Brief Paper

Minimum Thrust Levels for Spinning Drag-Free Satellites*†

Niveaux minimums de la Force Motrice pour des Satellites Rotatoires sans Résistance

Minimum Triebkraftniveaus für rotierende widerstandsfreie Satelliten

О минимальных уровнях движущей силе для вращающихся сателлитов
без сопротивления трения

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Summary—A drag-free satellite, which contains an internal unsupported proof mass, is shielded from external forces such as solar pressure. Thrusters force the satellite to follow the proof mass and hence the satellite follows an almost purely gravitational orbit. The dominant internal disturbing force is the mass attraction of the satellite on the proof mass. Spinning the satellite reduces this force and it has been investigated what the size of the thrusters must be in this case. This size is much smaller than originally thought. Its impact on some other features is shown.

1. Introduction

A DRAG-FREE satellite [1] (DFS) contains an internal unsupported proof mass. The proof mass is under the sole influence of gravitational forces because it is shielded by the other satellite from external forces such as solar pressure and atmospheric drag. A translation control system in the satellite senses the relative motion between the satellite and proof mass and actuates gas jets which force the satellite to follow the proof mass without touching it. The DFS follows a purely gravitational orbit. Therefore its orbit is substantially more predictable than conventional satellites. Stanford University in co-operation with the Applied Physics Laboratory at Johns Hopkins University has designed, built and launched a drag-free satellite for navigation. Application of the technique for geodesy investigations is also being studied.

In the drag free navigation satellite the largest disturbing force on the proof mass is due to the mass attraction of the satellite. It is anticipated that this will also be the dominant disturbance in future satellite designs. This disturbance is body fixed and spinning the satellite averages the effect of any body fixed force in the plane of spin. The orbit in-track direction is significantly more sensitive to disturbing

forces than other directions; therefore, spinning a drag-free satellite about an axis perpendicular to the in-track direction reduces the effect of mass attraction forces in the sensitive direction. Although the first drag-free satellite was not spinning, it is anticipated that future satellites will be spun.

Many factors enter into the sizing of the translational thrusters for any DFS. The most obvious requirement is that the thrust level be large enough to balance all anticipated drag forces. Other factors include the minimization of two-sided limit cycle propellant consumption [1], the desirability of having short transient settling times much faster than orbital dynamic time constants and the minimization of the total number of thruster firings over a satellite lifetime. An additional factor for spinning DFS's is the requirement that the thrusters be strong enough to capture the proof mass when initially on the cavity wall. In DFS designs now being considered, this last requirement is the governing one and could potentially dictate thrust levels considerably higher than is otherwise necessary. This paper has 2 primary contributions. The first is the analytical demonstration that thrust levels much lower than previously thought possible appear to be feasible. A general discussion of the requirements that initial capture place on thruster sizing is included. The second contribution is the study of the impact of the low thruster levels on a control phenomenon called 'trapping' [2]. Previous study of the phenomenon did not include the low thrust case since it required a substantially different approach to the analytical treatment and since the case was felt to be unimportant.

2. Initial capture

If a proof mass is initially caged against the side of the cavity in the plane of spin, the equatorial plane, and in an indentation as shown in Fig. 1, the control thrust required to 'lift' the proof mass off the cavity wall is

$$F_T > m\omega_s^2 r_w, \quad (1)$$

where

F_T = thrust magnitude,

m = mass of satellite,

ω_s = satellite spin rate,

r_w = cavity to proof mass gap.

This is based on counteracting the centrifugal force due to spin.

For the typical design values of $m = 100$ kg, $\omega_s = 0.1$ r/sec and $r_w = 1$ cm, the required thrust level is 10^{-2} N, which is 3 orders of magnitude larger than the drag force

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at an orbital altitude of 800 km.* This large thrust requirement is undesirable (a) because of the associated physical size and weight, (b) because any failures with the thruster open would quickly use up propellant and (c) because of the increased tendency for 2-sided limit cycles and their high propellant consumption.

FIG. 1. Proof mass and cavity schematic.

By aligning the caging rod parallel to the spin axis, there is theoretically no minimum thrust level. However, it is always possible that a large transient could inadvertently place the proof mass on the cavity wall in the equatorial plane. Recovery from this condition is mandatory.

