Line Tension and Wettability of Nanodrops on Curved Surfaces

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ABSTRACT: In this work we study the formation of nanodrops on curved surfaces (both convex and concave) by means of molecular dynamics simulations, where the particles interact via a Lennard-Jones potential. We find that the contact angle is not affected by the curvature of the substrate, in agreement with previous experimental findings. This means that the change in curvature of the drop in response to the change in curvature of the substrate can be predicted from simple geometrical considerations, under the assumption that the drop’s shape is a spherical cap, and that the volume remains unchanged through the curvature. The resulting prediction is in perfect agreement with the simulation results, for both convex and concave substrates. In addition, we calculate the line tension, namely, by fitting the contact angle for different size drops to the modified Young equation. We find that the line tension for concave surfaces is larger than for convex surfaces, while for zero curvature it has a clear maximum. This feature is found to be correlated with the number of particles in the first layer of the liquid on the surface.

INTRODUCTION

The line tension is a key property for understanding the behavior of nanodrops and, thereby, of great technological relevance for lithography techniques or in micro- and nanofluidics. On curved surfaces the line tension is of great significance for froth flotation, microporous solid, and condensation on nanorods. Due to the small magnitude of line tension, it can affect wetting properties at the nanoscale without having any effect on the micro- or macroscale. Understanding, and hence predicting, the line tension of nanodrops is nontrivial. Although there have been a number of theoretical and experimental studies on the subject, up to recently there was no consensus even on the sign nor on the order of magnitude.

The concept of line tension was introduced more than a century ago by Gibbs, who concluded that interactions at the three-phase contact line cannot be explained by surface free energies of each pair of phases alone. He defined the line tension as the excess free energy per unit length of a contact line of three phases, analogous to surface tension, which is the excess free energy per unit area. In 1937, Harkins managed to theoretically calculate the order of magnitude of line tension from the relation between latent heat of vaporization and the free, latent, and total energy of the three-phase contact line. In 1977, Pethica defined line tension for a liquid drop on an ideal (chemically and geometrically homogeneous) solid surface when it is in equilibrium with its own vapor, as illustrated in Figure 1. In eq 1, τ is the line tension, θ is Young contact angle, and γ, γ, and γ are the solid–liquid, solid–vapor, and liquid–vapor

\[
\cos \theta = \frac{\gamma_{SV} - \gamma_{SL}}{\gamma_{LV}} - \frac{\tau}{\gamma_{LV}} = \cos \theta_Y - \frac{\tau}{\gamma_{LV}}
\]

Figure 1. Definition of the various geometrical parameters used in this work, for (a) the convex surface and (b) the concave surface. Three different radii of curvatures can be identified: radius of surface curvature (R), radius of drop curvature (R), and radius of curvature of the contact line (R). Note that we define R such that for the convex surface it has a positive value and for the concave surface a negative value.

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surface tensions, respectively. In eq 1, we have not considered the effect of curvature of the liquid—vapor interface on the surface tension, because if these effects become comparable in magnitude to the line tension, then the measured \( \gamma \) cannot be considered as “pure” line tension, but as an apparent line tension.6,10

The magnitude of the line tension has been calculated from the free energy associated with the three-phase contact line using density functional theory11,12 or a model based on the interface displacement.13,14 Most of these theoretical analyses predict a value of the line tension in the range of 10⁻¹² to 10⁻¹¹ N. Experimental investigations show that the order of magnitude of an effective line tension varies from 10⁻⁵ to 10⁻¹² N with both positive and negative signs,6,15−19 which basically only shows how challenging it is to measure the line tension experimentally. The primary reason for this variation is the contact angle hysteresis caused by surface heterogeneities, either geometric or chemical, which are always present for an actual experimental situation. But also the extremely low size range of the drops makes it very difficult to measure the line tension experimentally. For example, in the case of water drops with a surface tension of 0.072 N/m and a line tension of 10⁻¹¹ N (the most consistent order of magnitude reported in literature), the line tension becomes significant only for contact line curvatures of around 5 nm.20

Studies have not been confined to flat surfaces. Extrand and Moon21 have investigated the dependence of contact angle on surface curvatures experimentally, however for drop sizes in the micro- and millimeter range. Marmur and Krasovitski22 have calculated the line tension on spherical surfaces from theoretical calculations, but it lacks validation from experimental or simulation data.

