EUROPEAN MECHANICS SOCIETY

EUROMECH

Colloquium 524

UNIVERSITY OF TWENTE.

Multibody system modelling, control and simulation for engineering design

February 27 – 29, 2012, Enschede, Netherlands

Program and abstracts

Editors: J.B. Jonker, W. Schiehlen, J.P. Meijaard and R.G.K.M. Aarts

EUROMECH Colloquium 524 - Multibody system modelling, control and simulation for engineering design - Program and abstracts

Editors: J.B. Jonker, W. Schiehlen, J.P. Meijaard and R.G.K.M. Aarts

ISBN : 978-90-365-3338-6

Scope

The colloquium addresses the method of multibody system dynamics for advanced technologies and engineering design for which a numerical efficient approach is crucial. In particular a designer can take significant advantage of model based dynamical analysis. The numerical methods applied for multibody systems dynamics have proved to offer solutions for the analysis of systems with interconnected rigid and flexible bodies subject to various loads and undergoing complex motion. While high accuracy can be obtained with extended models including a large number of degrees of freedom, there is a clear need for numerically efficient techniques that still offer an adequate level of accuracy. E.g. the optimisation of the design parameters usually implies that systems with varying parameters have to be analysed in a short time. The modelling techniques applied for this purpose should provide fast simulations of the relevant system's behaviour which exhibits often nonlinearities. Depending on the application area, multibody dynamic analysis has to be coupled to other relevant physical domains to address e.g. electrical and thermal effects or fluid-structure interaction. Mechatronic systems are usually modelled as multibody systems subject to sophisticated non-linear control. Model order reduction techniques are required for control design and to increase the computational speed.

The goal of this colloquium is to provide a platform for discussions on the relation between multibody systems analysis tools and the requirements needed for design.

Topics

- Numerically efficient multibody system dynamics techniques;
- Modelling for design and simulation;
- Design principles and contact problems;
- Underconstraint and overconstraint mechanical systems;
- Flexible multibody dynamics and reduced order modelling;
- Mechatronic design and compliant mechanisms;
- Parameter optimisation and manufacturing tolerances;
- Simulation for engineering design;
- Applications to engineering systems.

Multibody System Dynamics Colloquia

The method of multibody system dynamics and its applications belongs to a series of Colloquia promoted by the European Mechanics Society (EUROMECH) taking place in Paris, France (2001), Erlangen, Germany (2003), Halle, Germany (2004), Ferrol, Spain (2006), Bryansk, Russia (2008), Blagoevgrad, Bulgaria (2010) and Açores, Portugal (2011). Essential features of EUROMECH Colloquia are that they are specialized in content, small in size and informal in character. This type of scientific meetings has been found in practice to give good results and to meet a definite need. For further information visit http://www.euromech.org/.

Chairmen

Prof. Ben Jonker
Department of Mechanical Automation, University of Twente, Enschede, The Netherlands

Prof. Werner Schiehlen
Institute of Engineering and Computational Mechanics, University of Stuttgart

Scientific Committee

- Prof. Dr. Martin Arnold, Martin-Luther-University Halle-Wittenberg
- Prof. Olivier Brüls, University of Liège
- Prof. Dr.-Ing. Prof. E.h. Peter Eberhard, University of Stuttgart
- Prof.dr.ir. Just L. Herder, Delft University of Technology
- Dr.ir. Jaap P. Meijaard, University of Twente/Olton Engineering Consultancy

Organising Committee

- Dr.ir. Ronald Aarts, University of Twente
- Mrs. Martina Tjapkes, University of Twente

Supporting organisations

EUROPEAN MECHANICS SOCIETY

EUROMECH

UNIVERSITY OF TWENTE.

University of Twente

DEMCON

DEMCON advanced mechatronics BV

Controllab
your partner in mechatronic design

Controllab Products BV

em

Graduate School on Engineering Mechanics (EM)

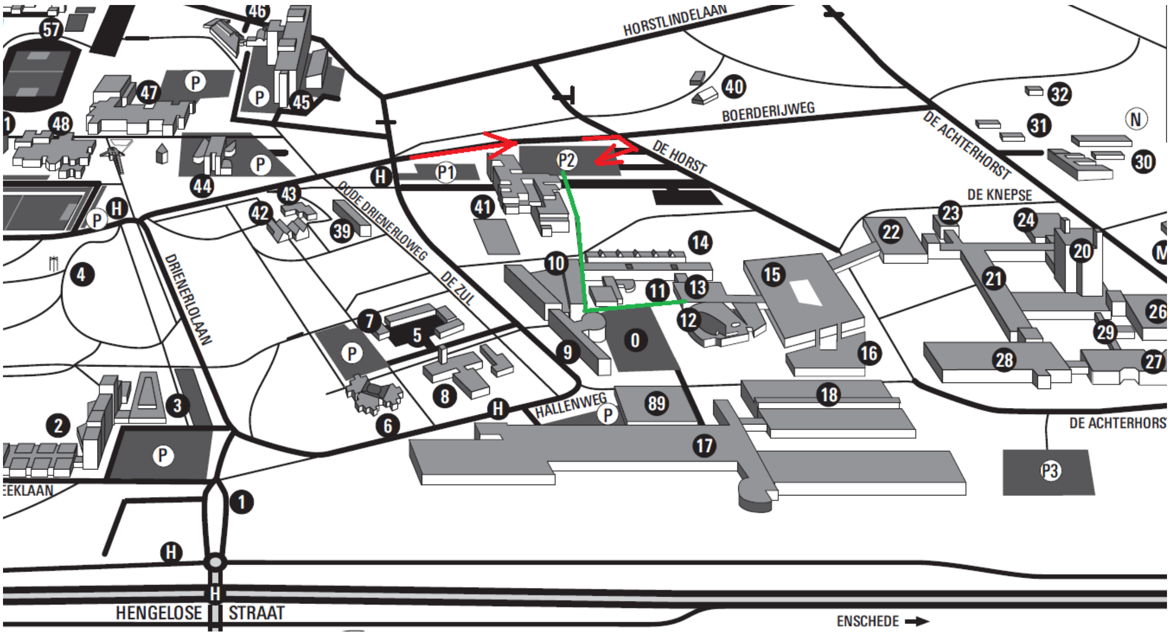
disc

Dutch Institute of Systems and Control (DISC)

Venue

The colloquium will take place at the University of Twente in Enschede, the Netherlands.

Twente is a region located in the Eastern part of the Netherlands, near the border with Germany. The University of Twente was founded as a university of technology in 1961 in order to increase the number of academic (university) engineers. The university campus is situated on the beautiful Drienerlo estate, between the two cities of Enschede and Hengelo. Modelled on the Anglo-Saxon example, the UT is the only real campus university in the Netherlands. The beautiful landscape of the UT campus, embellished with modern architecture, forms a unique environment for student life, sports and study. Up-to-date facilities offer an optimal environment for research. No doubt, the UT campus will offer an inspiring environment for this EUROMECH colloquium.



