in the mind concerning a certain issue or question. Most often, this concept is related to geographical or spatial aspects—hence the use of the "*map*" metaphor (Kuipers 1982)—but distinct interpretations exist in different scientific fields.

Ever since behavioural psychologist Tolman (1948) first put forward the original synonym "*cognitive map*", this concept has been studied, adopted and adapted in various disciplines such as cognitive psychology, behavioural geography, computer science, engineering, neuropsychology. For instance, our analysis of 305 references generated by entering the search term "*mental map*\*" in the ISI Web of Knowledge (Thomson Scientific 2008) shows that present sources can be assigned to 83 different subject areas. Inevitably, this has led to the attachment of multiple meanings to the concept and a proliferation of related terms such as: "(*spatial*) *mental model*", "*mental representation*", "*cognitive image*", "*cognitive collage*", "*mind map*", etc. Figure 1



Fig. 1 Force-directed network visualization (NWB Team 2006) of common mental map synonyms (*circles*), connected by their top 5 related subject area's (*squares*) in the ISI Web of Knowledge (Thomson Scientific 2008)

illustrates the most commonly used terms linked to their top 5 related subject areas. Even within one field, various metaphorical expressions are used. For instance, Barkowsky (2002) lists several notions for spatial mental knowledge processing, and according to Montello and Freundschuh (1995), up to 200 combinations of adjectives and nouns referring to cognition and space for describing environmental spatial knowledge are conceivable.

Because of mental map's varying contents and applications across different contexts and conditions, as well as its lack of a fixed and precise meaning, it has become an outstanding example of a fuzzy concept. But using such ill-defined constructs may lead to misunderstanding and misinterpretation. Moreover, its vagueness and ambiguity can hinder computational implementation. Although the mental map and its intuitive, virtual definitions might be sufficiently clear and self-explaining to use in human communication, definitions for the reconstruction of a knowledge universe require a far-reaching process of formalization in which mathematical logic plays a key role (Lucardie 1994). During this process, the meaning of a concept often appears to become less clear.

This is exactly what happened to the mental map concept in the travel demand research community to date. Clearly, the expression will be intuitively understood by most travel behaviour researchers. However, there is no generally accepted definition in this field—each author basically defines the notion closely to the task at hand—let alone a universally applied method to take the concept into account in computational applications such as travel demand models. Still its importance to understand travel behaviour is widely recognized (Chorus and Timmermans 2009; Hannes et al. 2009a, b). Moreover, in theoretical travel demand modelling descriptions, the mental map is mentioned as a distinct behavioural factor, e.g. (Arentze and Timmermans 2000; Salvini and Miller 2005). However, measurement of this construct and putting the concept into operation in actual forecasting models proves to be problematic (Golledge and Gärling 2004), to say the least, partly due to its fuzzy nature. This paper seeks to resolve the definition problem and proposes a computational modelling framework.

Ogden and Richards' scheme (1966) is used to disentangle mental maps' multiple meanings. Herein, two major areas of research relevant to travel demand modelling are indicated and discussed in detail: spatial cognition and decision-making. First of all, since travel involves movement in space and time, there is an obvious spatial component to the execution of travel plans. Thus, individual's perception and comprehension of geographical space is a key factor to understand travel behaviour; the mental map is human's spatial knowledge base, incomplete and biased, regularly updated by travel experiences and foundation of various travel decisions at the same time (Weston and Handy 2004). This brings us to the second notion of the mental map stemming from decision theory and human reasoning: it conveys the mental representation of a decision problem; a temporarily generated mental model in someone's thought process including relevant choice factors and decision rules (Johnson-Laird 2004). As planning and executing activity schedules involves different choices such as destination, travel mode and route choices, spatial knowledge is anchored in this broader, general decision process. Both meanings of the mental map, i.e. representation of individual's spatial knowledge and mental model of the personal thought process related to travel decisions, are crucial to comprehend individual's travel behaviour.

In addition, relevance and use of both constructs to model individual travel demand are addressed, and two potentially valuable computational models are proposed and implemented: BIN and FCM. Both models are primarily conceived as an individual mental model of decision problems involved in daily activity and travel scheduling, embedding spatial cognitive factors. Location choice and travel mode choice are related to contextual, instrumental and psychological choice factors. This way, a decision network is drawn, including the valuation of constituent parts and the specification of applied choice rules. These two models are proposed, and opportunities and drawbacks of both approaches are listed in conclusion.

# 2 Meanings of mental maps

Ogden and Richards (1966) created a semantic triangle to show the indirect relationship between concepts or words (symbols), the things they represent (referents) and the thoughts they can convey (references) to disentangle how words work and to explain sources of misunderstanding in human communication. The top of the triangle in Fig. 2 shows some thoughts that someone might have when the symbol "*mental map*" is mentioned (lower left corner). These thoughts are also directly linked with the (likely) actual object (lower right corner). But the link between the concept name and the object is tenuous at best (hence the dashed line). Two people could use the same expression to indicate completely different things.



Unless both understand the ambiguity of language, the use of this symbol can cause misunderstanding and misinterpretation.