A capture technique proposed by the French drag-free satellite effort [3], primarily because of their optical sensor requirements, is to thrust along the spin axis with enough force to drive the proof mass to the point on the cavity wall pierced by the spin axis. The thrust magnitude required to carry this out is identical to that given in equation (1) for lifting the satellite away from the proof mass. This result can be verified by a force diagram shown in Fig. 2 which shows that at the equilibrium position of the proof mass:

FIG. 2. Axial thrust force diagram.

$$\tan \phi = \frac{\omega_s^2 r_w \sin \phi}{F_T/m}$$

and if $F_T/m > \omega_s^2 r_w$, the only possible solution is $\phi = 0$. This technique, therefore, also requires a very large control force, although only one thruster with the large magnitude, aligned with the spin axis, is required.

Another capture technique is to allow the proof mass to roll along the cavity wall, or 'unwind', until it is stationary with respect to an inertial reference and the centrifugal force disappears. No indentations are possible with this scheme, in fact, the cavity wall should be smooth.

Simplified unwinding analysis. The essential features of the unwinding are exhibited in a simplified analysis where the effect of the proof mass on the satellite is neglected, the proof mass is assumed to be a point mass, and the satellite mass center is at the cavity center. In this case, the equations of motion of the proof mass with respect to the

spinning satellite in polar co-ordinates are

$$\begin{cases} \ddot{r} - (\dot{\theta} + \omega_s)^2 r = f_r, \\ r\ddot{\theta} + 2\dot{r}(\dot{\theta} + \omega_s) = f_\theta, \end{cases} \quad (2)$$

where

r = radial position of the proof mass,

θ = angular position of the proof mass with respect to the rotating satellite,

f_r = radial component of the specific control force on the satellite,

f_θ = tangential component of the specific control force on the satellite.

These equations were derived by Lange [1] in cartesian co-ordinates and applied to cases where the proof mass did not contact the wall. We wish to discuss the behavior of the proof mass/satellite system while the proof mass is in contact with the cavity wall.

We further assume that the control law is linear and as given in [1].

In a cartesian reference system fixed to the satellite, using position co-ordinates x_b, y_b in the plane of rotation, the control law is

$$\begin{cases} f_x = -k_p[x_b + \gamma(\dot{x}_b - \omega_s y_b)], \\ f_y = -k_p[y_b + \gamma(\dot{y}_b + \omega_s x_b)]. \end{cases} \quad (2)$$

In polar co-ordinates, it becomes

$$\begin{cases} f_r = -k_p(r + \gamma\dot{r}), \\ f_\theta = -k_p\gamma r(\dot{\theta} + \omega_s), \end{cases} \quad (3)$$

where

k_p = position gain (sec^{-2}),

$\gamma = \frac{\text{velocity gain}}{\text{position gain}}$ (sec).

Suppose initially the proof mass is fixed on the cavity wall, $r = r_w$, $\dot{r} = 0$ and $\dot{\theta} = 0$. In this initial condition with $k_p < \omega_s^2$, or $F_T < m\omega_s^2 r_w$, $\ddot{r}$ cannot be < 0 and therefore the proof mass remains on the cavity wall ($\dot{r} = 0$). The velocity dependent terms in the control law are based on the inertial velocity terms; therefore, there is a tangential component of control force which causes movement in the tangential direction even while initially stationary with respect to the wall. The pertinent equation of motion in this case is

$$r\ddot{\theta} = -k_p\gamma r_w(\dot{\theta} + \omega_s). \quad (4)$$

The final value, assuming $\dot{r}$ remains zero and the equation valid, in reality, of course, at some time r becomes $< r_w$, is

$$\dot{\theta} = -\omega_s$$

which means the proof mass is no longer caught on the wall, but is fixed with respect to an inertial reference. Hence, any value of k_p , or F_T , will move the satellite away from the proof mass and capture will occur.