The larger part of program takes place at this campus:

- All presentations are in building Waaier, number 12 on the map. The entrance is in "Hal B", number 13 just next to it. From the entrance signs will guide you to the "van Hasselt" room, number WA3.
- The registration is directly near this room.
- The coffee breaks also take place in the immediate vicinity of this room.
- The lunch breaks and the Monday evening reception are in the Faculty club, number 42, which is at walking distance from the Waaier.
- The lab visit is in the Westhorst, building 22, which can be reached indoor.

Building Waaier is quite close to hotel Drienerburght, building 44. The distance to hotel De Broeierd is about 1 km. This hotel is just outside the lower left corner of the map selection shown above.

Enschede can be reached well by train. It has a convenient connection to Amsterdam Schiphol Airport. There are two trains every hour and the travel time is somewhat over two hours. Bus lines connect the campus with the railway stations of Hengelo and Enschede. Bus lines 1, 15 and 16 stop on the campus, e.g. bus stop "UT/De Zul" is the sign "H" near "P1" on the map.

When travelling by car, there are good connections to the Dutch and German motorways. On the campus you can park on parking area “P2”, see the red arrows on the map. From here there is a walking path to building Waaier, green line.

Registration

The registration desk is near the room with the presentations in building Waaier, number 12 on the map. Open hours are:

- Monday February 27 from 08:30 – 12:00.
- Tuesday February 28 from 08:30 – 09:00.
- Wednesday February 29 from 08:30 – 09:00.

Social Program

On Monday evening a reception will be organized in the Faculty Club, building 42 on the map.

On Tuesday afternoon the conference program will be dedicated to a social program which will take place in and around the Twickel Castle in Delden. It is located at about 11 km from the University (or 14 km from the centre of Enschede). The history of this castle and its beautiful surroundings starts in the 14th century. The estate covers more than 4,000 hectares and includes 150 farms with agricultural land and meadows, interspersed with moorland, fens and woods. The oak woods have long been famous, not just because of their beauty, but also because of the quality of the wood they produce. The characteristic farms can be recognised by their black-and-white shutters.

The visit to this castle will be concluded with a conference dinner in the restaurant of Hotel Carelshaven, which is near the Twickel estate. Transportation to and from these activities will be arranged. After the lunch a bus will depart from the Faculty Club and after the dinner we return to the University campus.

Program Monday February 27, 2012

8:30	**Registration**	
9:00	**Welcome and opening**	
	Session 1: Mechatronic design and compliant mechanisms (Jonker)	
9:30	Jan Vaandrager, Just L. Herder – University of Twente, Netherlands	A static balancer for a large-deflection compliant finger mechanism
10:00	Thomas Gorius, Robert Seifried, Peter Eberhard - University of Stuttgart, Germany	Approximate Feedforward Control of Flexible Mechanical Systems
10:30	Coffee break	
	Session 2: Modelling for design and simulation (Ambrósio)	
11:00	Dmitry Balashov, Oliver Lenord – Bosch Rexroth AG, Germany	Modeling the Multibody Dynamics with the D&C System Simulator
11:30	Roland Pastorino, Javier Cuadrado, Dario Richiedei, Alberto Trevisani – University of La Coruña, Spain & Universitá degli Studi di Padova, Italy	State Estimation Using Multibody Models and Unscented Kalman Filters
12:00	J. P. Meijaard – Olton Engineering Consultancy, Netherlands	Modelling and simulating the motion of a wire in a tube
12:30	Lunch break (Faculty club)	
	Session 3: Applications to engineering systems (Cuadrado)	
14:00	Johann Zeischka – MSC.Software GmbH, München, Germany	FEA based modelling of rolling bearings for high fidelity multibody system modeling
14:30	L. van de Ridder, J. van Dijk, W.B.J. Hakvoort, and J.C. Lötters – University of Twente, DEMCON, Bronkhorst High-Tech B.V., Netherlands	Influence of external damping on phase difference measurement of a Coriolis mass-flow meter
15:00	Rob Waiboer – ASML Netherlands B.V., Netherlands	Modelling of flexible dynamic links in Nano-Positioning Motion Systems
15:30	Coffee break	
	Session 4: Design principles and contact problems (Meijaard)	
16:00	M. Machado, P. Flores, D. Dopico, J. Cuadrado – University of Minho, Portugal & Universidad de A Coruña, Spain	Dynamic response of multibody systems with 3D contact-impact events: influence of the contact force model
16:30	Sara Tribuzi Morais, Paulo Flores, J.C. Pimenta Claro – Universidade do Minho, Portugal	A Planar Multibody Lumbar Spine Model for Dynamic Analysis
17:00	Cândida Malça, Jorge Ambrósio, Amilcar Ramalho – Polytechnic Institute, IDMEC-IST, University of Coimbra, Portugal	An Enhanced Cylindrical Contact Force Model for Multibody Dynamics Applications
17:30	Lab visit (building WestHorst)	
19:00	Reception (Faculty club)	

Program Tuesday February 28, 2012

	Session 5: Flexible multibody dynamics and reduced order modelling (Schiehlen)	
9:00	Peter Eberhard, Michael Fischer – University of Stuttgart, Germany	Model Reduction of Large Scaled Industrial Models in Elastic Multibody Systems
9:30	Frank Naets, Wim Desmet – KU Leuven, Belgium	Sub-System Global Modal Parameterization for efficient inclusion of highly nonlinear components in Multibody Simulation
10:00	R.G.K.M. Aarts, D. ten Hoopen, S.E. Boer and W.B.J. Hakvoort – University of Twente, DEMCON, Netherlands	Model order reduction of non-linear flexible multibody models
10:30	Coffee break	
	Session 6: Simulation for engineering design 1 (Eberhard)	
11:00	A.L. Schwab and J.P. Meijaard – Delft University of Technology, Olton Engineering Consultancy, Netherlands	A necessary condition for bicycle self-stability: steer toward the fall
11:30	M. Machado, P. Flores, J.P. Walter, B.J. Fregly – University of Minho, Portugal & University of Florida, USA	Challenges in using OpenSim as a multibody design tool to model, simulate, and analyze prosthetic devices: a knee joint case-study
12:00	Pavel Polach, Michal Hajžman – VZÚ Plzeň s.r.o., Czech Republic	Investigation of dynamic behaviour of inverted pendulum attached using fibres at non-symmetric harmonic excitation
12:30	Werner Schiehlen – University of Stuttgart, Germany	Dynamic loads in kinematically determined multibody systems
13:00	Lunch break (Faculty club)	
	Social program and dinner	
14:30	Departure bus (Faculty club)	
15:00	Visit to the Estate and Castle of Twickle http://en.twickel.nl/	
17:30	Bus trip castle to restaurant	
18:00	Hotel and restaurant Carelshaven http://www.carelshaven.nl/?rubriekid=1857	
	Bus trip to Enschede	

Program Wednesday February 29, 2012

Session 7: Numerically efficient multibody system dynamics techniques (Brüls)

9:00	Martin Arnold – Martin Luther University Halle-Wittenberg, Germany	(Re-)Starting a Generalized-α Solver for Constrained Systems with Second Order Accuracy
9:30	Junyoun Jo, Myoung-ho Kim, Sung-Soo Kim – Chungnam National University, Korea	Joint Coordinate Subsystem Synthesis Method with Implicit Integrator in the Application to the Unmanned Military Robot
10:00	Tommaso Tamarozzi, Wim Desmet – KU Leuven, Belgium	Efficient flexible contact simulation by means of a consistent LCP\generalized-α scheme and static modes switching

