ELIMINATION OF CIRCULAR ROUTES IN SPATIAL DYNAMIC REGRESSION EQUATIONS

H.J. BLOMMESTEIN*

Twente University of Technology, 7500 AE Enschede, The Netherlands

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Spatial lag operators play an important role in such fields as biometrics, geography and spatial econometrics. Higher-order spatial interactions can be represented by powering spatial lag operators. The paper discusses the conditions under which higher-order spatial lag operators will contain circular routes. Next, it is demonstrated that valid causal inference with the help of spatial dynamic regression models, necessitates the elimination of circular routes. It is shown that the ML-search procedure proposed by Ord (1975) and generalized by Blommestein (1983a), will generate spurious results in case of circular routes. A ML-procedure to obtain non-spurious estimated parameters in general dynamic spatial regression models is outlined. In addition some recent proposed approximate ML-search procedures [see Blommestein (1984b)], based on the spectral decomposition of a matrix, are modified in a similar way. The last section demonstrates empirically the purport of the theoretical results derived in the paper.

1. Introduction

The linear temporal lag operator L and the polynomial operator $\alpha(L)$ have been extensively used in econometrics, notably in distributed lag analysis [cf. Dhrymes (1981)], and in time series analysis [cf. Box and Jenkins (1976)].¹ In other fields such as spatial economics, geography, biometrics and regional science, the transformation L_s plays an important role.

Definition 1. Let V_R be a R -dimensional vector space, $K = \{x_1, \dots, x_R\}$ any basis of V_R , and L_s a linear transformation on V_R . Since every vector is a linear combination of the x_r , we have $L_s x_r = \sum_{r=1}^R w_{rr} x_{r+1}$ for $r \in \{1, \dots, R\}$. The

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¹The transformation L with properties summarized in definition (i), is called a temporal lag operator [cf. Dhrymes (1981)]. *Definition (i):* Let X be a set of all functions $x: N \rightarrow R$, where $N = \{0, \pm 1, \pm 2, \dots\}$ and R is the one dimensional Euclidean space. The transformation L is defined on the vector space X as $Lx(t) = x(t-1)$, $x \in X$. Since this is a linear transformation, L is said to be a *linear temporal lag operator*. The powers of this operator, $L^k, k \in \{0, 1, 2, \dots\}$ yield a set $L_r = \{L^0, L^1, L^2, \dots\}$. The set of all finite linear combinations of elements of L_r over the field of real numbers is a vector space, the vectors being polynomials in the temporal lag operator L . *Definition (ii):* The *temporal polynomial operator* of degree M is defined as $\alpha(L) = \sum_{j=1}^M \alpha_j L^j$.

set $\{w_{rr^1}\}$ of R^2 scalars defines the spatial operator L_s :

$$L_s = \begin{bmatrix} w_{11} & w_{12} & \dots & w_{1R} \\ \vdots & \vdots & & \vdots \\ w_{R1} & w_{R2} & \dots & w_{RR} \end{bmatrix}. \quad (1)$$

Remark. The set $\{w_{rr^1}\}$ is equivalent to the matrix of L_s in the coordinate system K .

In Blommestein (1984b) 3 types or forms of the spatial lag operator L_s were studied: (i) symmetric matrices, i.e., $L_s = L_s^T$, (iia) asymmetric matrices which are non-triangular and (iib) upper (lower) triangular matrices, i.e., $w_{rr^1} = 0$ for $r > r^1$ ($r < r^1$).

Definition 2. All three forms are assumed to be real *non-negative* ($R \times R$) matrices $L_s = [w_{rr^1}]$, i.e., $w_{rr^1} \geq 0$, $\forall r \neq r^1 \in \{1, \dots, R\}$; further it is assumed here that $w_{rr} = 0$, $\forall r \in \{1, \dots, R\}$.

The specification of L_s is dependent on the spatial (economic) phenomena to be analysed. For example, triangular L_s can be used for the study of Christallerian or Löschian central-place systems, while more general spatial interactions — such as spatial competition models — can be presented by dynamic regression models with (a)symmetric spatial lag operators.

