LONGITUDINAL PROPAGATION VELOCITY OF THE NORMAL ZONE
IN SUPERCONDUCTING WIRES

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Abstract

The longitudinal propagation of the normal zone in superconducting wires was experimentally investigated in order to evaluate existing analytical expressions which attempt to describe the propagation velocity in a more or less simple manner. The availability of a reliable expression is important for application in computer codes that calculate quench evolutions in superconducting magnets. We measured the propagation velocity as function of transport current and magnetic field in five different insulated NbTi superconductors having a copper, a copper-nickel or a mixed matrix. The comparison to calculated velocities using five existing models showed that large quantitative and qualitative differences exist.

Introduction

The resistance rate in a superconducting magnet after the occurrence of a quench is of a great importance for the design of an appropriate quench protection system. The usual method for determining the growth of the coil resistance in impregnated coils is to calculate the longitudinal velocity with which the normal zone in the superconductor expands after which the different transverse components of the quench propagation velocity can be calculated assuming an estimate for the ratio of transverse to longitudinal thermal conductivities. The velocities in either direction so obtained are used in computer codes that simulate, among other things, the growth of the normal conducting part of the magnet and the further expiration of a quench [1]. It is the aim of the experiments to compare and contrast experimentally found velocities with five theoretical models.

Present analytic formulae

The normal zone propagation is determined by the amount of heat production in the normal zone, the enthalpy and heat flow from the zone to the environment. The heat-balance equation in one dimension (the longitudinal z direction) that describes such a process and the temperature profile along the wire is given by:

$$c(T) \frac{\partial T}{\partial t} = \frac{\partial}{\partial z} (k(T) \frac{\partial T}{\partial z}) - \phi + q, \quad (1)$$

where c is the mean heat capacity, k the mean thermal conductivity, ϕ the heat flow in the transverse direction and q the heat production. While treating the case as one dimensional, the basic assumptions are that the temperature profile and the heat production are uniform over the cross-section. As a consequence the heat capacity and thermal conductivity are averaged for each cross-section. It is obvious that the one-dimensional treatment neglects the effects due to the uniform distribution of the filaments, the non-uniform current distribution as well as inhomogeneous matrices such as the presence of copper-nickel barriers.

After adoption of equation (1), the remaining assumptions concern the mechanisms of heat transfer and heat production. The term ϕ in (1) can include steady-state heat conduction expressed by a transfer

coefficient h [$W/m^2/k$], but also a transient component. The latter term can usually be neglected if the conductor has no direct contact with liquid helium. The reader is referred to Lvovsky [6] for more details about transient terms. Here the models assuming a constant heat transfer are treated which attempt to describe propagations in enclosed superconductors. This means $\phi = h \cdot p / A \cdot (T - T_0)$, where p is the perimeter and A the cross-section of the wire and T_0 the bath temperature. The heat production term q at current I is different in the three regions in the temperature profile that can be distinguished:

$$q = 0, \quad \text{superconducting state;} \quad (2)$$

$$q = \left(\frac{T - T_0}{T_c - T_0} \right) \rho \left(\frac{I}{A} \right)^2, \quad \text{current sharing state;} \quad (3)$$

$$q = \rho (I/A)^2, \quad \text{normal state;} \quad (4)$$

in which T_c is the field-dependent critical temperature, and ρ the mean resistivity of the wire.

Equation (1) can be solved analytically provided k and c are constant and taken at a certain intermediate temperature. Three authors solved the model neglecting the current-sharing state. They considered the joule heating to begin discontinuously at a certain transition temperature T_i . Then, the general solution of (1) using the appropriate boundary conditions is found in the form of a travelling wave with a reduced velocity v in terms of the Stekly parameter α , the reduced transition temperature $t_i = (T_i - T_0)/(T_c - T_0)$, and the reduced transport current $i = I/I_c(B)$. The physical propagation velocity V is now:

$$V = \left[\frac{hpk}{\alpha c} \right]^{1/2} \cdot v, \quad (5)$$

$$\text{where } v = (\alpha i^2 - 2t_i) / (t_i(\alpha i^2 - t_i))^{1/2}, \quad (6)$$

$$\text{and the Stekly parameter } \alpha = \rho I_c^2 / (Aph(T_c - T_0)). \quad (7)$$