10:30 Coffee break

Session 8: Underconstraint and overconstraint mechanical systems (Herder)

11:00	Andreas Scholz, Francisco Geu Flores, Andrés Kecskeméthy – University of Duisburg-Essen, Germany	Trajectory Planning Optimization of Mechanisms with Redundant Kinematics for Manufacturing Processes with Constant Tool Speed
11:30	Quintilio Piattoni, Giovanni Lancioni, Stefano Lenci, Enrico Quagliarini – Polytechnic University of Marche, Italy	Application of the Non-Smooth Contact Dynamics method to the analysis of historical masonries subjected to seismic loads
12:00	Marek Wojtyra and Janusz Frączek – Warsaw University of Technology, Poland	Overconstrained multibody systems – known and emerging issues

12:30 Lunch break (Faculty club)

Session 9: Parameter optimisation and manufacturing tolerances (Arnold)

14:00	Olivier Brüls and Valentin Sonneville – University of Liège, Belgium	Sensitivity analysis for flexible multibody systems formulated on a Lie group
14:30	Michael Burger, Klaus Dreßler, Michael Speckert – ITWM, Kaiserslautern, Germany	Calculating Input Data for Multibody System Simulation by Solving an Inverse Control Problem
15:00	Emmanuel Tromme, Olivier Brüls, Laurent Van Miegroet, Geoffrey Virlez, Pierre Duysinx – University of Liège, Belgium	A level set approach for the optimal design of flexible components in multibody systems

15:30 Coffee break

Session 10: Simulation for engineering design 2 (Schwab)

16:00	V. van der Wijk, S. Krut, F. Pierrot, J.L. Herder – University of Twente, Netherlands & LIRMM, France	Experimental results of a high speed dynamically balanced redundant planar 4-RRR parallel manipulator
16:30	Evtim Zahariev – Bulgarian Academy of Science, Bulgaria	Simulation of non-stationary vibration of large multibidy systems
17:00	Mario Acevedo, Marco Ceccarelli, Giuseppe Carbone, Daniele Cafolla – Universidad Panamericana, Mexico & University of Cassino, Italy	Complete dynamic balancing of a 3-DOF spatial parallel mechanisms by the application of counter-rotary counterweights

17:30 **Closure**

Model order reduction of non-linear flexible multibody models

R.G.K.M. Aarts[1]*, D. ten Hoopen[1], S.E. Boer[1] and W.B.J. Hakvoort[2]
[1] Faculty of Engineering Technology, University of Twente, Enschede, The Netherlands
[2] Demcon Advanced Mechatronics, Oldenzaal, The Netherlands

Keywords: Flexible multibody modelling, Reduced model order, Closed-loop simulations.

In high precision equipment the use of compliant mechanisms is favourable as elastic joints offer the advantages of no friction and no backlash. For the conceptual design of such mechanisms there is no need for very detailed and complex models that are time-consuming to analyse. Nevertheless the models should capture the dominant system behaviour which must include relevant three-dimensional motion and geometric non-linearities, in particular when the system undergoes large def ections.

In [1] we discuss a modelling approach for this purpose where an entire multibody system is modelled as the assembly of non-linear f nite elements. The elements' nodal coordinates and so-called deformation mode coordinates are expressed as functions of the independent (or generalised) coordinates $\boldsymbol{q}$. With these expressions the system's equations of motion are derived as a set of second order ordinary differential equations in terms of the kinematic degrees of freedom $\boldsymbol{q}$, see e.g. [2] and the references therein:

$$\bar{\boldsymbol{M}}(\boldsymbol{q})\ddot{\boldsymbol{q}} = \mathbf{D}_q\boldsymbol{\mathcal{F}}^{(x)T}\left(\boldsymbol{f} - \boldsymbol{M}\mathbf{D}_q^2\boldsymbol{\mathcal{F}}^{(x)}\dot{\boldsymbol{q}}\dot{\boldsymbol{q}}\right) - \mathbf{D}_q\boldsymbol{\mathcal{F}}^{(e)T}\boldsymbol{\sigma}, \tag{1}$$

where $\bar{\boldsymbol{M}}$ is the system mass matrix computed from the global mass matrix $\boldsymbol{M}$. The notations $\mathbf{D}_q\boldsymbol{\mathcal{F}}$ and $\mathbf{D}_q^2\boldsymbol{\mathcal{F}}$ denote so-called f rst and second order geometric transfer functions. The vector $\boldsymbol{f}$ are the nodal forces. Generalised stress resultants $\boldsymbol{\sigma}$ represent the loading state of each element. The sound inclusion of the non-linear effects at the element level appears to be very advantageous [2]. Only a rather small number of elastic beam elements is needed to model e.g. wire f exures and leaf springs accurately. Still it appeared that for a more complex compliant mechanism a rather large number of degrees of freedom is needed for an accurate model in the relevant frequency range [1].

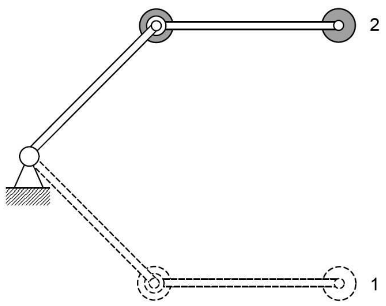
Model order reduction techniques have been studied by several authors as these techniques offer a method to reduce the number of degrees of freedom while an accurate description of the dominant dynamic behaviour may be preserved. In the present paper we propose to describe the vibrational motion as a perturbation of a nominal rigid link motion. For order reduction a modal reduction technique is applied by expressing the perturbations of the degrees of freedom $\delta\boldsymbol{q}$ as

$$\delta\boldsymbol{q} = \boldsymbol{V}\boldsymbol{\eta}, \tag{2}$$

where the elements of the vector $\boldsymbol{\eta}$ are the so-called principal coordinates and $\boldsymbol{V}$ is the modal matrix which is in general conf guration dependent. Applying modal reduction the number of principal coordinates $\boldsymbol{\eta}$ is reduced representing only a rather small number of low frequency modes. Although the non-linear equations (1) still need to be integrated, we expect a gain in computational eff ciency as large time steps can be applied in the absence of high frequent dynamic behaviour.

Consider the two-link f exible manipulator shown in Figure 1. This manipulator has been introduced as a benchmark by Schiehlen and Leister [3] and has been quoted in several papers. Some properties are given in the table next to the f gure. Joint angles $\phi_1(t)$ and $\phi_2(t)$ are prescribed with third order functions of time t moving from the initial to the f nal conf guration in 0.5 s. Different from the original benchmark, we don't include gravity in this paper.