Higher-order spatial interactions can be presented by powering spatial lag operators, that is, L_s^p , $p > 1$. To facilitate the interpretation of the information contained in L_s^p , consider the *incidence* matrix Q , corresponding to L_s . Next, conceive system (1) as a network, and the non-zero entries of the matrix Q as the vertices (nodes) of that network. If Q is raised to a certain power ($p \geq 2$), then the off-diagonal elements Q^p denote the total number of connecting paths (chains) between each pair of vertices (x_r, x_{r^1}). In general some of these paths will be *redundant*, also referred to as *circular routes*. Now, we are in the position to define the problem studied in this paper in the form of the following proposition and corollary.

Proposition 1. The use of higher-order ($p > 1$) spatial lag operators in spatial dynamic regression models will — in general — lead to spurious results, due to circular routes.

Corollary 1. Valid causal inference with the help of, for example, spatial dynamic regression equations, necessitates the elimination of circular routes.

The paper is structured as follows: Section 2 presents both a formal definition of circular routes and a method to eliminate redundant paths.

Further some useful general mathematical results for both triangular and non-triangular L_s are given. In section 3 the problem of redundant paths in spatial dynamic regression equations, is studied. A ML-method to obtain non-spurious estimated parameters in general dynamic spatial regression models, is outlined. An illustration of the elimination of circular routes in spatial regression models is presented in section 4.

2. Elimination of redundant paths in spatial lag operators

Definition 3. Let $r, r^1, r^{11}, \dots$ be arbitrary indices from the index set $\{1, 2, \dots, R\}$ of matrix L_s . A sequence $(r, r_1, r_2, \dots, r_{s-1}, r^1)$, for $s \geq 1$ and $r_0 = r$, from the index set $\{1, 2, \dots, R\}$ is said to form a *chain* of length s (or, equivalently, a s -step path) between the vertices or ordered pair (x_r, x_{r^1}) if $w_{rr_1} w_{r_1 r_2} \dots w_{r_{s-2} r_{s-1}} w_{r_{s-1} r^1} > 0$. In case $r = r^1$ such a chain is called a *cycle* of length s between r and itself.

Definition 4. A redundant chain or path (or, equivalently, *circular route*) is a chain in which at least one index number repeats in the sequence of indices.

Example. Construct an incidence matrix Q , by replacing all the positive entries of L_s by ones. The p th (≥ 1) power of Q , denoted by $Q^p = [q_{rr^1}^p]$, has as its (r, r^1) entry the total number of redundant and non-redundant p -step paths (i.e., chains of length p) from r to r^1 . Hence, the sequence $(r, r_1, r_2, \dots, r_{p-2}, r_{p-1}, r^1)$, corresponding to the matrix product $q_{rr^1}^p = \sum q_{rr_1} q_{r_1 r_2} \dots q_{r_{p-2} r_{p-1}} q_{r_{p-1} r^1}$, will contain redundant information in case of circular routes. For example, $(r, r_1, r_2, \dots, r_1, r_2, \dots, r_{p-2}, r_{p-1}, r^1)$.

Before stating general results on the occurrence of circular routes in powering L_s^p ($p \geq 1$), a few more concepts will be introduced [see e.g., Seneta (1973)].

Definition 5a. An index r is *essential*, if $r \rightarrow r^1$ and $r^1 \rightarrow r$. That is, r and r^1 communicate (corresponding to a positive entry in the matrices L_s^p and Q^p) which will be denoted as $r \leftrightarrow r^1$. *Essential classes* are obtained by subdividing all essential indices (denoted by E_i) such a way that all the indices belonging to one class communicate, but cannot lead outside the class, i.e., $E_{i1} \leftrightarrow E_{i11}$, $\forall i^1 \neq i^{11} \in \{1, \dots, N\}$.

Definition 5b. If $r \rightarrow r^1$ but $r^1 \nrightarrow r$ for some r^1 (corresponding with a zero entry in the matrices L_s^p and Q^p), then the *index* is called *inessential*. By subdividing all inessential indices which communicate with some index in such a way that all indices in a class communicate, are called *inessential classes* (denoted by I_j).

Definition 5c. Inessential and essential classes are called *self-communicating* classes.