Detailed unwinding analysis. The complete equations of motion of the drag-free satellite were derived by Lange [1]. In polar co-ordinates shown in Fig. 3, his results become

$$2(\omega_s + \dot{\theta})\dot{r} + r\ddot{\theta} = \frac{m_s + m_b}{m_s m_b} F_{s\theta} + f_\theta - \omega_s^2 r_{sc} \sin \theta, \quad (5)$$

$$\ddot{r} - (\omega_s + \dot{\theta})^2 r = \frac{m_s + m_b}{m_s m_b} F_{sr} + f_r + \omega_s^2 r_{sc} \cos \theta, \quad (6)$$

where

r = radial distance of the proof mass from the cavity center,

θ = angular position of the proof mass as shown in Fig. 3,

* Assumes $C_D = 2.2$ and area = 2.5 m^2 .

- m_s = mass of satellite,
 m_b = mass of proof mass,
 $F_{s\theta\theta}, F_{sbr}$ = components of the force of the satellite on the proof mass in polar co-ordinates,
 r_{sc} = mass center distance from cavity center along angular reference line as shown in Fig. 3.

FIG. 3. Polar co-ordinate frame.

The value of $F_{s\theta\theta}, r$ depends on whether the proof mass is in contact with the wall or not. When not in contact, the force is very low and not of interest here. When in contact, $F_{sbr} \leq 0$ and will be the value required to maintain $\dot{r} = \ddot{r} = 0$. For rolling without slipping, $F_{s\theta\theta}$ will be the value that produces the angular acceleration of the proof mass corresponding to $\ddot{\theta}$. Therefore

$$F_{s\theta\theta} = -I\ddot{\theta}/r_b^2, \quad (7)$$

where

I = moment of inertia of the proof mass about its rolling axis,

r_b = radius of proof mass.

Combining equations (5) and (7) and assuming that $\dot{r} = 0$, we obtain

$$\left[1 + \left(\frac{m_s + m_b}{m_s m_b}\right) \frac{I}{r_b^2}\right] r\ddot{\theta} = +f_\theta - r_{sc} \omega_s^2 \sin \theta. \quad (8)$$

To insure that it will always be possible to unwind, i.e. drive θ to $-\omega_s$, we require that

$$f_\theta > r_{sc} \omega_s^2 \quad (9)$$

which is much less restrictive than equation (1). For a linear control law given in equation (3), equation (8) reduces to

$$\left[1 + \left(\frac{m_s + m_b}{m_s m_b}\right) \frac{I}{r_b^2}\right] \ddot{\theta} + (\ddot{\theta} + \omega_s) k_p \gamma + \omega_s^2 \frac{r_{sc}}{r} \sin \theta = 0. \quad (10)$$

If $k_p \gamma > \omega_s(r_{sc}/r)$, as in equation (9), it can be easily verified that the solution of equation (10) after transients have died out is an oscillation about $\dot{\theta} = -\omega_s$. The amplitude of the oscillation is $\leq \omega_s^2 r_{sc}/k_p \gamma r$, which follows from equation (10) by substituting $\ddot{\theta} = 0$ and $\sin \theta = \pm 1$ since the maximum and minimum values of $\omega_s + \dot{\theta}$ are obtained at these values.

As in the previous simplified analysis, this condition will probably not be reached, depending on the values of certain parameters, because the control force will become strong enough to lift the proof mass off the wall. Considered in more detail, we have the following. Equation (6) can be written as

$$\ddot{r} = -k_p(r + \gamma\dot{r}) + r(\omega_s + \dot{\theta})^2 + \omega_s^2 r_{sc} \cos \theta + \frac{m_s + m_b}{m_s m_b} F_{sbr}. \quad (11)$$

As long as the proof mass is on the wall, F_{sbr} will cause $\ddot{r} = \dot{r} = 0$. In order to lift it off the wall, it is necessary that

$$-k_p r + r(\omega_s + \dot{\theta})^2 + \omega_s^2 r_{sc} \cos \theta < 0. \quad (12)$$

If we substitute for the extreme value of $\dot{\theta}$,

$$\omega_s + \dot{\theta} = \left(\frac{\omega_s^2 r_{sc}}{k_p r \gamma}\right), \quad (13)$$

then the condition becomes

$$-k_p + \omega_s^2 r_{sc} + \frac{\omega_s^4 r_{sc}^2}{k_p^2 r^2 \gamma^2} < 0 \quad (14)$$

which, together with $k_p \gamma > \omega_s(r_{sc}/r)$ which was obtained earlier, gives an underbound for k_p . In case r_{sc} reduces to zero, this underbound becomes zero, which is in agreement with the simplified analysis. For $r_{sc} > 0$ the underbound found is a conservative one.