*Email: R.G.K.M.Aarts@utwente.nl



Property	Node 1	Node 2
Joint mass m	1.0 kg	3.0 kg

Property	Link 1	Link 2
Length l	0.545 m	0.675 m
Density ρ	2700 kg/m^3	
Young's modulus E	$7.3 \cdot 10^{10}$ N/m^2	
Cross-sectional area A	$9.0 \cdot 10^{-4}$ m^2	$4.0 \cdot 10^{-4}$ m^2
Cross-sectional area moment of inertia I	$1.69 \cdot 10^{-8}$ m^4	$3.33 \cdot 10^{-9}$ m^4

Figure 1: Planar two-link manipulator: Initial conf guration (1) and f nal conf guration (2) with some of the parameters (adapted from [3]).

The motion of this manipulator has been computed with a non-linear model in which three f exible beam elements are used for each link. Each beam allows two bending modes yielding twelve dynamic degrees of freedom in total. After the joint angles have reached their f nal values, a vibration of the elastic links is observed that is dominated by the lowest natural frequency of approximately 3 Hz.

Next this simulation has been repeated with only a small number of time invariant modes that are computed with a modal analysis in the initial manipulator conf guration. The results in Fig. 2(a) show that with even only one mode the large scale motion at the tip is already described well. The detailed view near the upper extreme position reveals differences between the full order and reduced order simulations. Including the time invariant second mode improves the accuracy and only a negligible error remains. The computation time is reduced as no high frequency modes are present.

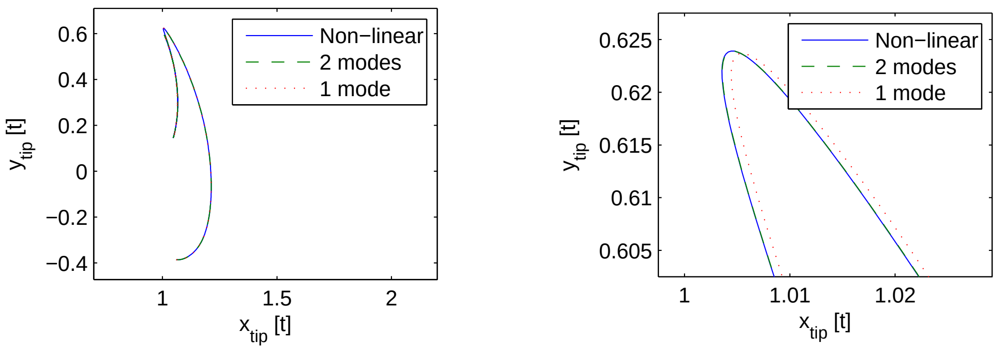


Figure 2: Motion of the manipulator tip during 0.7 s: Full view (left) and detailed view (right) near the upper extreme position.

The example illustrates the possibilities offered by the proposed order reduction according to Eq. (2) combined with the solution of the non-linear equation of motion (1). It should be noted that in this example the mode shapes do not vary much along the prescribed trajectory and the joint angles are prescribed. The application of the method to systems with controlled actuated joint angles and more signif cantly varying conf gurations is currently work in progress.

References

[1] R.G.K.M. Aarts, J. van Dijk, D.M. Brouwer and J.B. Jonker, *Application of flexible multibody modelling for control synthesis in mechatronics*, 10 pages in Multibody Dynamics 2011, ECCOMAS Thematic Conference, Ed. J.C. Samin, P. Fisette, Brussels, Belgium, July 4–7, 2011.

[2] J.B. Jonker, R.G.K.M. Aarts, and J. van Dijk, *A linearized input-output representation of flexible multibody systems for control synthesis*, Multibody System Dynamics, **21** (2) 99–122, 2009.

[3] W. Schiehlen and G. Leister, *Benchmark-Beispiele des DFG-Schwerpunktprogrammes Dynamik von Mehrkörpersystemen*, Universität Stuttgart, Institut B für Mechanik, Zwishenbericht ZB-64, 1991.

Complete dynamic balancing of a 3-DOF spatial parallel mechanisms by the application of counter-rotary counterweights

Mario Acevedo*
Escuela de Ingeniería, Universidad Panamericana
Augusto Rodin 498, Mixcoac,
03920 Mexico City, Mexico

Marco Ceccarelli, Giuseppe Carbone,
Daniele Cafolla†
Laboratory of Robotics and Mechatronics – DiMSAT – University of Cassino,
Via Di Biasio 43, 03043 Cassino (Fr), Italy

Keywords: Dynamic Balancing, Counter-Rotary Counter masses, Parallel Manipulators, Simulation.

Introduction

The balancing of mechanisms has been an important research topic for long time, see for instance [1] and [2] for a literature review. A mechanism is defined to be dynamically balanced or reactionless if, for any motion of the mechanism, there is no reaction force (excluding gravity) and moment on its frame at all times, as indicated for application to parallel manipulators in [3]. Dynamic balancing of mechanisms with multiple degrees of freedom has been addressed by a few authors with two main approaches to obtain a system that is not reactionless by design but capable of producing reactionless trajectories through careful trajectory planning, or to synthesize mechanisms that are reactionless by design (reactionless for any given trajectory).

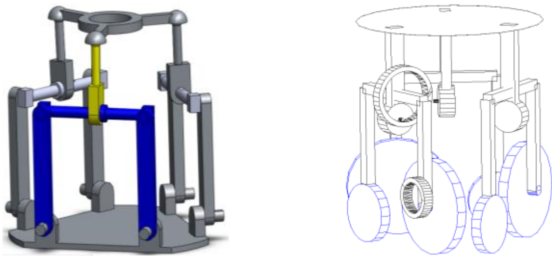
This work presents a complete dynamic balancing of a spatial parallel manipulator of three degrees-of-freedom, CaPaMan-2bis (Cassino Parallel Manipulator-2bis), [4], by application of Counter-Rotary Counterweights and by using properties of its architecture. To accomplish balancing objective the moving platform is replaced by a dynamic equivalent system of three point masses that are located at the points of attachment of the legs, and the mechanism is balanced by considering each leg independently. This fully parallel manipulator has three identical legs, each one composed by a four-bar mechanism (an articulated parallelogram) connected to the fixed base, and a link supported by the coupler that connects to the mobile platform. This last link can be seen as single pendulum and it can be transformed as a dynamic balancer by using a Counter-Rotary Counterweight in order to compensate the motion of the moving platform. In a second stage the articulated parallelogram is modified by adding a Counter-Rotary Counterweight and a single counterweight to achieve a dynamic balance the system.

As final result a new design is obtained for a complete dynamic balanced parallel manipulator, that has been validated by numerical results of dynamic simulations using ADAMS, a general purpose software for multi-body dynamics analysis.

Dynamic balancing of CaPaMan 2bis

The adopted strategy has been to replace the mass and inertia of the mobile platform by a set of three point-masses at the corresponding spherical joints of leg attachments, Fig. 1a. since their design the legs can be balanced independently. Then the upper link connected to the four-bar mechanism (having the point mass coming from the moving platform) is changed into a pendulum with a counter-rotation-element that works as a light-weight dynamic balancer with functions of counter-rotation and counterweight, [5]. Next this new component is taken as part of the coupler in the four-bar mechanism that is balanced again by a light-weight dynamic balancer and a counterweight as proposed in [6]. Fig. 1b shows the balance strategy by conceptual drawing.