Example. Consider a spatial system with 8 spatial units (regions, cities, etc.). Information on the interaction between those spatial units, stored in the (8×8) matrix L_s , is given below in the corresponding incidence matrix Q ,

$$\begin{array}{c}
 1 \\
 2 \\
 3 \\
 4 \\
 5 \\
 6 \\
 7 \\
 8
 \end{array}
 \begin{bmatrix}
 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0
 \end{bmatrix}
 \quad (2)$$

1 2 3 4 5 6 7 8

The essential classes corresponding to system (2) are $E_1 = \{3, 6\}$ and $E_2 = \{4, 8\}$, while the inessential classes are equal to $I_1 = \{1, 2\}$, $I_2 = \{5\}$ and $I_3 = \{7\}$.

Given the definitions introduced above the following results can easily be derived.

Lemma 1. All indices of triangular L_s fall into inessential non-selfcommunicating classes.

Proof. In case L_s is triangular we have, $r \rightarrow r^1$ and $r^1 \leftrightarrow r$ for all r^1 . Hence, all indices are inessential and it is not possible to construct inessential classes such that all indices in a class communicate. - Q.E.D.

Remark i. In case of an upper triangular L_s the last row of L_s is equal to $W_R = (0 \dots 0)$. Thus R is an index which leads to no index at all and is also called inessential.

Remark ii. The assertion that a triangular L_s does not possess (an) essential class(es) of indices also follows from a contradiction with lemma 1.1 in Seneta (1973), which states that a $(R \times R)$ non-negative matrix with at least one positive entry in each row possesses at least one essential class of indices.

Theorem 1. (i) Powering (real) non-negative symmetric and asymmetric non-triangular $(R \times R)$ matrices L_s^p ($p > 1$), will, in general, lead to the occurrence of circular routes.

(ii) Raising triangular L_s to a certain power does not lead to redundant paths, i.e., L_s^p ($p > 1$) has no circular routes.

Proof. (i) Raising L_s to higher powers ($p > 1$) means, in general, that the number of connecting paths (or chains) will increase with increasing p . This implies that more entries of L_s^p will be positive (i.e., $w_{rr_1}^p > 0$), while the number of redundant and non-redundant (p -step) paths [denoted by the (r, r^1) entry of the corresponding incidence matrix Q^p] increases as well.

(ii) Since L_s^p is reducible if L_s is reducible [cf. Lancaster (1969, Ex. 4, p. 281)] and noting that a triangular matrix L_s is reducible (which, is readily established from Lemma 1), yields the desired result. Q.E.D.

It may be necessary (see Corollary 1) to eliminate the circular routes of a matrix L_s^p ($p > 1$). Ross and Harary (1952) have constructed an algorithm for eliminating redundant paths of any given length. After determining a $(R \times R)$ matrix C_p , entirely consisting of redundant p -step paths, the following formulac is used to obtain a matrix W^p of non-redundant p -step paths, satisfying the requirements of a (row) stochastic matrix,

$$W^p = [(L_s^p - C_p) - \Delta(L_s^p - C_p)]S_p, \quad (3)$$

where $\Delta(X)$ is the diagonal operator, i.e., a $(R \times R)$ matrix whose principal diagonal is that of the $(R \times R)$ matrix X and zeros elsewhere; S_p is a $(R \times R)$ scaling matrix such that all rows of W^p sum to one.

Example. For $p=3$ we have

$$W^3 = [(L_s^3 - C_3) - \Delta(L_s^3 - C_3)]S_3, \quad (3')$$

where $C_3 = [L_s \Delta(L_s^2) + \Delta(L_s^2)L_s] - \tilde{M}$; $\tilde{M} = L_s \boxtimes L_s^T$ [$\boxtimes$ denotes the element-wise matrix multiplication operator due to Hadamard; see Halmos (1942)].

We conclude this section with a lemma.

Lemma 2. The spatial lag operator W^p as defined in eq. (3), is imprimitive.

Proof. By contradiction. Suppose W^p ($p \geq 1$) is primitive, then $W^p > 0$, some p [see e.g., Lancaster (1969)]. However, it is assumed that the diagonal elements of L_s are equal to zero, i.e., $w_{rr} = 0, \forall r \in \{1, \dots, R\}$ (see Definition 2). Further the diagonal operator $\Delta(\cdot)$ in eq. (3) ensures, that $\Delta(W^p)$ is equal to a square zero matrix. Q.E.D.