Moreover, frequent use of "mental map" synonyms furthers the confusion. Table 1 shows a comparison of likely synonymous topics in the ISI Web of Knowledge (2008), and the occurrence of each concept in the transportation subject area. Bearing Richards' lesson in mind that: "*context is the key to meaning*" (Griffin 1997, p. 58), the top 5 of subject area's in which each of these key words appears, is listed as well. All topics are current expressions in psychology and behavioural sciences, thus linking the subject to human beings. Apart from "*cognitive image*", the "*mental map*" phrasing is least often used in general, but geography is its second most important subject area. This shows the topic's relation to spatial aspects. Furthermore, in computer science, all combinations of "*mental/cognitive*" and "*map/model*" can be retrieved, a clear hint to its computational applications.

Of all conceivable meanings of the mental map, referents from two scientific fields are especially relevant to understand individual travel behaviour: human decision-making and spatial behaviour. Indeed, the movement from one place to another exists in geographical space and travelling involves making decisions such as when to go, where to go, how to get there.... Furthermore, a computation of mental maps in travel demand models requires an exact definition of these concepts. The conceptual diagram shown in Fig. 2 illustrates two distinct interpretations of the mental map. The next sections detail these referents in their research areas and summarize relevant references for travel behaviour.

# 2.1 Mental maps in geographical approaches

In geography, the "mental map" reflects an individual's spatial knowledge base. This is all location-specific information about the world stored in memory (see Fig. 3, right side). Some well-known techniques to explore people's knowledge about the environment are asking them to sketch a map of a certain area, to describe routes or to estimate distances. This way, content, structure and biases in mental maps are defined, and individual differences are determined. Reviews are numerous, e.g. Mark et al. (1999) provide a historical overview, Gould and White (1986) show different examples and Golledge and Stimson (1997) highlight various research aspects.

Not all geographic knowledge is relevant in everyday travel behaviour. Daily activity spaces are small compared to the likely extent of the spatial knowledge universe and relatively well known. Yet, imperfect information may affect the knowledge and appreciation of an area's accessibility, hence the considered destinations, transport mode options or route alternatives when planning a trip.

Executing travel plans, on the other hand, involves interactions with the environment. This entails updating, detailing or completing existing spatial knowledge. In cognitive psychology, spatial learning of large-scale environments is reflected in theories of spatial knowledge development. Siegel and White's model of evolving mental maps from landmark over route to survey knowledge is the dominant framework, but its stage-like development assumption is contested (see: Ishikawa and Montello 2006). Particularly, interesting for travel behaviour is the

"Mental map*"	"Cognitive map*"				
305 Publications	1,963 Publications				
In 83 subject areas, top 5	In 118 subject areas				
1. Computer Science (81)	1. Psychology (886)				
2. Geography (78)	2. Behavioural Sciences (816)				
3. Psychology (71)	3. Computer Science (628)				
4. Behavioural Sciences (64)	4. Neurosciences and Neurology (551)				
5. Environmental Sciences and Ecology (50)	5. Engineering (264)				
Yes, in transportation (4)	Yes, in transportation (14)				
"Mental Image*"	"Cognitive image*"				
2,134 Publications	39 Publications				
In 129 subject areas, top 5	In 47 subject areas				
1. Psychology (1,651)	1. Behavioural Sciences (22)				
2. Behavioural Sciences (1,547)	2. Psychology (21)				
3. Neurosciences and Neurology (1,059)	3. Neurosciences and Neurology (12)				
4. Ophthalmology (451)	4. Computer Science (8)				
5. Radiology and Medical Imaging (264)	5. Sports Science (7)				
Yes, in transportation (1)	No, not in transportation				
"Mental model*"	"Cognitive model*"				
2,664 Publications	2,758 Publications				
In 127 subject areas, top 5	In 128 subject areas				
1. Psychology (1,535)	1. Psychology (1,601)				
2. Behavioural Sciences (1,254)	2. Behavioural Sciences (1,509)				
3. Computer Science (794)	3. Computer Science (888)				
4. Engineering (437)	4. Psychiatry (597)				
5. Education and Educational Research (367)	5. Neuroscience and Neurology (596)				
Yes, in transportation (28)	Yes, in transportation (10)				
"Mental Representation*"	"Cognitive representation*"				
2,117 Publications	696 Publications				
In 117 subject areas, top 5	In 103 subject areas				
1. Psychology (1,634)	1. Psychology (528)				
2. Behavioural Sciences (1,498)	2. Behavioural Sciences (507)				
3. Neuroscience and Neurology (698)	3. Neurosciences and Neurology (182)				
4. Pediatrics (255)	4. Social Issues (135)				
5. Psychiatry (230)	5. Pediatrics (66)				
Yes, in transportation (4)	No, not in transportation				

 Table 1
 Comparison of likely mental map synonyms in the ISI Web of Knowledge (2008)

anchor point theory suggested by Golledge (1978), in which a hierarchical ordering of locations, paths and areas is based on the relative significance of each to the individual. Important elements of the daily activity space such as home, work, and

TRAVEL DECISIONS

**GEOGRAPHICAL SPACE** 



Fig. 3 Mental map referents related to travel behaviour

shopping serve as initial primary anchor points for further spatial knowledge acquisition. Anchor points of the daily activity space form the basis of a skeletal mental map structure. Additional anchor points may include commonly recognized elements of the environment, such as well-known landmarks, nodes, routes, edges and districts, which generally constitute the "image of the city" (Lynch 1960). Travelling between these places adds to the development of areal concepts such as neighbourhoods, regions, etc.

