COMPARISON OF SUBGRID-MODELS IN LES OF THE COMPRESSIBLE MIXING LAYER

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1. Introduction

Large Eddy Simulation (LES) of turbulent flow forms a field of intensive research. Whereas a Direct Numerical Simulation (DNS) solves all scales present in the flow, LES solves only the large scales directly and models the effect of the small scales, represented by the turbulent stress, with a subgrid-model. For testing subgrid-models two approaches can be distinguished, *a priori* tests and *a posteriori* tests. In this paper we will focus on *a posteriori* tests, in which actual Large Eddy Simulations are performed and the results are compared with results from filtered DNS-data. The flow we consider is the temporal mixing layer at low Mach number. Models incorporated into the tests are: the Smagorinsky model [1], the gradient model [2], the similarity model [3], the dynamic eddy-viscosity model [4] and the dynamic mixed model [5].

The paper is organised as follows. In section 2 we will present the filtered Navier-Stokes equations and the numerical scheme. Results are presented in section 3, which contains the description of the DNS and the comparison of the results for several subgrid-models. Section 4 contains the conclusions.

2. Mathematical formulation and numerical method

The partial differential equations which govern a compressible flow are the Navier-Stokes equations, representing conservation of mass, momentum and energy. In Large Eddy Simulation of turbulent flows these equations are filtered in order to reduce the amount of scales to be solved. The filter operation extracts the large scale part $\bar{f}$ from a flow variable f . For compressible flows, Favre [6] introduced a related filter operation, $\bar{f} = \rho \bar{f} / \bar{\rho}$,

where ρ denotes the fluid density. The filtered Navier-Stokes equations contain so-called subgrid-terms, which cannot be expressed in the filtered flow variables, and have to be modelled with subgrid-models. In this paper we will focus on the modelling of the subgrid-term in the momentum equation, which contains the turbulent stress tensor, defined as

$$\bar{\rho}\tau_{ij} = \overline{\rho u_i u_j} - \overline{\rho} \bar{u}_i \bar{u}_j / \bar{\rho} = \overline{\rho} (\widetilde{u_i u_j} - \bar{u}_i \bar{u}_j), \quad (1)$$

where $\bar{\mathbf{u}}$ is the Favre-filtered velocity vector. This turbulent stress tensor has several algebraic properties which can be used in the construction and qualification of subgrid-models [4, 7]. Expressions for the subgrid-terms in the energy equation can be found in ref. [8]. They will not be considered here, as *a posteriori* tests have shown that for the mixing layer at low convective Mach numbers the modelling of these terms has no effect on the Large Eddy Simulation [8]. In total five models for the turbulent stress tensor τ_{ij} as it appears in the subgrid-term in the momentum equation will be investigated and compared in this paper.

The numerical method which will be used to solve the filtered Navier-Stokes equations is based on the method of lines. The explicit time integration is performed with a compact-storage four-stage Runge-Kutta method with second-order accuracy. We use a fourth-order accurate spatial discretization for the convective terms which is robust with respect to odd-even decoupling [9]. The viscous terms are discretized with a standard cell-vertex finite volume method which is second-order accurate [10].

3. Results of simulations

In this section we present results of Large Eddy Simulations using several subgrid-models. The results are compared with results of filtered Direct Numerical Simulations.

In order to simulate a three-dimensional temporal mixing layer we solve the flow equations in a cubic geometry $[0, L] \times [-\frac{1}{2}L, \frac{1}{2}L] \times [0, L]$. Periodic boundary conditions are imposed in the x_1 - and x_3 -direction, while in the x_2 -direction free-slip boundary conditions are used. The basic initial velocity profile is the hyperbolic tangent profile $u = \tanh x_2$. The initial temperature profile follows from the Busemann-Crocco law [11] and the initial pressure distribution is uniform. The reference length L_R is half the initial vorticity thickness, while the other reference scales are the initial upper stream values. The length L of the domain is set equal to four times the wavelength of the most unstable mode according to linear stability theory [12]. Superimposed on the mean profile we put a three-dimensional large amplitude perturbation, as described by Sandham *et al.* [13]. Here we do not study compressibility effects, so a low convective Mach number

Figure 1. Comparison of subgrid-models: the evolution of the total kinetic energy (a) and the momentum thickness (b). Filtered DNS (o), LES with the zero model (solid), Smagorinsky model (*), gradient model (+), similarity model (dashed), dynamic model (dotted) and dynamic mixed model (dashed-dotted).