Remark. It was assumed that W^p is irreducible.

In the next section, some of the results presented above, will be used for the analysis of dynamic spatial regression models.

3. Elimination of circular routes in spatial dynamic regression models

In Blommestein (1983b) spatial versions of two procedures for determining the dynamic structure in spatial regression models, viz. common factor analysis (COMFAC) and economic factor analysis (ECONFAC), are proposed. A major objective of both procedures is to mitigate the risk of misspecification due to the exclusion of variables and or lags. For this reason both COMFAC and ECONFAC start with a fairly general model (i.e., the unrestricted model or maintained hypothesis), such that the 'true' model is nested within it. Suppose that this general model can be expressed as a class of linear spatial dynamic regression equations,

$$\gamma(L_s) \text{vec } Z_s = E_s, \quad (4)$$

in which $\gamma(L_s)$ is a $(R \times (R(K+1)))$ matrix consisting of $K+1$ $(R \times R)$ polynomial matrices $\gamma_j(L_s^{n_j})$ in $L_s^{n_j}$ of orders $n_j, j \in \{0, 1, \dots, K\}$; $L_s^{n_j}$ is the n_j th order spatial lag operator $Z_s = [YX_1 \dots X_K]$ a $(R \times (K+1))$ matrix with spatial observations; $E_s \sim \text{NID}(0, \sigma_{E_s}^2)$ a $(R \times 1)$ vector with white noise error terms; and 'vec' a vectorization operator. Both COMFAC and ECONFAC employ a sequential testing framework for reducing the order of dynamics in system (4) [i.e., imposing restrictions on $\gamma(L_s)$] for all $K+1$ variables separately or simultaneously, until a test statistic exceeds a chosen critical value [see Blommestein (1983b) for details].

The log-likelihood function for model (4) can be written as [see Blommestein (1983a)]

$$l(Y) = (-R/2) \ln(2\pi\sigma_{E_s}^2) - (1/2\sigma_{E_s}^2) [AY - X\beta]^T [AY - X\beta] + \ln|A|, \quad (5)$$

where $|A| = |I_R - \rho_1 L_s^1 - \rho_2 L_s^2 - \dots - \rho_p L_s^p|$ is the Jacobian of the transformation from E_s to Y . Let L_s^1 have $\lambda_1, \dots, \lambda_R$ as its eigenvalues. The characteristic polynomial of A can then be written as [see e.g., Blommestein (1984b)]

$$|A| = \prod_{r=1}^R (1 - \rho_1 \lambda_r - \rho_2 \lambda_r^2 - \dots - \rho_p \lambda_r^p). \quad (6)$$

Define $\omega = \sigma_{E_s}^2$; $D = AY$; $Z = X^T X$; and $M = I_R - XZ^{-1}X^T$. Setting $\partial[l(Y)]/\partial\beta$ to zero gives

$$\hat{\beta} = Z^{-1}X^T D. \quad (7)$$

Similarly, $\partial[l(Y)]/\partial\omega=0$ and using (7) yields

$$\hat{\omega}=(1/R)D^TMD. \quad (8)$$

Substituting (7) and (8) in (5) gives an objective function of the form

$$l(\rho, \hat{\beta}, \hat{\omega}) = \text{const} - (R/2) \ln \hat{\omega} + \ln |A|. \quad (9)$$

The value of $|\rho| < 1$ that maximizes $l(\rho, \hat{\beta}, \hat{\omega})$ can be found by a direct search procedure, using expression (6) [see for details appendix A in Blommestein (1983a)].

In case L_s is non-triangular, expression (5), and thus expression (9), will contain circular routes (cf. Theorem 1). Consequently the ML-estimates $\hat{\rho}$ and $\hat{\beta}$ are contaminated by redundant information. Using the procedure proposed by Ross and Harary [cf. expression (3)] yields lag operators W^p with non-redundant p -step paths. Substituting W^p in A gives $\tilde{A} = [I_R - \rho_1 W^1 - \rho_2 W^2 - \dots - \rho_p W^p]$. However, it should be emphasized that W^p is *not* simply the matrix W raised to the p th power! For this reason expression (6) should appropriately be modified,

$$|\tilde{A}| = \prod_{r=1}^R (1 - \rho_1 \Theta_{1r} - \rho_2 \Theta_{2r} - \dots - \rho_p \Theta_{pr}), \quad (6')$$

where $\{\Theta_{n_j r} | r=1, \dots, R\}$ is the spectrum of the matrix W^{n_j} with eliminated redundant n_j -step paths (from $L_s^{n_j}$).