There are some specific theoretical accounts with regard to travel behaviour and the geographic notion of the mental map, e.g. (Golledge and Gärling 2003; Weston and Handy 2004). A major part of related empiric research is focused on way finding and navigation. The impact of the mental map on travel decisions prior to route choices such as transport mode decisions, destination choices and activity scheduling is less well documented. Most recent examples: based on distance estimates and estimates of the relative distance to pairs of commonly known destinations, Mondschein et al. (2008) show that individual differences in spatial knowledge (hence individual differences in accessibility) are related to previous travel experiences and differences in transport mode use; Chorus and Timmermans (2009) examined, stated and revealed mental map quality and find similar evidence of better spatial knowledge for people who travel by car or bike (active transport modes) than people who travel by bus (a passive mode); in qualitative interviews about individual's daily activity travel, Hannes et al. (2008, 2009a, b) descry limited spatial awareness in the scheduling process due to fixed travel routines.

### 2.2 Mental maps and decision-making

The second referent of the mental map relevant for individual travel behaviour is its notion of temporary mental representation of a travel decision problem (see Fig. 2, right side). This specific meaning builds upon the seminal work of Hayes-Roth and Hayes-Roth (1979), and it is related to the general research area of mental models of deduction, e.g. (Johnson-Laird 2004). Faced with a decision problem, individuals

explore and evaluate alternative courses of action, taking personal contexts, means and goals into account. Therefore, a temporary and situation-specific reduction of reality is created in mind. Obviously, the decision context in which they operate and the knowledge they rely on exceeds mere spatial characteristics. Most common techniques to elicit mental representations are thinking aloud methods (Someren et al. 1994) and laddering (Neimeyer et al. 2001). The latter technique links thought processes to the core values and beliefs of an individual.

In decision theory in general, the predominant paradigm is expected utility theory founded in von-Neumann and Morgenstern's utility theorem (Hansson 2005). Here, a decision is considered to be a choice out of certain options, depending on the probability of occurrence and a valuation of alternatives. This implies a considerate, informed decision-maker, prone to a high degree of rationality, as opposed to approaches that account for bounded rationality (Simon 1990), intuition (Plessner et al. 2008) or uncertainty and lack of information of the decision-maker (Frederick 2002; A. Tversky and Kahneman 2002). Most often, the decision-making mechanisms in the latter behavioural approaches are fast and frugal heuristics (Gigerenzer et al. 1999). For a theoretical account on behavioural decision-making in travel behaviour, the reader is referred to Svenson (1998).

This dichotomy in theoretical approaches of decision-making applies to the different types of decisions that characterize individual travel as well. On the one hand is the repetitive nature of trips (e.g. commuting, chauffeuring kids to school, grocery shopping) likely to render (once) conscious decisions script-based or habitual behaviour (Gärling and Axhausen 2003). On the other hand is activity scheduling (including choices of destinations, travel modes and routes) likely to entail the coordination of competing goals and intentions (e.g. amongst household members) in a complex environment (e.g. traffic-jams, opening hours), similar to complex planning problems (Gärling et al. 1997).

The mental representation of both types of travel decisions can be modelled as a causal network. Only recently, Arentze et al. (2008) have developed the Causal Network Elicitation Technique (CNET) method to elicit individual's mental representations of intertwined spatio-temporal travel decisions, and they have tested the method in a complex shopping-trip planning experiment. Kusumastuti et al. (2008) have applied this method in a less restricted setting. On the other hand, Hannes et al. (2009a, b) have proposed a model of the mental repertoire of fixed scripts and routines in daily activity travel. The computational model developed further in this paper builds on these exemplary applications.

#### 3 Modelling mental maps

The understanding of both referents of the mental map and their relationship is vital for its adequate computation and any further integration in an agent-based, activitybased model of travel demand. The scheduling engine in such a model produces individual calendars of activities and its related travel decisions, such as where to go and how to get there. Representation of decision-making mechanisms in current activity-based models of travel demand can be divided into two distinct approaches (Algers et al. 2005), reflecting mainstream perspectives in decision theory. There are econometric, discrete choice models based on random utility maximization (RUM) on the one hand (the predominant format), and computational process models (CPM) comprising a set of scheduling rules and decision heuristics on the other hand. Thus, the meaning of the mental map related to decision-making is implicitly present in the model system designs.

To date, attempts to take the geographical connotation of the mental map into account in these models are few. Golledge and Gärling (2004) review some early endeavour. In addition, Sivakumar and Bhat (2006) integrate causes of individual heterogeneity such as spatial cognitive factors (learning, preference...) in an econometric model of location choice for non-work activity in specific vector functions and the error term of a random utility maximization-based model structure. An interesting separate computational conceptualization of the geographical mental map and spatial learning is developed by Arentze and Timmermans (2003). This model, based on Bayesian beliefs networks, is framed in the ALBATROSS CPM model system as part of the research effort to model various aspects of individual learning and adaptation in urban environments.

The computational models of the mental map proposed in the next sections focus on structuring and quantifying individual mental representations of travel-related decision problems. In these models, geographical mental map aspects are part of all choice considerations. Both the BIN and the FCM are mathematical elaborations of decision networks, elicited by means of a structured interview protocol based on the CNET method (Arentze et al. 2008). In these interviews, respondents are asked to list all their thoughts and considerations when making a decision in such a way that individual decision networks can be drawn. The next section elaborates the structure and content of the decision networks. Then, a numerical example is provided for both model types, and first empiric results are shown.