The choice of the transition temperature has led to three models. The consequences of these models for the reduced velocity v , the minimum propagation current (also called cold-and recovery current) i_m ($v=0$) as well as the full recovery current i_f ($v=-\infty$) are given in the next table.

	i_m	i_f	t_i	T_i	v_{max}
Cherry [2]	$\sqrt{2/\alpha}$	$1/\sqrt{\alpha}$	1	T_c	$\frac{\alpha-2}{\sqrt{\alpha-1}}$
Keilin [3]	$\frac{\sqrt{1+2\alpha}-1}{\alpha}$	$\frac{\sqrt{1+4\alpha}-1}{2\alpha}$	1-i	T_{cs}	-
Dressner [4]	$\frac{\sqrt{1+8\alpha}-1}{2\alpha}$	$\frac{\sqrt{1+16\alpha}-1}{4\alpha}$	1-1/2	$\frac{T_{cs}+T_c}{2}$	$\frac{2(\alpha-1)}{\sqrt{2\alpha-1}}$
Turck [5]	$\frac{\sqrt{1+8\alpha}-1}{2\alpha}$	$\frac{1}{\sqrt{\alpha}}$	solving a third diff. equation		$2\sqrt{\alpha-1}$
Lvovsky [7]	$\frac{\sqrt{1+8\alpha}-1}{2\alpha}$	$\frac{1}{\sqrt{\alpha}}$	approximation of numerical results		$\frac{T_c}{2(T_i)} 2.5\sqrt{\alpha-1}$

¹⁾ provided physical parameters c , k taken at T_c .

Fig. 1. Temperature profile and the heating and cooling curves vs. temperature, assuming a constant h .

dependence of the heating and cooling curves. For convenience the calculated velocities according to five models are shown in Fig. 2.

Cherry and Gittleman [2] describe the solution where the joule heating starts step-wise at the critical temperature. This leads to propagation velocities which are too small and a minimum propagation current which is too large. Keilin et al. [3] predict the velocity when the joule heating starts at the current-sharing temperature T_{cs} . The found velocities are obviously too large and the minimum propagation velocity is too small.

Fig. 2. Comparison of calculated velocities according to five models (—) and measured data (o) for a certain wire and a field of 6 tesla.

A more realistic behaviour is given by Dresner [4]. He takes the transition temperature as the mean value between the current sharing and the critical temperature and finds a correct minimum propagation current but a too large full-recovery current. For α less than 20, Dresner states a correction factor on v of $(1+0.561\alpha^{-1.45})$. The fourth model as described by Turck et al. [5] includes equation 3 in the current-sharing region. The third differential equation thus obtained introduces, compared to Dresner's model, an increase of the velocity in the vicinity of the critical current. The solution is:

$$v = \frac{\alpha i^2 + i - 2}{(\alpha i^2 - 1)^{1/2}} + (\alpha - 1)^{1/2} \left[2 - 2 \left(\frac{1-i}{1-i_m} \right)^{1/2} - \left(\frac{i-i_m}{1-i_m} \right) \right] \quad (8)$$

It can be seen from Fig. 2 that the formulae of Dresner and Turck coincide in the main current range between the minimum propagation current and 80% of the critical current. Lvovsky proposed an analytical expression that approximates his numerical solution [7]. He stated:

$$v = v_{\max} \left[1 - \left\{ 1 - \frac{v_1}{2} \ln \left(1 - \frac{i-i_m}{1-i_m} \right) \right\}^{-2} - v_1 (1 - 0.4v_1) * \left(\frac{i-i_m}{1-i_m} \right)^2 \left(\frac{1-i}{1-i_m} \right) \right] \quad (9)$$

where

$$v_1 = (1-i_m)/v_{\max} * \left(\frac{\partial v}{\partial i} \right)_{i_m}$$

and

$$\left(\frac{\partial v}{\partial i} \right)_{i_m} = \frac{4-i_m}{i_m(1-i_m)} * \left[1 + \left(\frac{i_m}{2(1-i_m)} \right)^{1/2} / \operatorname{tg} \left(\frac{i_m}{2(1-i_m)} \right)^{1/2} \right]^{-1}.$$

This solution gives a minimum propagation current which is identical to those of Dresner and Turck. However the maximum reduced velocity v following this formulae is larger than that given by Turck.

Since Lvovsky takes the physical parameters at T_c and Turck at T_{av} , the deviation between the maximum physical velocities is a factor $(T_0/T_{av})^{2.5}$. This difference can be clearly seen in Fig. 2.