$M_R = 0.2$ is used [8], while $Re = 50$. The DNS-database has been obtained by solving the Navier-Stokes equations on a fine grid with 192^3 cells.

We will compare the results of Large Eddy Simulations for several subgrid-models with the filtered DNS-results. The Large Eddy Simulations presented have been performed on a grid with 32^3 cells, which is considerably coarser than the DNS-grid, and a top-hat filter with filter width $\Delta = L/16$ has been used. Thus the filter width equals twice the grid-spacing of the coarse grid, denoted by h . The LES and filtered DNS will be compared with respect to the time-evolution of the total kinetic energy, E , and the momentum thickness, d .

In figure 1, E and d obtained from the filtered DNS and Large Eddy Simulations for several subgrid-models have been plotted. For a 'perfect' LES, the curves should coincidence with those of the filtered DNS. To obtain a point of reference, also a LES with a 'zero model' (a coarse grid DNS using a filtered initial state) has been performed. The performance of a specific subgrid-model is considered to be bad if the deviations from the filtered DNS are comparable to or larger than the errors corresponding to the coarse grid DNS. In such a case the incorporation of the subgrid-model does not make sense. The discrepancy between the simulation with the zero model and the filtered DNS is quite large, illustrating that there is something to improve upon.

The Smagorinsky model [1] gives worse predictions for E and d than

the zero model provides. The total kinetic energy E is observed to exhibit a characteristic behaviour: in the transitional regime of the simulation the dissipation of energy is far too large, while it is far too low afterwards (figure 1a). This model yields such an excessive dissipation in the transitional regime that transition to turbulence is hindered, which can also be inferred from the slow growth of the momentum thickness (figure 1b).

Therefore we next turn to models which are less dissipative. The first is the gradient model by Clark *et al.* [2] which is based on a Taylor series expansion of the filtered velocity. In *a priori* tests it turned out that this model correlates very well with the turbulent stress tensor. However, a simulation based on this model requires a 'limiter', which prevents the kinetic energy to increase. The results can certainly not be called excellent. Comparing this gradient model with the zero model, the evolution of E predicted by the gradient model is not better, while for the momentum thickness the prediction is even less accurate.

Next we discuss the performance of the scale-similarity model by Bardina *et al.* [3]. Although the correlations found in the *a priori* tests were slightly lower than for the gradient model, the Large Eddy Simulations with the similarity model yield considerably better results, since the similarity model does not require a limiter in order to prevent instabilities.

The fourth model considered is the dynamic eddy-viscosity model developed by Germano [4], where the eddy viscosity is dynamically obtained from the local structure of the flow. This dynamical adjustment solves the major short-coming of Smagorinsky's model. Indeed the results are much better than those of the Smagorinsky model. With respect to the E -evolution, incorporation of the dynamic model improves the results compared to the zero model, but for the evolution of d no improvement is obtained. If we compare the results of the similarity model and the dynamic model, the first model is preferred.

Finally, we discuss the results of the dynamic mixed model [5], which is a combination of the similarity model and the dynamic eddy-viscosity concept. The evolution of E is more accurately predicted by the mixed model than by the similarity and dynamic eddy-viscosity models. The evolution of the momentum thickness is almost identical for the dynamic mixed and the similarity model. This indicates that for the momentum thickness the improvement over the zero model results is mainly due to the similarity part of the dynamic mixed model, whereas the eddy-viscosity part of the model does not seem to affect this quantity. In fact, when compared to the zero model, the mixed model closely approaches the filtered DNS-results for the E -evolution, while for other quantities the predictions are reasonable and certainly not worse than for the other models. This dynamic procedure can also be used to stabilise the gradient model [14].

4. Conclusions

Summarising the results, the Smagorinsky and gradient model (with a limiter) have to be rejected for actual LES in the mixing layer. The similarity, dynamic eddy-viscosity and dynamic mixed model are in general better than the zero model. Out of these last three models the dynamic mixed model seems most promising. Similar conclusions were found for different numerical methods, e.g. a pseudo-spectral method. We also compared the results of several numerical methods for the dynamic mixed model using different Δ/h ratios. The combination of a fourth-order accurate discretization and $\Delta = 2h$ is to be preferred when considering a balance between accuracy and computational efficiency. Furthermore, in several cases it was observed that the discretization error partially cancels the modelling error. These results will be presented in a separate paper [9].

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