### 3.1 Decision networks

A decision network or influence diagram is a graphical structure in which decisionmaker's problems can be formulated and knowledge of experts can be incorporated (Shachter 1986). Decision problems (e.g. a transport mode choice) are analysed by mapping all related considerations (represented as nodes in the graph) and their relationships (represented as directed arcs). Figure 4 shows a simple example of a decision network and its components. Three types of nodes can be distinguished: decision nodes (shown as a boxes), chance nodes (ovals) and utility nodes (diamond shaped). Decision nodes represent the choice options available to the decisionmaker (e.g. car, bus or bike for the transport mode choice). Chance nodes represent all aspects that the individual takes into account when making the decision. In this case, these aspects can be elements of the decision context (in grey) that impact the decision (e.g. the weather, pressure of time). They can be instrumental characteristics of the choice options (in yellow) related to a particular context (e.g. shelter, speed). Or they can be specific evaluative components (bright green) of the decision (e.g. comfort, efficiency). The latter subtype of chance nodes is related to the third type of *utility nodes*; i.e. the dimensions or meanings of utility that people pursue.



Fig. 4 Decision network and its components

They represent the situation-specific goals, while instruments represent the means to achieve certain goals.

The arcs in the graph have different meanings, based on their destinations. In his seminal work, Pearl (1988, p. 307) defines this as follows: "Arcs pointing to utility and chance nodes represent probabilistic or functional dependence... They do not necessarily imply causality or time precedence, although in practice, they often do. Arcs into decision nodes imply time precedence and are informational, i.e., they show which variables will be known to the decision-maker before the decision is made". In our example, when making a transport mode choice, the contextual node "Weather" and its states "Rain or No rain" leads to the consideration of "Shelter" on the one hand. The node "Weather" is called the parent node for the node "Shelter". Beliefs about these elements are known prior to the decision. On the other hand, given a certain context, specific evaluative aspects are taken into account (e.g. having the "Comfort" of staying dry because of rain). In a decision-making process, each choice option is evaluated in terms of their contribution to the specific value that is desired.

In this conceptualization, each set of related contextual, instrumental, evaluative and utility aspects connected to a decision node, constitutes a cognitive subset. The cognitive subset is the smallest building block of a mental representation. It captures individual's associations per specified goal or value during the interview's elicitation process. If no specific context is mentioned, the contextual node is regarded as the normal or default situation. In that case, related values represent decision-maker's general goals, applicable in any situation. Each decision can consist of multiple cognitive subsets. Moreover, different interrelated decisions can be represented in one decision network. The link between decision nodes represents time precedence in the decision-making process.

This description of the decision network illustrates our theoretical point of departure with regard to travel-related decision-making: individual travel behaviour is goal-oriented. Associations of considerations can be represented as means-end-chains, akin to the model introduced by Gutman (1982) in marketing and consumer research. Context is of key importance to understand decision outcomes (Shafir 2007). After mentally evaluating possible courses of action in a certain situation, eventually, the choice options that have the highest overall utility will be chosen.

The inclusion of specific spatial mental map elements in the decision network is illustrated in our example. First of all, considered location choice alternatives represent perceived choice options, based on (limited and biased) spatial knowledge. This spatial knowledge includes perceived characteristics of places (such as crowdedness) relevant to achieve specific personal goals. Moreover, in the transport mode decision, the consideration of "Travel Time" implies an appraisal of the time distance that can be covered by different transport modes.

#### 3.2 Bayesian inference networks

One possible computational implementation of a decision networks is a BIN. The addition of numerical values to each node and the technique of Bayesian propagation through the network enable a calculation of the overall utility of each choice option (Winkler 1972). Furthermore, by changing the values of different contextual variables (e.g. no rain and no time pressure, versus rain and time pressure), the outcome of various scenarios can be inferred. Figure 5 shows an example of the calculation and inference of one cognitive subset.

Each node in the BIN has some defined states. These states correspond to classes or options of the variable. For example, the variable "Weather" has the options: rain, no rain. Each chance node (contexts, instruments and values) has a related conditional probability table (CPT) that describes the probability of occurrence of each state of that specific node for each possible combination of states of the parent nodes.

The CPT of a node that has no parents is simple. It only contains the chance distribution across the states of this node, e.g. the "No evidence" table of the context node "Weather" in Fig. 5 shows estimated chances that it will rain as fifty-fifty. The CPT of a node with one or more incoming links (child node) is more complicated, because values have to be specified for each state of the node, taking each state of the parent node(s) into account. In the CPT of "Shelter" in the example, the probability that a vehicle with a shelter will be chosen when it rains is 1, opposed to the probability of 0 for a vehicle without a shelter. In case it doesn't rain, the decision-maker believes that chances are fifty-fifty to have a vehicle with a



Fig. 5 Input and output of a Bayesian inference network

shelter. Finally, each value in the CPT of the node "Comfort" has to be interpreted as an answer to the question similar for the value in the first cell: "Suppose that it is raining and you choose to make the trip by car, what is the chance that you will experience all benefit of comfort in that situation?"

The utility node and the decision nodes have values expressed as utilities. The utility node contains the linear utility function in a conditional utility table (CUT). In our example, the partial utility function is estimated in the CUT at 0 for the parent state "No comfort", and ends at 100 for the state "All comfort". In case of multiple partial utilities in a decision network, each utility node can have a different weight in the decision. Weight values can be assessed in a questionnaire by means of a stated preference experiment.