The physical model parameters

A serious handicap with calculating propagation velocities is the lack of precise knowledge of the various physical properties of the wire materials. Many authors fit calculated velocities to measured ones and ascribe the fitting factor to the large uncertainties in the model constants such as the thermal conductivity and heat capacity. Here we attempt to compute them using h , the heat transfer coefficient, as the only fitting parameter.

The measured quantities are the velocity V , the wire diameter D , the wire resistivity as function of field $\rho(B)$, the critical current $I_c(B)$, the current I and the applied field B_{ext} . The magnetic field on the wire is given by $B_{ext} + f_D * I$, where the latter term represents the self-field and the field due to adjacent wires. For the critical current the formulation of Lubell was adopted: $T_c(B) = T_c(B=0) * (1-B/14.5)^{0.59}$. The current sharing temperature $T_{cs}(B, I) = T_0 + (T_c(B) - T_0) * (1-i)$ and the average transition temperature $T_{av} = (T_c + T_{cs})/2$. The heat capacity is $c(T, B) = [(6.75 * f + 50.55) * T + (97.43 * f + 69.81 * B) * T] / (1+f)$, which includes a field-dependent term [8]. The mean thermal conductivity is eliminated from the calculation by using the Wiedemann-Franz Law stating $k = L_0 * T / \rho(B)$, where $L_0 = 2.44 * 10^{-8} \text{ W}\Omega/\text{K}^2$.

The temperature dependent parameters are taken at the proposed transition temperatures which are given as $T_c(B)$, $T_c(B, I)$ and $T_{av}(B, I)$ by Cherry, Keilin and Dresner respectively. In Turck's model they are also taken at $T_{av}(B, I)$, while Lvovsky uses them at $T_c(B)$.

A more serious problem arises with the critical current which has a large influence on the velocity especially at low magnetic fields. The effect of the currently applied definitions such as $1 \mu\text{V}/\text{cm}$, $10^{-13} \Omega\text{m}$ and $10^{-11} \Omega\text{m}$ may result in deviations as large as 1 to 5%. Therefore the ultimate accuracy of the calculated velocities will be about the same percentage. In our calculations the 10^{-11} criterion has been applied.

Measurements on five superconductors

The propagation velocities are determined by measuring the time-lag between the start of the voltage rise at two adjacent voltage taps on the wire. During the measurement the current is kept almost constant by using a choking coil with an appropriate inductance, which value is chosen in such a way that the wire can not burn out due to excessive heating. This protection is very important in wires with a copper-nickel matrix, since, for example, at a current density of $1000 \text{ A}/\text{mm}^2$ the maximum time constant is 2 ms. Approximately one meter of the sample is wound on a former (dia. 70 mm), covered with about 1 mm STYCAST 2850FT resin and

inserted in a 7 tesla magnet. In order to prevent an undesired heat exchange the sensing wires are of nichrome. So we obtain an enclosed superconductor with no direct helium contact. The maximum current during the tests was 400 A and the field was varied between 0 and 6 tesla.

The specifications of the investigated wires are given in table 1.

Table 1. Specifications of the tested NbTi superconductors

no	manufac- turer	matrix	diam. [μm]	number fil.	CuNi/Cu/NbTi ratio	Fig.
1	MCA	Cu	686	2070	0/1.80/1	3
2	MCA	Cu	279	367	0/1.25/1	4
3	MCA	CuNi	127	114	1.35/0/1	5
4	MCA	CuNi(*)	295	574	1.10/0/1	6
5	ALSTHOM	Cu/CuNi	300	14496	0.83/1.16/1	8

* Cu₃ONi bulk and Cu₁₀Ni shell.

Note that all types of matrices are represented. The accuracy of the measured velocities is within 2%.

The calculated velocities using these assumptions were already shown in Fig. 2. The differences between Keilin's and Cherry's formulae with those of Dresner and Turck are reduced by approximately 50% if the constants c and k are taken at T_{av} . The effect of using the mean heat capacity between T_{cs} and T_c instead of the capacity at T_{av} is 1 or 2 percent and thus negligibly small.