The ML-search procedure, using objective function (9), becomes computationally more demanding if p (i.e., the highest order of the spatial lag operator) increases. For this reason approximate ML-procedures, based on the spectral decomposition of a matrix, were proposed in Blommestein (1984a).

Note first that the spatial lag operator L_s (L_s non-triangular) can be decomposed as follows:

$$L_s = \sum_{r=1}^R \lambda_r G_r, \quad (10)$$

where $\{\lambda_r\}$ is the set of eigenvalues or spectrum of L_s , and G_r the spectral components. Powering (10) yields

$$L_s^{n_j} = \sum_{r=1}^R \lambda_r^{n_j} G_r, \quad n_j \geq 1. \quad (11a)$$

Eq. (11a) can be used for a first approximation of A and or $\ln |A|$. Suppose

$L_s^{n_j} \approx G_1$, $\forall n_j \geq 1$, i.e., $\sum_{r=2}^R \lambda_r^{n_j} G_r$ is dropped from (11a), then A from (5) can be approximated as $A \approx I_R - \psi G_1$, ($\psi = \sum_{j=1}^P \rho_j$) and $\ln |A|$ as

$$\ln |A| \approx \ln |I_R - \psi G_1| = \sum_{r=1}^R [\ln (1 - \psi \mu_r)], \quad (12)$$

where $\{\mu_r\}$ is the spectrum of G_1 . Noting that G_1 is idempotent and the rank of G_1 is equal to one, then $\mu_1 = 1$, $\mu_2 = \mu_3 = \dots = \mu_R = 0$. This means that (12) further simplifies to

$$\ln |A| \approx \ln (1 - \psi \mu_1) = \ln (1 - \psi), \quad |\psi| < 1. \quad (12')$$

The relative error of approximating L_s^p by G_1 is λ_2^p . Suppose this error is considered too large. A reduction of the relative error can be obtained by taking $L_s^j \approx G_1 + \lambda_2^j G_2$, and so on.

However, in view of our discussion above on circular routes, (11a) will contain redundant information. Obviously, the spectral decomposition of a spatial lag operator W^{n_j} with non-redundant n_j -step paths is equal to

$$W^{n_j} = \sum_{r=1}^R \Theta_{n_j r} G_{n_j r}. \quad (11b)$$

A first approximation of $\ln |\tilde{A}|$, using the first spectral component, is equal to expression (12'). Clearly, further approximations should be based on (11b); e.g., $W^3 \approx G_{31} + \Theta_{32} G_{32}$.

The next section demonstrates empirically the purport of Proposition 1 and Corollary 1.

4. Estimation of spatial dynamic regression equations

Consider the following specifications, chosen in Blommestein (1984b) as the maintained hypothesis in a COMFAC analysis of some models selected by Cliff and Ord (1981) for a spatial econometric analysis of the Irish economy:

$$Y = \delta_0 I + \delta_1 X_1 + \delta_2 L_s^1 X_1 + \delta_3 L_s^2 X_1 + \delta_4 L_s^1 Y + \delta_5 L_s^2 Y + E, \quad (13)$$

where Y is a $(R \times 1)$ vector with observations on the percentages of gross agricultural output (in value terms) of each of the $R (= 26)$ countries in Eire consumed by itself; X_1 is a $(R \times 1)$ vector with values of an index of arterial road accessibility [see Cliff and Ord (1981) for details]; L_s^1 is a first order $(R \times R)$ spatial lag operator, whereby each non-zero off-diagonal element represents interaction between a pair (r, r^1) of counties and all the diagonal

elements are zero; $E \sim \text{NID}(0, \sigma_e^2)$, i.e., a $(R \times 1)$ vector with white noise error terms.

The value of $\rho = (\delta_4 \delta_5)^T$ that maximizes the objective function $l(\rho, \hat{\beta}, \hat{\omega})$ [compare eq. (9)] is given in table 1. The characteristic polynomial is equal to $\ln|A| = \sum_{r=1}^R [\ln(1 - \delta_4 \lambda_r - \delta_5 \lambda_r^2)]$.

