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An LMI Approach to Multiobjective Robust Dynamic Output-Feedback Control for Uncertain Discrete-Time Systems

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1 Abstract

This note discusses an LMI framework to the design of robust multiobjective dynamic output-feedback controllers for discrete-time systems with structured uncertainty. The control objectives considered are guaranteed $\mathcal{H}_2$ norm, guaranteed $\mathcal{H}_\infty$ norm and regional pole-placement. The uncertainty that can be dealt with by the proposed approach is allowed to have a quite general structure – it is just assumed to be such that the state-space matrices of the uncertain system belong to a given convex set.

2 Introduction

Much attention has been focused on controller and filter design based on LMIs in the last decade due to the recent development of computationally fast and numerically reliable algorithms for solving convex optimization problems subject to LMI constraints. Due to the well-known separation theory, in the case when no uncertainty is present these filters can directly be coupled with their dual state-feedback controllers to yield optimal output-feedback controllers. For uncertain systems, trying to solve the coupled problem of output-feedback controller design for system with structured uncertainty one immediately faces a nonlinear, non-convex problem.

3 Outline of the approach

In [1] a novel approach to the design of guaranteed-cost robust $\mathcal{H}_2$ and $\mathcal{H}_\infty$ dynamic output-feedback controllers was proposed. It is well-known that both objectives define a nonlinear, non-convex problem. To circumvent this difficulty, a two-stage design approach is proposed. First, a multiobjective robust state-feedback is designed, represented by the state-feedback gain matrix F, and second, the matrix F is fixed constant in the design of the other matrices of the dynamic output-feedback controller. Although the second step remains non-convex, it is shown that by restricting the Lyapunov function for the closed-loop system to have a certain block-diagonal structure, this problem can be recast into an LMI feasibility problem. The conservatism that is sacrificed by imposing this structural constraint on the Lyapunov function is, however, well justified by the ability of the approach to explicitly deal with structured uncertainties in the system.

The approach discussed here makes use of the results obtained in [1], and focuses on the design of *multiobjective* dynamic output-feedback controllers for discrete-time systems with structured uncertainties. A sufficient condition, based on LMIs, to the existence of solution to the following mixed $\mathcal{H}_2/\mathcal{H}_\infty$/pole-placement control problem is proposed

$$\begin{array}{ll} \mathcal{H}_2 \text{ objective:} & \sup_{\Delta} \|L_2 T_{cl}^{\Delta}(z) R_2\|_2^2 < \gamma_2, \\ \mathcal{H}_\infty \text{ objective:} & \sup_{\Delta} \|L_\infty T_{cl}^{\Delta}(z) R_\infty\|_\infty^2 < \gamma_\infty, \\ \text{Pole-placement:} & \lambda(A_{cl}^{\Delta}) \in \mathcal{D}, \ \forall \Delta. \end{array} \tag{1}$$

given any $\gamma_2 > 0$, and $\gamma_\infty > 0$, where $T_{cl}^{\Delta}(z)$ is denoted the closed-loop transfer function, Δ denotes the (structured) uncertainty in the system, and the matrices L_2, R_2, L_∞, and R_∞, are used to select the desired input-output channels in the mixed control objective above. The superscript Δ denotes dependence on the uncertainty. The complex region $\mathcal{D}$, in which the closed-loop eigenvalues, denoted as $\lambda(A_{cl}^{\Delta})$, are required to lie, is assumed to have the form

$$\mathcal{D} = \{z \in C : \ L + zM + \bar{z}M^T < 0, \ L = L^T\}.$$

Due to the fact that the system of LMIs, which implies the constraints (1), is affine in both γ_2 and γ_∞, one may also wish to consider the optimization problem

$$\min_{\gamma_2, \gamma_\infty} \alpha_2\gamma_2 + \alpha_\infty\gamma_\infty \text{ subject to (1).}$$

for given positive numbers α_2 and α_∞. The approach has been tested on a case study with an aircraft model with six uncertain parameters.

References

[1] S. Kanev and M. Verhaegen, An LMI Approach to Multiobjective Robust Dynamic Output-Feedback Control for Uncertain Discrete-Time Systems, *submitted to Automatica*, December 2001.