* Email: mario.acevedo@up.edu.mx
† Email: ceccarelli@unicas.it; carbone@unicas.it



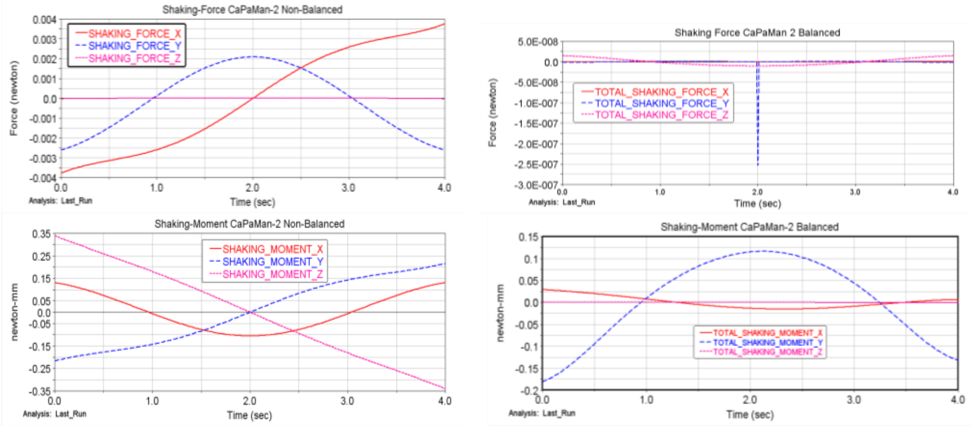
a) b)

Figure 1: Illustrative designs of CaPaMan 2bis: a) original design; b) proposed balanced design

Dynamic balancing has been accomplished by imposing the usual shaking force balance condition, indicating that the general centre of mass of the systems remains static, and the shaking moment balance condition as a constant global angular momentum during motion.

Simulation results

A new design has been obtained and is presented as an ADAMS model in Fig. 1b (crown wheels of the gear systems are shown only in one leg). A general third-order polynomial motion (4 seconds) has been applied to only one leg for a simple motion of platform rotation. Fig. 2 shows computed results that indicate the achieved balanced operation..



a) b)

Figure 2: Numerical results of balancing CaPaMan 2bis: a) original design; b) balanced design

References

[1] G. G. Lowen, F. R. Tepper and R. S. Berkof, *Balancing of Linkages - An Update*, Mechanism and Machine Theory, **18**(3), pp. 213–220, 1983.

[2] V. Arakelian, M. Dahan and M. R. Smith, *A Historical Review of the Evolution of the Theory of the Balancing of Machines*, pp. 291–300 in Proceedings of the International Symposium on History of Machines and Mechanisms - HMM2000, Ed. Ceccarelli, M., Italy, 2000.

[3] C. Gosselin, *Static Balancing and Dynamic Balancing of Parallel Mechanisms*, pp. 291–300 in Smart Devices and Machines for Advanced Manufacturing, Ed. , Wang, L. and Xi, J., London, 2008.

[4] E. E. Hernandez-Martinez, L. Conghui, G. Carbone, M. Ceccarelli, C. S. Lopez-Cajun, *Experimental and Numerical Characterization of CaPaMan 2bis Operation*, Journal of Applied Research and Technology, **8**(1), pp-101-119, 2010

[5] J. Herder and C. Gosselin, *A Counter-Rotary Counterweight (CRCW) for Light-Weight Balancing*, DETC2004-57246, ASME 2004 Design and Engineering Conference, September 28-October 2, Salt Lake City, Utah, USA, 2004.

[6] V. van der Wijk and J. Herder, *Synthesis of Dynamically Balanced Mechanisms by Using Counter-Rotary Countermass Balanced Double Pendula*, ASME Journal of Mechanical Design, **131**(11), 2009.

(Re-)Starting a Generalized-α Solver for Constrained Systems with Second Order Accuracy

Martin Arnold*
Martin Luther University Halle-Wittenberg
NWF II – Institute of Mathematics
D - 06099 Halle (Saale), Germany

Keywords: Efficient time integration, Generalized-α methods.

Abstract

Generalized-α time integration methods were originally designed for large scale problems in structural mechanics [1] but may be used as well for constrained systems of moderate dimension that are typical of multibody dynamics [2]. For more than a decade, the method has been used successfully in industrial simulation software that is based on a finite element approach in multibody dynamics [3]. In terms of numerical effort and stability, these Newmark type integrators are considered to be an interesting alternative to more classical ODE and DAE time integration methods in multibody numerics that are discussed, e.g., in [4]. For constrained systems, the Newmark like update formula

$$\mathbf{q}_{n+1} = \mathbf{q}_n + h\mathbf{v}_n + (0.5-\beta)h^2\mathbf{a}_n + \beta h^2\mathbf{a}_{n+1}\,, \tag{1a}$$

$$\mathbf{v}_{n+1} = \mathbf{v}_n + (1-\gamma)h\mathbf{a}_n + \gamma h\mathbf{a}_{n+1} \tag{1b}$$

for position coordinates $\mathbf{q}$ and velocity coordinates $\mathbf{v}$ and the generalized-α update scheme

$$(1-\alpha_m)\mathbf{a}_{n+1} + \alpha_m\mathbf{a}_n = (1-\alpha_f)\ddot{\mathbf{q}}_{n+1} + \alpha_f\ddot{\mathbf{q}}_n \tag{1c}$$

for the auxiliary vectors $\mathbf{a}_n$ are coupled to equilibrium equations $\mathbf{M}\ddot{\mathbf{q}} = \mathbf{f} - \mathbf{B}^\top\boldsymbol{\lambda}$ and constraints $\boldsymbol{\Phi} = \mathbf{0}$ at $t = t_{n+1}$:

$$\mathbf{M}(\mathbf{q}_{n+1})\ddot{\mathbf{q}}_{n+1} = \mathbf{f}(t_{n+1},\mathbf{q}_{n+1},\mathbf{v}_{n+1}) - \mathbf{B}^\top(\mathbf{q}_{n+1})\boldsymbol{\lambda}_{n+1}\,, \tag{1d}$$

$$\mathbf{0} = \boldsymbol{\Phi}(\mathbf{q}_{n+1})\,. \tag{1e}$$

In (1), the method parameters α_f, α_m, β, γ are chosen to satisfy the order condition $\gamma = \frac{1}{2} + \alpha_f - \alpha_m$, see [1], and the time step size h is for the moment considered to be constant for all time steps.

Following the principles of classical mechanics, the $n_{\boldsymbol{\Phi}}$ constraints (1e) are coupled to the dynamical equations (1d) by constrained forces $-\mathbf{B}^\top(\mathbf{q})\boldsymbol{\lambda}$ with $\mathbf{B}(\mathbf{q}) := (\partial\boldsymbol{\Phi}/\partial\mathbf{q})(\mathbf{q})$ and Lagrange multipliers $\boldsymbol{\lambda}(t) \in \mathbb{R}^{n_{\boldsymbol{\Phi}}}$. All other forces and moments of the system are summarized in vector $\mathbf{f} = \mathbf{f}(t,\mathbf{q},\mathbf{v})$. The mass matrix $\mathbf{M}(\mathbf{q})$ is assumed to be non-singular and symmetric, positive definite.