3. Trapping

In the previous trapping analysis [2], it was assumed that $k_p/\omega_s^2 \gg 1$ in the development of criteria. In Section 2 it was pointed out that it is possible to size the thrusters so that $F_T < m\omega_s^2 r_w$. For a linear control law, this implies that $k_p/\omega_s^2 < 1$. For a controller with saturation and/or a deadband, it is possible that $k_p/\omega_s^2 < 1$ when $F_T < m\omega_s^2 r_w$, although not necessary. Therefore, the trapping criteria are extended to include the cases where $k_p/\omega_s^2 \approx 1$ and < 1 .

Trapping is defined to be a stable equilibrium of the controlled system which is maintained by a non-zero control force level while no external disturbing forces are acting on the satellite. Important parameters in the study of trapping are the mass center location and the deadband size and shape. In fact, the amount of mass center offset from the proof mass sensor null point at which trapping occurs has been used [2] as a measure of the susceptibility of a particular design to trapping.

Square deadband. The equations of motion in rectangular coordinates, as defined in Section 2, neglecting any disturbance forces are

$$\begin{cases} \ddot{x}_b - \omega_s^2 x_b - 2\omega_s \dot{y}_b = f_{cx} - \omega_s^2 x_s, \\ \ddot{y}_b - \omega_s^2 y_b + 2\omega_s \dot{x}_b = f_{cy} - \omega_s^2 y_s, \end{cases} \quad (15)$$

where (x_s, y_s) is the mass center location with respect to the control center. The control law with a square deadband is

$$\begin{cases} f_{cx} = -k_p e_{ex}, \\ f_{cy} = -k_p e_{ey}, \end{cases} \quad (16)$$

$$\begin{cases} e_{ex}' = x_b + \gamma(\dot{x}_b - \omega_s y_b), \\ e_{ey}' = y_b + \gamma(\dot{y}_b + \omega_s x_b), \end{cases} \quad (17)$$

where the relationship between e_{ex}, e_{ey} and the e_{ex}', e_{ey}' defined by equation (17) is illustrated in Fig. 4(a). Figure 4(b) shows the deadband in the e_{ex}', e_{ey}' plane and defines regions I and II.

In region I, the control becomes

$$\begin{cases} e_x = x_b + \gamma(\dot{x}_b - \omega_s y_b) - d_x, \\ e_y = y_b + \gamma(\dot{y}_b + \omega_s x_b) - d_y. \end{cases} \quad (18)$$

FIG. 4(a). Deadband nonlinearity description.

FIG. 4(b). Deadband region description.

When steady state (ss) solutions to equations (15) and (18) exist and the resulting e_x', e_y' is in region I, trapping exists and (18) is a valid equation. Combining (15), (16) and (18) and setting $\ddot{x}_b, \ddot{y}_b, \dot{x}_b, \dot{y}_b = 0$, we obtain

$$\begin{bmatrix} e_x' \\ e_y' \end{bmatrix}_{ss} = \frac{\begin{bmatrix} 1 & -\eta \\ \eta & 1 \end{bmatrix} \begin{bmatrix} 1-\alpha & -\eta\alpha \\ \eta\alpha & 1-\alpha \end{bmatrix} \begin{bmatrix} x_s - \alpha d_x \\ y_s - \alpha d_y \end{bmatrix}}{(1-\alpha)^2 + \eta^2 \alpha^2}, \quad (19)$$

where

$$\alpha \triangleq k_p/\omega_s^2, \quad \eta \triangleq \gamma\omega_s.$$

For e_x', e_y' to be in region I, i.e.

$$\left. \begin{aligned} e_{x_{ss}}' &\geq d_x, \\ e_{y_{ss}}' &\geq d_y. \end{aligned} \right\} \quad (20)$$