Finally, based on the input values in the network, calculated outcomes are represented as overall utility values for the decision options in the decision nodes. From a decision-making perspective, the option with the highest overall utility will be chosen. BIN can handle uncertainties, e.g. the "No evidence" table for "Weather". Moreover, beliefs can be updated when evidence for certain chance nodes becomes available and is entered in the network. For instance, in "Scenario 1" of the context node "Weather", the decision-maker knows for sure that it is not raining. In that case, the outcome of the Bayesian propagation through the network shows that the transport mode "Bike" has the highest overall utility. While in

"Scenario 2", evidence of rain is entered in the network, and the according calculation of the network shows the highest overall utility for "Car". This calculation and inference of the Bayesian network involves different steps, detailed here for the "No evidence" option of the simple example shown in Fig. 5.

First, the joint probability  $(P_i)$  of "Shelter" is calculated as follows:

$$\begin{aligned} P_{j}^{\text{shelt}} &= \left(P^{\text{norain}} \cdot P^{\text{norain-shelt}}\right) + \left(P^{\text{rain}} \cdot P^{\text{rain-shelt}}\right) = \left(0.5 \times 0.75\right) + \left(0.5 \times 1\right) = 0.75\\ P_{j}^{\text{noshelt}} &= 1 - P_{j}^{\text{shelt}} = 0.25 \end{aligned}$$

Next, the joint probability 
$$(P_j)$$
 of "Comfort" is calculated as follows:  

$$P_j^{\text{allcomf}} = (P^{\text{car}} \cdot P^{\text{norain}} \cdot P^{\text{car-norain-allcomf}}) + (P^{\text{car}} \cdot P^{\text{rain}} \cdot P^{\text{car-rain-allcomf}}) + (P^{\text{bus}} \cdot P^{\text{rain}} \cdot P^{\text{bus-rain-allcomf}}) + (P^{\text{bus}} \cdot P^{\text{rain}} \cdot P^{\text{bus-rain-allcomf}}) + (P^{\text{bike}} \cdot P^{\text{rain}} \cdot P^{\text{bus-rain-allcomf}}) + (P^{\text{bike}} \cdot P^{\text{rain}} \cdot P^{\text{bike-rain-allcomf}}) + (1/3 \times 0.5 \times 0) + (1/3 \times 0.5 \times 0) + (1/3 \times 0.5 \times 0) + (1/3 \times 0.5 \times 1) + (1/3 \times 0.5 \times 0) = 0.555$$

$$P_j^{\text{nocomf}} = 1 - P_j^{\text{allcomf}} = 0.445$$

Then, the partial utility  $(U_p)$  of "Comfort" for decision alternatives and different contexts is derived as follows:

$$\begin{split} U_{\rm p}^{\rm car-rain} &= \left(U^{\rm allcomf} \cdot P^{\rm car-rain-allcomf}\right) + \left(U^{\rm nocomf} \cdot P^{\rm car-rain-nocomf}\right) \\ &= (100 \times 1) + (0 \times 0) = 100 \\ U_{\rm p}^{\rm car-norain} &= \left(U^{\rm allcomf} \cdot P^{\rm car-norain-allcomf}\right) + \left(U^{\rm nocomf} \cdot P^{\rm car-norain-nocomf}\right) \\ &= (100 \times 0.5) + (0 \times 0.5) = 50 \\ U_{\rm p}^{\rm bus-rain} &= \left(U^{\rm allcomf} \cdot P^{\rm bus-rain-allcomf}\right) + \left(U^{\rm nocomf} \cdot P^{\rm bus-rain-nocomf}\right) \\ &= (100 \times 0.83) + (0 \times 0.17) = 83 \\ U_{\rm p}^{\rm bus-norain} &= \left(U^{\rm allcomf} \cdot P^{\rm bus-norain-allcomf}\right) + \left(U^{\rm nocomf} \cdot P^{\rm bus-norain-nocomf}\right) \\ &= (100 \times 0) + (0 \times 1) = 0 \\ U_{\rm p}^{\rm bike-rain} &= \left(U^{\rm allcomf} \cdot P^{\rm bike-rain-allcomf}\right) + \left(U^{\rm nocomf} \cdot P^{\rm bike-rain-nocomf}\right) \\ &= (100 \times 0) + (0 \times 1) = 0 \\ U_{\rm p}^{\rm bike-norain} &= \left(U^{\rm allcomf} \cdot P^{\rm bike-norain-allcomf}\right) + \left(U^{\rm nocomf} \cdot P^{\rm bike-norain-nocomf}\right) \\ &= (100 \times 0) + (0 \times 1) = 0 \\ U_{\rm p}^{\rm bike-norain} &= \left(U^{\rm allcomf} \cdot P^{\rm bike-norain-allcomf}\right) + \left(U^{\rm nocomf} \cdot P^{\rm bike-norain-nocomf}\right) \\ &= (100 \times 0) + (0 \times 1) = 0 \end{split}$$

Finally, expected utility  $(U_e)$  of decision alternatives can be calculated:

$$U_{e}^{car} = \left(P^{rain} \cdot U_{p}^{car-rain}\right) + \left(P^{norain} \cdot U_{p}^{car-norain}\right) = (0.5 \times 100) + (0.5 \times 50) = 75$$
$$U_{e}^{bus} = \left(P^{rain} \cdot U_{p}^{bus-rain}\right) + \left(P^{norain} \cdot U_{p}^{bus-norain}\right) = (0.5 \times 83) + (0.5 \times 0) = 41.5$$
$$U_{e}^{bike} = \left(P^{rain} \cdot U_{p}^{bike-rain}\right) + \left(P^{norain} \cdot U_{p}^{bike-norain}\right) = (0.5 \times 0) + (0.5 \times 100) = 50$$

Deringer

Specialized software is available to calculate and infer BIN, such as Hugin, Netica and many more (Murphy 2005). They can deal with complex networks involving multiple interrelated decisions and cognitive subsets. Thus, BIN allows a computational representation of a mental map of a decision problem.