With an appropriate scaling of the corrector equations [5], the fixed step size implementation of (1) works well for reasonable time step sizes $h > 0$. The convergence analysis for $h \to 0$ proves stability and second order convergence if the stability conditions $\alpha_m < \alpha_f < 1/2$ and $\beta > 1/4 + (\alpha_f - \alpha_m)/2$ are satisfied, see [6] and the extension to the generalized-α Lie group integrator of Brüls and Cardona [7] in [8]. Because of numerical damping, the generalized-α method (1) shows a favourable long-time behaviour. But in a short transient phase, the method may suffer from large errors that are damped out rapidly, see Fig. 1.

*Email: martin.arnold@mathematik.uni-halle.de

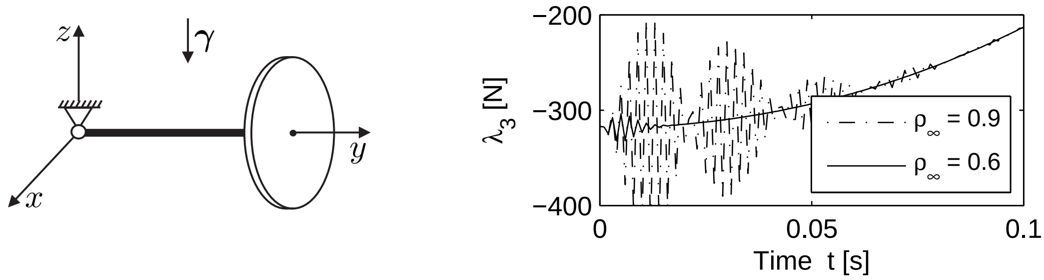


Figure 1: Spurious transient oscillations: Generalized-α method applied to benchmark Heavy top.

In a recent joint work with O. Brüls (Liège) and A. Cardona (Santa Fe), this order reduction was studied in detail for a Lie group integrator [9]. The problem is fixed by perturbed initial values $\mathbf{v}_0$.

In the present paper, we study the extension of these results from the constrained case to (very) stiff unconstrained systems. Furthermore, the efficient (re-)initialization after discontinuities and a variable step size algorithm with improved accuracy will be discussed.

References

[1] J. Chung and G. Hulbert. A time integration algorithm for structural dynamics with improved numerical dissipation: The generalized-α method. *ASME Journal of Applied Mechanics*, 60:371–375, 1993.

[2] A. Cardona and M. Géradin. Time integration of the equations of motion in mechanism analysis. *Computers and Structures*, 33:801–820, 1989.

[3] M. Géradin and A. Cardona. *Flexible Multibody Dynamics: A Finite Element Approach*. John Wiley & Sons, Ltd., Chichester, 2001.

[4] M. Arnold, B. Burgermeister, C. Führer, G. Hippmann, and G. Rill. Numerical methods in vehicle system dynamics: State of the art and current developments. *Vehicle System Dynamics*, 49:1159–1207, 2011.

[5] C. Bottasso, O.A. Bauchau, and A. Cardona. Time-step-size-independent conditioning and sensitivity to perturbations in the numerical solution of index three differential algebraic equations. *SIAM J. Sci. Comp.*, 29:397–414, 2007.

[6] M. Arnold and O. Brüls. Convergence of the generalized-α scheme for constrained mechanical systems. *Multibody System Dynamics*, 18:185–202, 2007.

[7] O. Brüls and A. Cardona. On the use of Lie group time integrators in multibody dynamics. *J. Comput. Nonlinear Dynam.*, 5:031002, 2010.

[8] O. Brüls, A. Cardona, and M. Arnold. Lie group generalized-α time integration of constrained flexible multibody systems. *Mechanism and Machine Theory*, doi:10.1016/j.mechmachtheory.2011.07.017, 2011.

[9] M. Arnold, O. Brüls, and A. Cardona. Improved stability and transient behaviour of generalized-α time integrators for constrained flexible systems. Fifth International Conference on Advanced COmputational Methods in ENgineering (ACOMEN 2011), Liège, 14-17 November 2011, 2011.

Modeling the Multibody Dynamics with the D&C System Simulator

Dmitry Balashov*
Bosch Rexroth AG
Rexrothstr. 3
97816 Lohr am Main
Germany

Oliver Lenord
Bosch Rexroth AG
Rexrothstr. 3
97816 Lohr am Main
Germany

Summary

The D&C System Simulator software is developed and deployed at the Bosch Rexroth AG to analyze complex multi-domain engineering systems and to predict their hydraulic, electric or mechanical behavior. The underlying numerical model and key features of the multi body dynamics module that is a part of the entire software package are outlined in this paper. The multi body 3D model is assumed as an arbitrary system of rigid bodies connected by mechanical joints and subjected to mechanical loads. Multiple kinematic chains, over constrained systems and models with active control units are featured by our simulation tool. The minimum dimension order-N algorithm [1] is used to calculate the multi body dynamic response. A simulation example is presented.

What is the D&C System Simulator?

Development of the dynamic simulation software is motivated by the need of accurately modelling a variety of industrial products and prototypes at the Bosch Rexroth AG such as hydraulic and pneumatic actuators, chain conveyors, rail transportation and linear motion systems, production platforms, wind turbines, solar plants and industrial robots. Typical problems related to the performance and functionality verification; analysis of deficiencies; tuning and evaluation of the optimum design parameters; noise, vibration and harshness issues can be analyzed and resolved using the sophisticated software tool. This reflects a main challenge to develop the dynamic simulation software D&C System Simulator which is suitable for the multi disciplinary models with standard hydraulic components such as pumps or valves from one side and mechanical subsystems with arbitrary geometry and topological configuration from the other side.

The simulator core accounts a set of libraries with standard components matching different physical domains: hydraulic, electric, 1D mechanic, analog, logic, digital and 3D multi body. An interactive model set up is split into two separate steps: drawing the 1D diagram with e.g. hydraulic pipelines and a control circuit and creation of the 3D multi body subsystem. The both models are coupled via the input/output ports implementing an interface between the mechanical state variables and the feedback actuating forces. Graphical set up of the 1D diagram is provided by positioning icons associated with the model components and drawing the connectors between them in a manner that is very similar to the Matlab/Simulink. A 3D multi body model can be generated using a graphical user interface with space representation of the mechanical parts connected by joints and optionally applied motion generators. The items causing forces or torques can be attached to the selected points on the bodies.

From a mathematical point of view dynamics of the entire multi domain model is governed by the common system of differential algebraic equations (DAE) while a time integration of this DAE system is performed by the solver object. The runtime simulation core (the model libraries, the solver and their interfaces) is implemented in native C++ using an object oriented approach while the dialogs and graphical user interface controls are implemented in C#. A sophisticated 3D graphics modeling and animation facility is developed using the Open GL graphics library.