Now the results will be discussed consecutively. First both wires with a copper matrix (1 and 2) are considered. In Figures 3 and 4, the comparison between measured data and the models of Dresner and Turck is shown. As already shown in Fig. 2, these models fit qualitatively and quantitatively very well. At 4 and 6 tesla with the thick wire, and at 2, 4 and 6 tesla with the thin wire the velocities were measured up to the critical current, as indicated in the figures. Note that both models almost coincide between the minimum propagation current and 80% of the critical current.

Fig. 3. Wire 1, table 1, Cu matrix. Comparison between measured velocities and the formulae of Turck and Dresner. Parameters: B_{ext} 0-6 T, $h = 650 \text{ W/m}^2/\text{K}$.

Fig. 4. Wire 2, table 1, Cu matrix. As in figure 3. $h = 850 \text{ W/m}^2/\text{K}$.

The extra increase of the velocity near the critical current as given by Turck cannot be proven by this experiment if we remind the limited accuracy of the critical current. Moreover, this difference has no practical sense because superconductors usually do not operate near the critical current. Further it appears that at zero external field and large currents the models underestimate in both wires the velocity with 10%. Note that the minimum propagation current and the slope at this point is correctly given at all fields. Obviously a constant heat transfer coefficient, which is 650 and 850 $\text{W/m}^2/\text{K}$ for these wires respectively, can describe the field-dependent minimum propagation current as given in table 1 very well.

Wire 3 and 4 have a high-resistivity matrix of CuNi. This implies a bad thermal conductivity as well, which may have its influence on the qualitative behaviour of the velocity. As a consequence the heat transfer coefficient h has almost no effect on the velocities at least in the practical range of 0 to 10 $\text{W/m}^2/\text{K}$. In Fig. 5 the results for wire 3 are shown. It

Fig. 5. Wire 3, table 1, CuNi matrix. Measured velocities at 0, 1 and 2.5 tesla, compared to Dresner's and Turck's formulae. $h = 650 \text{ W/m}^2/\text{K}$.

Fig. 6. Wire 4, table 1, velocities between 0 and 6 T compared to Dresner's and Turck's formulae. $h = 650 \text{ W/m}^2/\text{K}$.

is surprising to see that there is only a weak field dependency, and that the measured velocities increase almost linear with current up to 80% of I_c although in both models they are non-linear. The results of wire 4 indicate a similar behaviour, see figure 6. Nor does any of the other three models provide a good solution as illustrated in Fig. 7. Therefore, we must conclude that the five models under consideration cannot explain the propagation velocity in this type of conductors in a satisfactory manner though the formulae of Dresner and Turck can be used to get an indicative value with an accuracy of about 30%.

Fig. 7. Wire 4. Comparison of all measured data with all five models computed at 4 tesla. The measured data at 4 T are indicated by (o).

Finally the results of wire 5 are considered. They are pictured in Fig. 8. It is a superconductor for ac applications and therefore it has a mixed matrix whereby the filaments are surrounded by CuNi while the core is made of Cu with CuNi barriers. The influence of the copper content is clear in spite of the low copper content of 28%. A significant minimum propagation current is present and it is well explained by Dresner's formula. The fitting heat transfer coefficient is $500 \text{ W/m}^2/\text{K}$.

Fig. 8. Wire 5, table 1, mixed matrix. Measured velocities between 0 and 6 tesla. Comparison to Turck's and Dresner's formulae. $h = 650 \text{ W/m}^2/\text{K}$.

Conclusions

Propagation velocities were measured as function of applied field and current in enclosed superconductors. It appears that the models of Dresner and Turck provide correct velocities in the case of wires having a copper matrix. The extra velocity rate in the vicinity of the critical current as stated in Turck's model could not be proven though in a few cases such an increase was measured. The absence of a well defined critical current causes considerable errors which prohibits a definite conclusion on this point.

The assumptions in the models of Cherry and Keilin are unrealistic and cause large deviations. It is surprising to note that the formula of Lvovsky which actually is an analytical approximation of a numerical solution underestimates the velocity in the main current range while near the critical current the velocity is much too large. The minimum propagation current and the velocity rate near this current, as function of the field but using a constant heat transfer coefficient, is well described by Dresner's and Turck's model. The velocities near the critical current are finite.

The formulae of Dresner and Turck have a very limited accuracy in the case of wires with a copper-nickel or a mixed matrix. They can be used to estimate the velocity within approximately 30%. For this type of wires with a bad thermal conductivity it is recommended to solve the set of heat-balance equations in two dimensions.

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