Apart from theoretical and experimental research, there have also been a number of studies on the line tension and measurement of contact angle using molecular dynamics simulations.23−26 Such simulations have the advantage that the line tension can be calculated with relatively large accuracy, and also the heterogeneities can be well-controlled. Shi and Dhir24 studied the behavior of the contact angle of drops on a plane solid substrate as a function of temperature and force field parameters, using a simple Lennard-Jones model (as described in the next section), as well as a more advanced model potential for water. Ingemertsén and Toxvaerd25 have calculated the contact angles for drops of different sizes and observed that the contact angle from MD simulations disagrees with the Young contact angle for nanodrops which have a very small contact angle. Weijts et al.25 have analyzed the effect of line tension by measuring contact angles of droplets on a plane substrate by varying the droplet size and found that the line tension decreases with increasing \( \theta_y \).

In the present work, we have extended the simulation by Weijts et al.25 to curved substrates. Simulations have been performed in 3D, that is, for spherical drops, and in quasi-2D (where one dimension is considerably smaller than the other two), to which we refer as cylindrical drops. Cylindrical drops give the value of \( \theta_y \) as the contact line is free from any curvature and hence the contact angle does not change with the size of the drop. The effect of the curvature on the contact line can then be predicted by calculating the contact angle for spherical drops of different sizes and comparing it with \( \theta_y \). In this way, we have systematically studied the effect of surface curvature on the magnitude of the line tension and wettability of nanodrops on curved surfaces.

### NUMERICAL METHOD

**Molecular Dynamics Simulations of Nanodrops.** Molecular dynamics (MD) simulations were performed to simulate the drop on a solid substrate for which we used the open source code GROMACS.27 Two kinds of particles were used in the simulations: solid substrate particles, which are held fixed in a fcc lattice setting during the whole simulation, and liquid/vapor particles, which are free to move and in the equilibrium-state form in a liquid drop on the solid substrate, with a vapor phase filling the remaining volume. The interaction between the particles is described by Lennard-Jones potential:

\[
\phi_{LJ}(r) = 4\epsilon \left( \frac{r_x}{\sigma_x} \right)^{12} - \left( \frac{r_x}{\sigma_x} \right)^{6}
\]

in which \( \epsilon \) is the interaction strength between particles \( i \) and \( j \) and \( \sigma \) is the characteristic size of particles, which is set to a value \( \sigma = 0.34 \) nm for all interactions. The potential is truncated at a relatively large cutoff radius of \( \tau_c = 5\sigma \). The time step for updating the particle velocities and positions was set at \( dt = \sigma \sqrt{m/(\epsilon \sigma^3)} / 400 \), where \( m \) is the mass of the particles and \( \epsilon_{LL} = 3 \) kJ/mol is the Lennard-Jones interaction parameter for the liquid phase. Simulations have been performed in an NVT ensemble where the temperature is fixed at 300 K, which is below the critical point of the Lennard-Jones parameters (\( \epsilon \), \( \sigma_{LL} \)) that we have used. Periodic boundary conditions have been employed in all three dimensions. We have studied two different kinds of systems to examine the effect of line tension: quasi-2D and 3D. In quasi-2D, the system size in one dimension is substantially less than the size of the system in the other two dimensions. The typical dimension of the system is 10.5\( \sigma \) in the \( x \)-direction and around 150\( \sigma \) in the \( y \)- and \( z \)-directions, where the \( x \), \( y \), and \( z \) directions are defined in Figure 1. In 3D, the system size in all three directions is of the same order of magnitude. The system size is such that in all cases the distance between a drop and its neighboring image is at least 80\( \sigma \). In all simulations, the overall number density is kept constant. The equilibrium contact angle (or wettabilility) of the liquid drop on the solid substrate was varied by changing the interaction strength between solid and liquid particles \( (\epsilon_{SL}) \), from 1.0 to 2.0 kJ/mol. In all simulations the liquid drop was found to be in equilibrium with its own vapor, where the liquid and vapor density was found to be in close agreement with the theoretical result as obtained from the equation of state of the Lennard-Jones fluid.28