* Email: Dmitry.Balashov@boschrexroth.de

Multi body system model library

According to the minimum dimension order-N algorithm [1], topology analysis once performed defines the time-invatiant minimal spanning tree and the corresponding set of primary joints. Using the associated with these joints minimal co-ordinates, an absolute motion of the bodies is obtained by superimposing the relative motions at the primary joints in a joint-by-joint recursive procedure while the co-ordinates of the secondary joints are calculated as a solution of the algebraic constraint equations. Elimination of the redundant algebraic equations is based on analysis of the range of the constraint matrix in the case of an over constrained system.
A sophisticated set of system tests is worked out in order to validate the numerical algorithms implemented in the MBS simulator core. A verification procedure compares the calculated data versus simulations of exactly the same physical models obtained with the MSC ADAMS. Performance benchmarking performed with the closed loop test systems and the default solver settings indicates a good overall performance level compared to the MSC ADAMS.
An application example of the hydraulic actuated hexapod platform is shown shown in Figure 1. A position of the top platform is defined by the length of six legs connecting the platform and the base via six prismatic actuators and twelve universal hinges. An actuator circuit diagram incorporates the hydraulic differential cylinder connected with the proportional directional valve which is used to control the leg's elongation by tracking the differential input of the target and the measured signal.

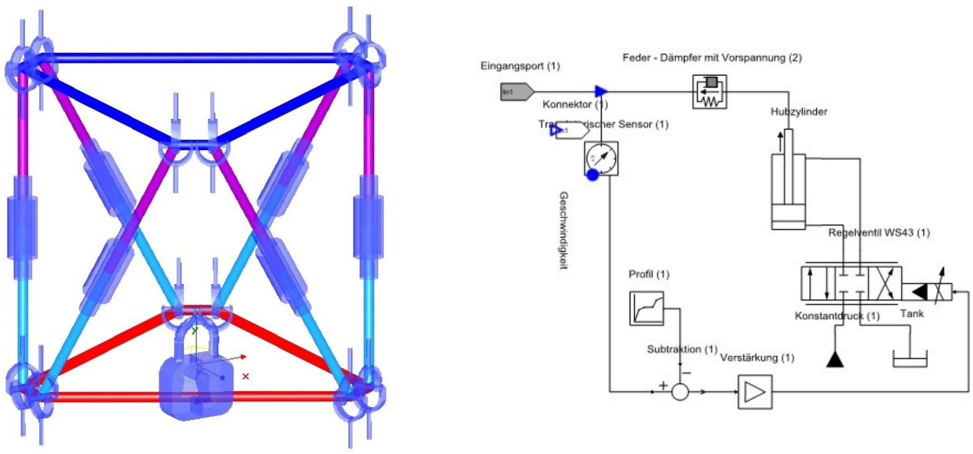


Figure 1: Hexapod model: 3D view of the MBS model (left) and actuator circuit (right).

Advantages of the D&C System Simulator are: a) runtime evaluation of the multi body dynamic response is based on the numerically efficient order-N algorithm; b) significantly shorter time to create a multi domain model (compared to the MSC ADAMS) because of no need to implement a time wasting co-simulation interface; c) virtual models of the manufacturing products are directly available in the hydraulic model library; d) company wide deployment without extra costs as well as flexibility in the extensions and bug fixes is provided by the in-house software development.
The simulation core of the MBS library has been already extended in order to support modelling of the flexible multi body dynamics and collision detections. The modal reduction techniques, Krylov-subspace based and Gramian matrix based [1] are used for the elastic multi body analysis. The featured V-clip collision detection algorithm for polyhedral objects [2] is also implemented in the source code. The impact modelling is provided by the spring-damper reaction model.

References

[1] M. Lehner, *Modellreduktion in elastischen Mehrkörpersystemen*, Shaker, 2007.
[2] Mirtich B., *V-Clip: Fast and Robust Polyhedral Collision Detection*, ACM Transactions on Graphics - TOG, **17**(3), pp. 177–208, 1998.

Sensitivity analysis for flexible multibody systems formulated on a Lie group

Olivier Brüls* and Valentin Sonneville†
University of Liège
Department of Aerospace and Mechanical Engineering
Chemin des Chevreuils 1, Building B52,
4000 Liège, Belgium

Keywords: Lie group formalism, sensitivity analysis, direct differentiation method, adjoint variable method, optimization.

The sensitivity of the dynamic response of a multibody system is a key information in gradient-based design optimization and optimal control problems. The present contribution addresses the computation of the sensitivities for systems which naturally evolve on a Lie group and not on a linear space.

The Lie group framework offers a number of advantages for the analysis of systems with large rotations variables [4, 5], e.g. for finite element models of systems with rigid bodies, kinematic joints, beams and shells. Firstly, the equations of motion are derived and solved directly on the nonlinear manifold, without an explicit parameterization of the rotation variables, which leads to important simplifications in the formulations and algorithms. Secondly, displacements and rotations are represented as increments with respect to the previous configuration, and those increments can be expressed in the material (body-attached) frame. Therefore, geometric nonlinearities are automatically filtered from the relationship between incremental displacements and elastic forces, which strongly reduces the fluctuations of the iteration matrix during the simulation [3].

Classical sensitivity analysis methods include finite difference methods, semi-analytical approaches or automatic differentiation. Semi-analytical approaches have interesting properties in terms of accuracy, robustness and computational cost. They have been successfully exploited for dynamic systems evolving on a linear parameter space, for which classical ODE or DAE solvers are available [1, 2]. However, to the best of our knowledge, the sensitivity analysis of dynamic systems on a Lie group has not been addressed in literature and deserves some particular investigations.

In a previous work [6], a direct differentiation method was proposed for systems evolving on $SO(3)$, the group of finite rotations. Here, the study is extended to a more general class of dynamic systems with kinematic joints, whose equations of motion have the structure of a DAE on a Lie group. It is shown that the nonlinearity of the Lie group and of the time integration formulae need to be carefully treated for the development of accurate sensitivity analysis algorithms.

A broad class of semi-analytical methods, including the direct differentiation method and the adjoint variable method, is discussed in the presentation. The main properties of those methods, which are well-known for problems on a linear space, are also observed for problems on a Lie group. Accurate sensitivity analysis algorithms are established and implemented in a simple way, exploiting the compact and elegant Lie group formalism. Their performance is studied for academic examples as well as for the optimization of a vehicle multi-link suspension mechanism.

*Email: O.Bruls@ulg.ac.be
†Email: V.Sonneville@ulg.ac.be

References

[1] D. Bestle and P. Eberhard. Analyzing and optimizing multibody systems. *Mechanics of Structures and Machines*, 20:67–92, 1992.

[2] D. Bestle and J. Seybold. Sensitivity analysis of constrained multibody systems. *Archive of Applied Mechanics*, 62:181–190, 1992.

[3] O. Brüls, M. Arnold, and A. Cardona. Two lie group formulations for dynamic multibody systems with large rotations. In *Proceedings of the IDETC/MSNDC Conference*, Washington D.C., U.S., August 2011.

[4] O. Brüls and A. Cardona. On the use of Lie group time integrators in multibody dynamics. *ASME Journal of Computational and Nonlinear Dynamics*, 5(3):031002, 2010.