However, a few drawbacks can be mentioned. First of all, the BIN has to be a directed acyclic graph in order to be able to calculate the system. Although forward propagation (calculating utilities from defined probabilities) and backward propagation (calculating probabilities from defined goals) through the network is possible, it is primarily a static system. The inability to incorporate cyclic structures is likely to limit its application for the representation of all human reasoning, although simulations of learning and adaptation are feasible. In future applications, this could be useful for the representation of spatial learning (on the level of the model's parameters) and habit formation as the outcome of repeated considered choices (Danner et al. 2007).

In addition, calculating BIN is restricted to linear propagation, and the types of relationships between variables are limited, while in actual decision-making, different types of relationships and various complex correlations might be needed to represent true decision-making mechanisms. Moreover, probability estimations in BIN are discrete values. In our application, individuals have to estimate these probabilities one by one. The correctness of such subjective probability estimates is debatable (Fox and Clemen 2005), especially for hazy, value laden concepts.

A final comment is related to the future integration of individual representations in an agent-based travel demand model. Merging BIN involves combining models qualitatively and quantitatively (del Sagrado and Moral 2003). The qualitative part represents the structure of the network and the dependencies among the variables. A clear challenge here is to avoid cycles. The quantitative part represents the CPT and CUT tables. When a set of BIN has to be combined in one model, an incremental process is a first choice. However, according to the order of BIN additions, the result of the integration process is different. So an open problem is how to determine the order when merging BIN. Recently, Ravi Kishan et al. (2009) and Jiang et al. (2005) tackled this problem. In Ravi Kishan et al. (2009), a framework is proposed and the problem of cycle generation is solved by assuming that the user can give a target variable ordering. But such an approach is problematic when applying this framework to interrelated travel decisions, since research has shown that the order of destination and transport mode decisions varies across respondents (Kusumastuti et al. 2010). Jiang et al. (2005) propose some heuristics when the user cannot give an ordering, but the application constraints remain and cycles cannot be avoided. A solution could be to make a distinct model for every ordering, i.e. a distinct model for every skeleton in an AB model of travel demand. To overcome some of these practical problems, the FCM technique is proposed.

# 3.3 Fuzzy cognitive maps

Fuzzy logic, its heuristics and good sense rules, and neural networks and its learning heuristics are combined in FCM (Stylios and Groumpos 1999). Building on cognitive maps introduced by political scientist Robert Axelrod (1976) in the 1970s

to analyse social decision-making problems in socio-economics and politics, Kosko (1986) developed FCM by enhancing cognitive maps with fuzzy reasoning. In soft knowledge domains such as politics, organization theory, international affairs and the like, system concepts and causal relationships are fundamentally fuzzy, as is the meta-system language. FCM allow reasoning with hazy concepts and hazy degrees of causality. Therefore, its application to model mental representations of travel behaviour decision-making in general, and the evaluation of related individual goals and values herein in specific, is appropriate.

Graphically represented, a FCM is a network of labelled nodes and labelled arrows (see Fig. 6). Nodes represent concepts that define a system, and their connecting arcs represent causal relationships. Unlike the BIN graph, cyclic connections are allowed in this depiction. Because of this characteristic, FCM can represent evolving systems in subsequent iterations. Although the mental map model proposed in this paper does not use this property to the full, this characteristic might offer opportunities to consider spatial learning and habit formation.

To transform a decision network (e.g. the BIN of a cognitive subset shown in Fig. 5) into a FCM (such as shown in Fig. 6), the structure of the BIN has to be adjusted to the structure of a FCM. This step involves a qualitative interpretation of the BIN. In this case, there are two transformations. The first conversion makes all nodes binary by splitting multi-value nodes, e.g. in the example shown in Figs. 5 and 6, the former "mode" and "comfort" node are split. The second transformation involves removing redundant nodes that represent a one-to-one linear construct. Therefore, the "shelter" and "utility" node disappear in the example; "shelter" is an essential part of each transport mode option, influencing the weight from the transport mode to its comfort level, while the utility of each transport mode equals the comfort level in this example.

To quantify a FCM, values are associated with its nodes and links (weighted arcs). All the values in the graph are fuzzy, so concepts take values in the range between [-1, 1] and the weights of the arcs are in the interval [-1, 1]. Concepts are fuzzy variables with two crisp states -1 and 1. A value 0 means there is no evidence for a state. According to Yan (2007), there are three possible types of causal relationships that express the type of influence from one concept to the others:



Fig. 6 Fuzzy cognitive map of a cognitive subset

positive (e.g. increasing the value in the child node leads to an increase in the value in the parent node), negative (e.g. an increase leads to a decrease or vice versa) and no causality (an increase or decrease in the value in the child node has no effect on the parent node).