Table 1
Estimated spatial dynamic regression models.

	Model (13)		Model (13a)		Model (13b) ML	Approximation (12')	Approximation (12'')
	ML	OLS	ML	OLS			
δ_0	-17.32	+8.79	-13.39	-7.61	-20.42	-15.97	-19.05
δ_1	+ 0.0008	+0.0023	+ 0.0027	+0.0025	+ 0.0028	+ 0.0027	+ 0.0028
δ_2	- 0.0067	-0.0032	- 0.0034	-0.0042	- 0.0027	- 0.0031	- 0.0027
δ_3	+ 0.11	-0.0039	+ 0.0049	+0.0035	+ 0.0074	+ 0.0057	+ 0.0064
δ_4	+ 0.59	+0.098	+ 0.46	+0.67	+ 0.66	+ 0.64	+ 0.60
δ_5	- 0.11	+1.74	+ 0.04	+0.34	- 0.75	- 0.08	- 0.19

Since L_s^2 contains redundant information, the ML-estimates $\hat{\delta} = (\hat{\delta}_0 \dots \hat{\delta}_5)^T$ are not reliable. Meaningful results can be obtained by substituting $\tilde{A}$ and $\ln|\tilde{A}|$ in (9) for A and $\ln|A|$ respectively. Estimation results,² referred to as model (13a), are reported in table 1. The differences are striking and are therefore a nice illustration of Proposition 1 and Corollary 1. To demonstrate the importance of the Jacobian in (5), and hence in (9), OLS estimates were also calculated, by setting $A = I_R$ (I_R is the identity matrix), $X = [X_1 L_s^1 X_1 L_s^2 X_1 L_s^1 Y L_s^2 Y]$ and $\ln|A| = 0$ in (5).

Similarly, OLS-results were obtained but now with eliminated circular routes [i.e., the OLS counterpart of model (13a)]. From the comparison of the results in table 1 it can be concluded that: (i) the inclusion of the Jacobian term in (5) and (9) is important to offset the inconsistency of the OLS-estimates, (ii) the presence of redundant information seriously affects both ML- and OLS-results. To show the importance of the use of the correct Jacobian term [i.e., without redundant paths; see (6')], ML-estimates were calculated, using $\tilde{A}$ in (9), but with Jacobian term $\ln|\tilde{A}|$ instead of $\ln|A|$. The results [referred to as model (13b); see table 1] speak for themselves.

Finally, approximate ML-estimates were calculated, using the first spectral component for approximating $\ln|\tilde{A}|$ [see (12')], i.e., $\ln|\tilde{A}| \approx \ln[1 - (\delta_4 + \delta_5)]$.

A reduction in the error of approximating $\ln|\tilde{A}|$ was obtained by using two spectral components:

$$\ln|\tilde{A}| \approx \sum_{r=1}^2 \ln[1 - (\delta_4 \Theta_{1r} + \delta_5 \Theta_{2r})]. \quad (12'')$$

²The lag operators L_s^p and W^p ($p \geq 1$) satisfy the requirements of a (row) stochastic matrix. Hence the largest eigenvalue is equal to one in all calculations.

These estimates depart considerably from the 'true' ML-estimates in model (13a). As has been noted in Blommestein (1984a) this is not very surprising, since the eigenvalues $\{\theta_{n_{j1}} \geq \theta_{n_{j2}} \dots \geq \theta_{n_{j26}}\}$ decline very slowly in magnitude for approximating $\ln|\tilde{A}|$, implies that only a small part of the information contained in W^{n_j} is captured.

5. Concluding remarks

This paper clearly demonstrates the importance of eliminating circular routes in spatial dynamic regression models. Failing to remove redundant paths from spatial lag operators will result in seriously distorted ML-estimates. This in turn affects the outcomes of various research activities, using these contaminated estimation results, or (test) statistics calculated from them. For example, specification search procedures like COMFAC [see Blommestein (1983b)] may result in incorrect model choices. In case model (4), or nested versions thereof, are hypothetical representations of spatial causal economic relationships in quantitative form, spurious falsifications may occur. Etcetera.

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