In the simulations, the line tension is evaluated in terms of the tension length (\( l \)) defined as \( l = -\tau_y \gamma_{LV} \), which is of the order of magnitude of the molecular scale. We have calculated the magnitude of the line tension length along the lines of Weijts et al.,25 by measuring the equilibrium \( \theta_y \) for different drops and fitting a straight line to \( \cos \theta_y \) vs \( 1/R \), where the slope is then equal to \( l \). Repeating these calculations for various values of the LJ parameters then gives the tension length (\( -\tau_y \gamma_{LV} \)) as a function of \( \theta_y \).

Two kinds of surface curvatures were used in this study as shown in Figure 2: a curved outward (or convex) surface defined as a positive curvature and a curved inward (or concave) surface defined as a negative curvature. In order to keep the overall number density constant, we have scaled the system dimensions while increasing the number of particles in the simulations. We have also scaled the radius of curvature of the surface according to the system dimensions. The surface curvature scales with \( n^{-2/3} \) in the case of quasi-2D and with \( n^{-1/3} \) in the case of 3D, where \( n \) is the number of moving particles in the simulation. Note that owing to the discrete nature of particles and the relatively small system size, the curvature of the solid substrate is not smooth but consists of steps, with a step height equal to the particle diameter, as shown in Figure 2. Because of these finite steps, the three-phase contact line will be in contact with different crystallographic axes in each simulation. Different crystallographic axes exhibit different surface energies which may lead to a slight change in the contact angle.29,30 We have ignored this effect as we have averaged the contact angle for different equilibrium profiles which means that the contact angle calculated from our simulations is average over different crystallographic axes. We have also ignored the effect of surface curvature.
reconstruction as solid particles remain fixed in an fcc lattice during the whole simulation. This occurs either with less stable metal surfaces, semiconductor surfaces at very high temperatures, or polymer surfaces with polar groups.31−33 We are not dealing with polar polymer groups or high temperatures, so it is justified to ignore this effect.

Initially, the liquid particles are set in a fcc configuration close to the solid substrate and free to move from then on at the prescribed temperature (see Figure 2). After equilibrium has been reached, i.e., after around $5 \times 10^7$ time steps, the density field is calculated by averaging over typically 1,000,000 time steps (which corresponds to roughly 2 ns) taking into account the fluctuation of the center of mass of the droplet. The radius of curvature of the droplet is then obtained by fitting a sphere (circle in 2D) to the isodensity contour of 0.5 of the normalized density field, $\rho^*(r) = \frac{\rho(r) - \rho_L}{\rho_V - \rho_L}$, where $\rho_V$ and $\rho_L$ is the bulk vapor and liquid density, respectively. Since the liquid very near to the solid substrate is subject to layering, we have excluded the density field in the range of $2\sigma$ from the substrate for the circular cap fitting. From the intersection of the circular fit with the substrate, the contact angle and volume of the drop are evaluated (see Figure 3). Note that we have split the time interval over which we measured into 10 subsets and calculated the average of each subset in order to evaluate a standard deviation, from which the error bars in the results of Figures 4, 5, and 6 were obtained.