Equation (19) shows, with some algebra, that

$$\left. \begin{aligned} [1-\alpha(1+\eta^2)] x_s - \eta y_s &\geq (1-\alpha) d_x - \eta \alpha d_y, \\ \eta x_s + [1-\alpha(1+\eta^2)] y_s &\geq \eta \alpha d_x + (1-\alpha) d_y. \end{aligned} \right\} \quad (21)$$

These inequalities describe a region of mass center locations for which trapping will occur in region I. Figure 5 depicts the region, the shaded area, for the case $\alpha = \frac{1}{2}$, $\eta = 1$. Note from equation (21) that, in general, the region boundary line slopes are given by

$$\left. \begin{aligned} l_1 &= \frac{1-\alpha(1+\eta^2)}{\eta}, \\ l_2 &= \frac{-\eta}{1-\alpha(1+\eta^2)} \end{aligned} \right\} \quad (22)$$

and the location of the perpendicular intersection of the two boundaries is given by the equality solution of equation (21) and is, with $d_x = d_y$

$$\left. \begin{aligned} x_s^* &= \frac{1+\eta}{1+\eta^2} d_x, \\ y_s^* &= \frac{1-\eta}{1+\eta^2} d_x. \end{aligned} \right\} \quad (23)$$

FIG. 5. Mass center locations causing trapping (region I).

For region II, $f_{cy} = 0$, therefore

$$\begin{bmatrix} e_x' \\ e_y' \end{bmatrix}_{ss} = \frac{\begin{bmatrix} 1 & -\eta \\ \eta & 1 \end{bmatrix} \begin{bmatrix} 1 & -\eta\alpha \\ 0 & 1-\alpha \end{bmatrix} \begin{bmatrix} x_s - \alpha d_x \\ y_s \end{bmatrix}}{1-\alpha} \quad (24)$$

and if the resulting solution satisfies

$$e_x' > d_x \quad \text{and} \quad |e_y'| < d_y \quad (25)$$

a trapping equilibrium exists. Equations (25) and (24) yield the criteria, with $d_x = d_y$,

$$x_s - \eta y_s > d_x, \quad (26)$$

$$\left. \begin{aligned} \eta x_s + [1-\alpha(1+\eta^2)] y_s &< d_x(1-\alpha+\eta\alpha), \\ \eta x_s + [1-\alpha(1+\eta^2)] y_s &> d_x(\alpha-1+\eta\alpha). \end{aligned} \right\} \quad (27)$$

Figure 6 depicts the region for the same case as Fig. 5, i.e. $\alpha = \frac{1}{2}$, $\eta = 1$.

FIG. 6. Mass center locations causing trapping (region II).

It has been pointed out [2] that any equilibrium solution in region II is unstable for $\alpha > 1$. Equations (26) and (27) additionally show that no equilibrium exists with $\alpha > 1$ which makes the stability question somewhat academic.

The closest distance, normalized with respect to d_x , between either shaded area, region I or II, and the origin will be defined to be the trapping susceptibility, r^* . For $\alpha < 1$, the minimum normalized distance between the line given by equation (26) and the origin is $1/\sqrt{1+\eta^2}$. Furthermore, the point of closest approach always satisfies the inequalities (27), therefore $1/\sqrt{1+\eta^2}$ is the minimum distance to the region II area. Since the region I area is always adjacent to the region II area and the 90° corner at x_s^*, y_s^* is coincident with a region II corner, as shown in Figs. 5 and 6, it can be seen that the trapping susceptibility is

$$r^* = 1/\sqrt{1+\eta^2} \quad \text{if } \alpha < 1 \quad (28)$$

For $\alpha > 1$, r^* is the minimum normalized distance to the region I area or the line with slope l_2 in Fig. 5. For $\alpha \gg 1$, the line is horizontal and we have

$$\left. \begin{aligned} r^* &= \frac{|y_s^*|}{d_x} = \frac{\eta-1}{1+\eta^2} \quad (\eta > 1), \\ r^* &= 0 \quad (\eta \leq 1). \end{aligned} \right\} \quad (29)$$

This case agrees exactly with the results given in [2]. Figure 7 shows r^* for several values of α covering the entire range. For $0 < \alpha < 1$, r^* is independent of α and represents trapping in region II, equation (28), while for $\alpha > 1$, r^* is a continuous function of α and represents trapping in region I, equations (22) and (23). Note that for $\alpha > 1$, there is some value of η below which trapping will occur without any mass center offset ($r^* = 0$). It can be shown that this critical value of η is given by

$$\eta = (\alpha-1)/\alpha. \quad (30)$$

FIG. 7. Trapping susceptibility for square deadspaces.