In the example FCM in Fig. 6, the value of the concept "weather" is either "1" (rain), "0" (unknown) or "-1" (no rain). "Car", "bus" and "bike" are "1" (present) when calculating their respective comfort level, while the values of "comfort" related to each transport mode represent the calculated outcome of the FCM, hence the actual decision base. The weights of the FCM in Fig. 6 can be derived from the values of the BIN in Fig. 5, representing the answers to different survey questions. Assuming a linear relationship f(x) = x, these questions and answers can be rewritten as a set of equations to derive the weight values, as follows:

$$C_1^{\text{rain}} = f(w_1 \cdot C_4^{\text{rain}} + w_4 \cdot C_5^{\text{car}}) = f(w_1 \times 1 + w_4 \times 1) = 1 \text{ and} \\ C_1^{\text{norain}} = f(w_1 \cdot C_4^{\text{norain}} + w_4 \cdot C_5^{\text{car}}) = f(w_1 x - 1 + w_4 \times 1) = 0.5,$$

hence  $w_1 = 0.25$  and  $w_4 = 0.75$ ;

$$C_2^{\text{rain}} = f(w_2 \cdot C_4^{\text{rain}} + w_5 \cdot C_6^{\text{bus}}) = f(w_2 \times 1 + w_5 \times 1) = 0.83 \text{ and}$$
  

$$C_2^{\text{norain}} = f(w_2 \cdot C_4^{\text{norain}} + w_5 \cdot C_6^{\text{bus}}) = f(w_2 \times -1 + w_5 \times 1) = 0,$$

hence  $w_2 = 0.415$  and  $w_5 = 0.415$ ;

$$C_{3}^{\text{rain}} = f\left(w_{3} \cdot C_{4}^{\text{rain}} + w_{6} \cdot C_{7}^{\text{bike}}\right) = f\left(w_{3} \times 1 + w_{6} \times 1\right) = 0 \text{ and } \\ C_{3}^{\text{norain}} = f\left(w_{3} \cdot C_{4}^{\text{norain}} + w_{6} \cdot C_{7}^{\text{bike}}\right) = f\left(w_{3} \times -1 + w_{6} \times 1\right) = 1,$$

hence  $w_3 = -0.5$  and  $w_6 = 0.5$ .

To compute and reason within a fuzzy cognitive map, its algebraic representation is used. As Yan (2007, p. 76) explains: "It consists of a 1 × n state vector A which influences the values of the n concepts and a n x n weight matrix W which gathers the weights of  $W_{ij}$  of the interconnections between the n concepts of the FCM. The matrix W has n rows and n columns where n equals the total number of distinct concepts of the FCM and the matrix diagonal is zero since it is assumed that no concept causes itself. The value of each one concept is influenced by the values of the connected concepts with the appropriate weights." Table 2 shows the weight matrix representation of the Fuzzy Cognitive Map shown in Fig. 6.

From/to	$C_1$	$C_2$	$C_3$	$C_4$	$C_5$	$C_6$	$C_7$	Notation
<i>C</i> <sub>1</sub>	0	0	0	0	0	0	0	$C_1$ : Comfort car
$C_2$	0	0	0	0	0	0	0	$C_2$ : Comfort bus
<i>C</i> <sub>3</sub>	0	0	0	0	0	0	0	$C_3$ : Comfort bike
$C_4$	0.25	0.415	-0.5	0	0	0	0	$C_4$ : Weather
$C_5$	0.75	0	0	0	0	0	0	$C_5$ : Car
$C_6$	0	0.415	0	0	0	0	0	$C_6$ : Bus
<i>C</i> <sub>7</sub>	0	0	0.5	0	0	0	0	C <sub>7</sub> : Bike

Table 2 Weight Matrix representation of a Fuzzy Cognitive Map

The value  $A_i$  for each concept  $C_i$  can then be calculated as follows (1):

$$A_{i} = f\left(\sum_{\substack{j=1\\j\neq i}}^{n} A_{j}W_{ji}\right)$$
(1)

In this formula,  $A_i$  is the activation level of concept  $C_i$ ,  $A_j$  is the activation level of concept  $C_j$ , and  $W_{ji}$  is the weight of the interconnection between  $C_j$  and  $C_i$ , and f is a threshold function. For example with f(x) = x when it rains (weather = 1) and the car is chosen (car = 1), the obtained activation level of comfort is 1, since  $A_1 = f(0.25 \times 1 + 0.75 \times 1) = 1$ .

This way, the transport mode options can be ranked according to their comfort level. In this case, the outcomes are similar to the outcome of the BIN calculation. In both cases, it can be assumed that the final decision will equal the highest ranked choice option. These calculated decision outcomes are predictions of the transport mode choice in various circumstances. They can be validated further by asking the decision-maker about their actual choices given certain predefined scenarios.

However, the formulas above are but one implementation of FCM. In fact, every conceivable aggregator, operator or evaluator can be used in FCM. In this paper, the equal weighted sum is chosen as operator in every node.

#### 3.4 Comparing BIN and FCM: first empiric results

To illustrate the prospects of both modelling approaches, the first results of an empiric study are shown in this section. In December 2009, a computer-based survey (CB-CNET) was developed based on qualitative research (Kusumastuti et al. 2010) to gather mental representation data of 221 respondents regarding their location choice (zone 1, 2 or 3) and their transport mode choice (car, bus or bike) when planning a fun shopping activity. Figure 7 shows the resulting decision network of one respondent, picked randomly from the sample.

Table 3 compares the calculated outcomes of the corresponding BIN and FCM for the shopping location decision in four scenarios (with various evidence entered) and in one scenario without evidence. The first five rows give the raw figures from both models, the last five rows rank the decisions from most preferred (1) to least preferred (3) based on the model calculation.