- RESULTS AND DISCUSSION

Wettability of Cylindrical Nanodrops on Curved Surfaces. Figure 4 shows the Young contact angle as a function of the surface curvature, for three different values of the liquid−solid interaction strength. It can be clearly seen that the contact angle is unaffected by the surface curvature, which is consistent with the experimental observations by Extrand and Moon21 for microscopic drops. Note that Wolansky and Marmur34 also showed theoretically that the contact angle is independent of the shape of the surface, unless line tension is considered. This means that curvature effects are only prominent in nanoscale systems where the line tension is appreciable as shown in the next section. Figure 5 shows that also the drop volume does not change with the surface curvature, which is expected since the volume is set by the condition of liquid−vapor equilibrium, which to first order is not affected by the surface curvature. In fact, the straight line in Figure 5 is the volume of the liquid drop evaluated from the bulk vapor−liquid equilibrium calculated from a highly accurate equation of state of the LJ fluid for the given temperature and overall number density.28
as found in the simulations could be attributed to the effect of the liquid–vapor surface and the solid substrate, which are not accounted for in bulk phase equilibrium. Figure 6 shows the variation of the radius of drop curvature ($R_d$) with surface curvature ($1/R_s$). The simulation results are found to be in very good agreement with the prediction for $R_d$ that follows from straightforward geometric relations for the drop volume as a function of the contact angle and surface curvature. In this, we take the volume of the drop as calculated from the equation of state, and the contact angle for a planar surface as reference values.

**Line Tension of Nanodrops on Curved Surfaces.** The line tension of LJ nanodrops has been calculated by fitting the modified Young equation, eq 1, to the simulation data. To this end, different sized LJ drops were simulated in 3D on the curved surface, and the cosine of the contact angle is plotted against the inverse of the radius of the curvature of the three-phase contact line, $1/R$, as shown in Figure 7. The slope of the straight line fitted through the data points then gives the line tension length $l = -\tau/f_{LV}$. To double check, in Figure 7, we also show the result for $\cos \theta$ for the quasi-2D system (cylindrical drop), which as expected is not changing with the curvature of the drop because the contact line is free of any curvature and the line tension does not have any effect on it. According to eq 1, the two lines in Figure 7 should intersect at zero surface curvature. However, we find a small offset, which can most likely be contributed to the fact that the exact position of the liquid–vapor interface is not well-defined. That is, there is a smooth transition between the two distinct phases and many logical definitions are available to calculate the position of the interface. Hence quantities which are derived from the interface location, such as contact angle, volume, and radius of curvature of the drop, etc., will slightly vary depending on the definition, which may result in a slight offset from eq 1. Note also that, for these typical values of the contact angle, only a slight error in $\theta$ of say 1% leads to errors of 3% in $\cos \theta$, which leads even to larger errors in the extrapolated value of the line fit.

Figure 8 shows the magnitude of the line tension length as a function of the surface curvature normalized by $V_{drop}^{1/3}$, where $V_{drop}$ is the volume of the drop as calculated from the equation of state for Lennard-Jones fluid. We find a clear maximum in the case of the planar surface which is compatible with the theoretical predictions; however, surprisingly we find that $l$ is much higher in the case of concave surfaces (negative curvature) as compared to convex surfaces (positive curvature). From Figure 8, we can infer that the magnitude of the surface curvature does not have a very strong effect on the line tension length but it is strongly dependent on the sign of the surface curvature. We investigated this finding by analyzing the arrangement of particles very close to the surface. In Figure 9, we have plotted the variation of density of particles and the absolute number of particles in a drop as a function of distance from the surface. Both quantities are evaluated from the time-averaged number of particles for concentric spherical shells or layers of thickness 0.1σ. The variation in the absolute number of particles is determined by averaging the number of particles in each layer over time. Density in each layer is then calculated by dividing the absolute number of particles in each layer by the volume of that layer. It can be seen that the amplitude of the oscillations in the density is much larger in the case of the planar surface as compared to the curved ones, yet the difference in density oscillations between positive and negative surface curvature is small. However, the variation in the number of particles in the drop as a function of distance from the surface clearly shows the difference between the three types of surface particles.