Circular deadband. By defining

$$r \triangleq \sqrt{(e_y^2 + e_x^2)},$$

$$r_d \triangleq \text{deadband radius},$$

the control law becomes

$$\left. \begin{aligned} -\frac{f_{cx}}{k_p} &= e_x = e_x' - \frac{e_x'}{r} r_d \\ -\frac{f_{cy}}{k_p} &= e_y = e_y' - \frac{e_y'}{r} r_d \\ e_x &= 0 \\ e_y &= 0 \end{aligned} \right\} \begin{aligned} &\text{if } r > r_d \\ &\text{if } r \leq r_d. \end{aligned} \quad (31)$$

Combining equations (15), (17) and (31) and setting $\ddot{x}_b = \ddot{y}_b = \dot{x}_b = \dot{y}_b = 0$, we obtain

$$\frac{1}{\alpha} (z_b - z_e) = z_b c \left(1 - \frac{r_d}{|z_b| |e|} \right), \quad (32)$$

where

$$z_b = x_b + jy_b,$$

$$z_e = x_e + jy_e,$$

$$c = 1 + j\eta,$$

$$e = e_x' + je_y',$$

$$j = \sqrt{-1}.$$

Noting that $e = cz_b$, we can solve (32) directly for the mass center distance from the origin.

$$|z_e| = |(1/c - \alpha)|e| + \alpha r_d|. \quad (33)$$

We wish to find the minimum value of $|z_e|$ for which $|e| > r_d$. In other words,

$$r^* = \min_{|e| > r_d} \left[\frac{1}{r_d} \left| \left(\frac{1}{c} - \alpha \right) |e| + \alpha r_d \right| \right]. \quad (34)$$

After some algebra we get 2 cases

(a) for $\alpha > 1$

$$r^* = \frac{\alpha \eta}{\sqrt{([1 - \alpha(1 + \eta^2)]^2 + \eta^2)}}. \quad (35)$$

Note that as $\alpha \rightarrow \infty$, this reduces to $r^* \rightarrow \eta/(1 + \eta^2)$, which was the case considered in [2].

(b) for $\alpha \leq 1$

$$r^* = 1/\sqrt{1 + \eta^2}. \quad (36)$$

Figure 8 shows the trapping susceptibility, r^* , for several values of α as given by equations (35) and (36).

The stability of the trapped equilibrium points was investigated in [4] for the circular deadband and it was found that if

$$(1 - r_d/r) [\alpha^2(\eta^2 + 1) - \alpha] + 1 - \alpha > 0 \quad (37)$$

FIG. 8. Trapping susceptibility for circular deadspaces.

the equilibrium was a stable one. This was obtained by linearizing the equations of motion and insuring that all terms in the characteristic equation were positive. Equation (37) shows that α must be > 1 for the equilibrium to be a stable one, hence the r^* in Fig. 8 for $0 < \alpha < 1$ represents an unstable equilibrium and the system is not in a trapped condition.

4. Conclusions

In spinning DFS designs where the centrifugal acceleration on a proof mass located on the cavity wall is larger than the control acceleration, proof mass capture is possible with smooth cavity walls. Capture occurs by allowing the proof mass to roll along the walls until the centrifugal acceleration becomes lower than the control acceleration. The lower limit of control acceleration was found to be based on the mass center offset from the cavity center and is given in equation (9).

In view of control forces which are lower than previously thought possible, the trapping phenomenon was extended to these cases. Figures 7 and 8 show that, for both round and square deadbands, the trapping susceptibility parameter, r^* , can have a much higher value than for the previously reported [2] case of $\alpha = \infty$. Since higher values of r^* indicate a reduced likelihood of trapping, i.e. more c.m. motion is tolerable before trapping occurs, it is concluded that in low thrust, $k_p/\omega_s^2 \approx 1$ or < 1 , DFS's, trapping is not as severe a design constraint as in the higher thrust cases [2].

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