For the shopping location decision, both models yield similar results. However, in the BIN model, calculations in all choice options are coupled, i.e. an increase in a value in one choice option results in different values of the other choice options, while choice options can be modelled independently in the FCM representation. For instance, the "FCM zone 2" column is the least preferred in all scenarios and its value remains "0" throughout the calculations, accordingly. In the BIN representation, changing evidence for zone 1 and zone 3 changes the values for zone 2.

The model results for the transport mode decision are shown in Table 4. As for the shopping location decision, both models perform similarly in their current conceptualization. Scenario 4 represents a choice situation in which the car is not available. In this case, the bike is chosen by the respondent. However, the overall





	BIN zone 1	BIN zone 2	BIN zone 3	FCM zone 1	FCM zone 2	FCM zone 3
Values						
Scenario 1	46.98	19	46.06	4.125	0	3.7
Scenario 2	51	26	40	4.25	0	3.1
Scenario 3	43	26	46	3.27	0	3.62
Scenario 4	41	20	40	3.875	0	3.46
No evidence	50	26	49	4	0	3.58
Rank						
Scenario 1	1	3	2	1	3	2
Scenario 2	1	3	2	1	3	2
Scenario 3	2	3	1	2	3	1
Scenario 4	1	3	2	1	3	2
No evidence	1	3	2	1	3	2

Table 3 Comparison of BIN and FCM outcomes for the shopping location decision

Table 4 Comparison of BIN and FCM outcomes for the transport mode decision

	BIN car	BIN bus	BIN bike	FCM car	FCM bus	FCM bike
Values						
Scenario 1	53	22	37	2.9	0	1.3
Scenario 2	48	30	38	3.4	1.5	1.8
Scenario 3	48	30	38	3.4	1.5	1.8
Scenario 4	35	24	41	1.5	1.4	2.7
No evidence	52	20	39	3.2	1.6	2.0
Rank						
Scenario 1	1	3	2	1	3	2
Scenario 2	1	3	2	1	3	2
Scenario 3	1	3	2	1	3	2
Scenario 4	2	3	1	2	3	1
No evidence	1	3	2	1	3	2

utility of the car is still higher than the overall utility of the bus, although this choice option is unavailable. In FCM, this problem can be solved by adjusting the evidence weights or by choosing a threshold function with anchoring behaviour in case "no car" evidence is shown, representing true decision-making. Such reasoning cannot be accommodated in BIN without fundamental changes of the model structure.

In future research, the algebraic cognitive map representation will be used to calculate distances or similarities between respondents' maps and to cluster and aggregate cognitive maps. Based on this, it will be possible to calculate a centroid cognitive map and make a corresponding centroid BIN for every cluster. This BIN can be used for reasoning by using its forward and backward reasoning properties.

Besides this, more fuzzy techniques will be introduced. These techniques help to improve the representation of the measured information and to model human decision-making more realistically. Until now respondents' answers are treated as true numbers, but such estimations are unreliable (Fox and Clemen 2005). Fuzzy concepts and operators akin to the fuzzy nature of human language and human reasoning may overcome this problem.

While there is specialized commercial software available to calculate BIN, FCM require the development of specific software tools due to the large degree of freedom in its system design. To visualize, calculate and infer the travel decision mental maps by means of FCM as explained in this paper, a specific software tool is under development (León Espinosa et al. 2009).

#### 4 Conclusions and discussion

In this paper, different meanings of the mental map relevant for the understanding of individual travel behaviour are addressed. Two notions of the mental map are of particular interest for modelling travel demand. In geographical approaches, the mental map is an umbrella term for an individual's spatial knowledge. From a decision-making perspective, the mental map is the temporary mental representation of a decision problem. The latter interpretation of the mental map is the point of departure for two computational implementations: BIN and FCM.

Although both models serve a similar purpose in representing the mental map of a decision problem, they differ with regard to basic theorem, network representation and parameter types that can be taken into account. BIN relies on Bayesian probability theory. For its compilation, decision nodes, chance nodes and utility nodes have to be defined and structured in a decision network and several parameters have to be estimated. Chance node values are based on conditional probabilistic assessments, while utility nodes and calculated decisions are deterministic. FCM on the other hand, combine the robust properties of fuzzy logic and neural networks (Kosko 1986). Here, knowledge is typically represented in a symbolic manner and states, processes, policies, events, values and inputs are related in an analogous manner. Moreover, FCM can incorporate cyclic structures, offering additional opportunities with regard to the representation of human reasoning. In addition, individual FCM can be merged more straightforward than BIN to establish a depiction of all individual mental maps in a single graphical model. However, FCM's versatility comes at the cost of sacrificing information and utility theories.

Since the objective of this paper is to illustrate methods to model the mental map of travel-related decision problems quantitatively rather than conducting an exhaustive comparison between modelling methods, clearly, there is scope for conducting more rigorous evaluations in the future. This paper outlines the conditions required to make such comparisons fair to both approaches, while current and future research efforts concentrate on further large-scale empirical testing of the modelling concepts illustrated here. Therefore, a computer-based survey is developed that replaces the time-consuming oral CNET interview protocol. Besides this, the integration of mental maps in an activity-based, computational process model of travel demand is examined. Preliminary conclusions with regard to the integration of the CNET interview protocol

to fun shopping-related travel decisions, are discussed in (Kusumastuti et al. 2010). The approach shows promising results for the improvement of behavioural realism and forecasting accuracy of travel demand models.

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