![Figure 6](image6.png)  
*Figure 6. Radius of curvature, ($R_d$) of the nanodrop on a solid substrate as a function of surface curvature ($1/R_s$).*

![Figure 7](image7.png)  
*Figure 7. Results from the simulation for the contact angle on a concave surface with $V_{drop}^{1/3}/R_s = -0.2$ as a function of the curvature of the base circles ($\sigma/R$). The circles are the results for spherical drops, while the squares are for cylindrical drops. Straight lines are the linear fit through the data points. The slope of the straight line for spherical drops gives the line tension length $l = -\tau/f_{LV}$.*

![Figure 8](image8.png)  
*Figure 8. Variation of $l$ as a function of $V_{drop}^{1/3}/R_s$. Note that smaller moduli of $V_{drop}^{1/3}/R_s$ are not possible due to the graininess of the surface particles.*
surfaces. The amplitude of the first peak is maximal in the case of a flat surface, then followed by the peak of negative and positive surface curvature, respectively. This trend is directly correlated with the magnitude of the line tension length, which suggests that more particles in layers close to the surface imply a larger line tension. Note that the absolute number of particles and the density of particles as a function of distance are related to each other by the volume of each layer. Since the density of particles is almost the same for both curvatures, this implies that the volume of layers very close to the surface is larger in the case of a concave surface. We have also analyzed the structure of the particles very near to the surface by evaluating radial distribution functions or pair correlation functions for particles in the first two layers as shown in Figure 10. The magnitude of the second peak in the radial distribution function is marginally larger in the case of curved surfaces as compared to the plane surface, but there is no difference between positive and negative surface curvatures which indicates that it is the absolute number of particles in the layers near to the surface which is responsible for the decrease in magnitude of the line tension and not the relative arrangement of the particles. From Figure 11, we can see a direct proportionality between the line tension length and the amplitude of the first oscillation of a variation of number of particles with the distance from the surface.

The actual quantity of interest, the line tension, can be calculated by multiplying the line tension length by the liquid-vapor surface tension ($\gamma_{LV}$). Kirkwood and Buff showed that the latter can be calculated by integrating the difference between normal and tangential components of the pressure tensor across the interface,

$$\gamma_{LV} = \frac{1}{2} \int_0^L [p_n(z) - p_t(z)] \, dz$$

We have used this procedure to calculate $\gamma_{LV}$ from independent molecular dynamics simulation of liquid molecules in equilibrium with its own vapor and for a planar interface. Note that the factor $1/2$ in eq 3 is the correction for the extra interface that is present due to periodic boundary conditions. We again emphasize that we have not considered surface tension as a function of interface curvature. The radius of curvature of a nanodrop of 7 nm is required to change the surface tension by 5% for the Lennard-Jones particles that we are simulating. For our Lennard-Jones parameters, i.e., $\epsilon_{LL} = 3.0$ kJ/mol, this procedure gave a value of $\gamma_{LV} = 4.1893 \times 10^{-3}$ N/m. Using this value for the surface tension, the order of magnitude of the line tension is coming in the vicinity of $10^{-12}$ N which is very close to the theoretical prediction and many experimental findings.

The maximum value of the line tension is around $13 \times 10^{-12}$ N for planar surface, and the minimum value is $0.07 \times 10^{-12}$ N, which is almost zero, in the case of a convex surface.

**CONCLUSIONS**

Molecular dynamics simulations were performed for liquid drops on curved surfaces. The Young contact angle was found to be constant with surface curvature which is consistent with previous experimental and theoretical predictions. The volume
of the drop is also found to be independent of the curvature of the surface, and the radius of curvature of the drop is well-predicted by simple geometric relations by keeping contact angle and volume constant. The magnitude of the line tension is calculated by MD simulation and found to be comparable with theoretical values. The line tension strongly depends on the sign of the surface curvature: its value is much larger in the case of negative curvature (concave surface) as compared to positive curvature (convex surface), while it reaches a maximum for zero curvature. This trend is found to be correlated with the number of particles in the initial layers of the drop. The relative arrangement of particles near the surface is found to be the same which means that the number of particles near the three-phase contact line is the only significant factor which is responsible for altering the magnitude of the line tension on curved surfaces.

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**Notes**

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