[5] O. Brüls, A. Cardona, and M. Arnold. Lie group generalized-α time integration of constrained flexible multibody systems. *Mechanism and Machine Theory*, (48):121–137, 2012.

[6] O. Brüls and P. Eberhard. Sensitivity analysis for dynamic mechanical systems with finite rotations. *International Journal for Numerical Methods in Engineering*, 74(13):1897–1927, 2008.

Calculating Input Data for Multibody System Simulation by Solving an Inverse Control Problem

Michael Burger* Klaus Dreßler†

Michael Speckert‡

Fraunhofer Institute for Industrial and Financial Mathematics (ITWM)
Fraunhofer Platz 1, 67663 Kaiserslautern, Germany

Keywords: Multibody Systems, Control, Inverse Problems.

In order to simulate a multibody system (MBS) model of a real mechanical system, input data (or drive-signals) are needed to excite the numerical model in such a way that the simulated loads are as close as possible to the loads that act on the real system under operational conditions. If such input data is available, numerical system simulation of MBS models can be used very efficiently in many application areas. For instance, in vehicle engineering, during the development process of a full vehicle or of specific components, different designs and constructions can be analyzed and optimized by numerical simulation of a computer (MBS) model, see [3]. However, such input data with suitable properties is often not available. In case of vehicle engineering, a convenient example for input data is a digital road profile, which has an additional very desirable property: it is invariant w.r.t. the vehicle model. That is, a digital road profile can be used to excite different vehicles, it does not depend on a specific vehicle. Unfortunately, however, digital road profiles are hard to obtain; of course, a real road can be measured and digitalized, but this is costly and time-consuming, requires complex sensor techniques.

In contrast to this, during a typical test-track drive of a prototype vehicle, a lot of quantities *within* the vehicle are measured and stored comparably easy and by default without additional effort. Whence, the obvious task arise to derive input data with suitable properties, e.g., a road profile, on the basis of typically measured inner vehicle quantities, such as accelerations of specific components or the wheel forces and torques that act on the vehicle's spindles.

This task leads to the following mathematical problem formulation, cf. [1]. Assume that there is a real mechanical system, e.g., a full vehicle, and a mathematical description as MBS model, i.e., the corresponding equations of motion, e.g., in the following well-known form:

$$\begin{aligned} M(q)\ddot{q} &= f(t, q, \dot{q}, u) - G^T(q)\lambda \\ 0 &= g(q), \end{aligned} \tag{1}$$

with generalized coordinates $q \in \mathbb{R}^{n_q}$, Lagrange multipliers λ, a positive definite mass matrix $M(q)$ and $G(q) := \partial g/\partial q$ being of full row rank. The vector f subsumes all acting forces and torques and, in addition to that, it included the dependence on the desired, but *unknown*, input quantity $u \in \mathbb{R}^{n_u}$. Last, not least, suppose that the measured quantities, denoted by $z_{REF} : [0;T] \to \mathbb{R}^{n_z}$ as functions of time, correspond to system outputs, defined by

$$z_{out} := h(t, q, \dot{q}, u), \tag{2}$$

*Email: Michael.Burger@itwm.fraunhofer.de
†Email: Klaus.Dressler@itwm.fraunhofer.de
‡Email: Michael.Speckert@itwm.fraunhofer.de

where h is a smooth vector function. With these notion, we are faced with the following inverse control problem, fig. 1:

$$\text{Find } u \in \mathcal{D} \text{ such that } \|z_{out} - z_{REF}\| = \|h(t, q, \dot{q}, u) - z_{REF}\| \to \text{ min}, \tag{3}$$

where $(q, \dot{q})$ is the solution of eq. (1) with the input u and $\mathcal{D}$ is a suitable domain for input functions.

In this contribution, we will present an operator-theoretic framework to precisely formulate the above inverse control problem. We introduce an input-output-operator P_h that maps each input u the corresponding output,

$$P_h(u) = z_{out}. \tag{4}$$

We discuss some properties of this operator like continuity and differentiability and interpret these notions in the context of perturbations. These results are derived and proven in [1]. The inverse control problem from above leads to the question wether or not the input-output-operator is invertible at z_{REF}. We discuss several approaches to (computationally) solve the inverse control problem - for a detailed discussion we also refer to [1].

We end with a numerical case study, in which the operator-theoretic framework as well as some of the computational solution methods is applied to compute a virtual road profile for full-vehicle simulation, see also [1, 2]. In this case study, a specific subsystem technique is introduced that allows to reduce the inverse-control problem to a subsystem of moderate complexity when compared to the full vehicle MBS model, which is built up in a commercial software tool. This subsystem approach is briefly sketched and discussed, a detailed description, discussion and proofs can be found in [1].

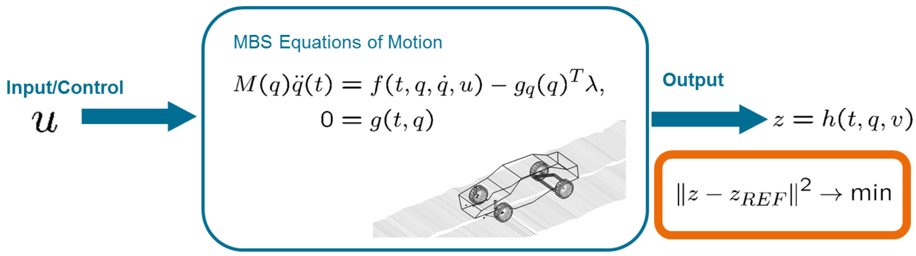


Figure 1: Input-Output Configuration

References

[1] M. Burger. *Optimal Control of Dynamical Systems: Calculating Input Data for Multibody System Simulation. Dissertation. TU Kaiserslautern. 2011. To appear.*

[2] M. Burger, K. Dressler, and M. Speckert. Invariant Input Loads for Full Vehicle Multibody System Simulation. In *Multiobody Dynamics 2011 ECCOMAS Thematic Conference*, Brussels, 2011.

[3] K. Dressler, M. Speckert, and G. Bitsch. Virtual test rigs. In C. Bottasso, P. Masarati, and Trainelli, editors, *Multibody dynamics 2007, Eccomas Thematic Conference*, Milano, Italy, 25-28 June.

Model Reduction of Large Scaled Industrial Models in Elastic Multibody Systems

Peter Eberhard*
Institute of Engineering
and Computational Mechanics
University of Stuttgart

Michael Fischer
Institute of Engineering
and Computational Mechanics
University of Stuttgart

Keywords: model order reduction, elastic multibody systems, LU-decomposition, high performance computing, out-of-core solution.

Introduction

The description of the dynamical behavior of mechanical systems is of great interest in the development process of technical products. If rigid body movements and additional elastic deformations have to be concerned, the method of elastic multibody systems (EMBS) is used. With the floating frame of reference formulation, the movement of an elastic body is separated into a huge nonlinear motion of the reference frame and a small elastic deformation with respect to this reference frame.

The discretization of the elastic body with finite elements provides a linear time-invariant second order multi input multi output (MIMO) system

$$\begin{aligned} \boldsymbol{M}_e \cdot \ddot{\boldsymbol{q}}(t) + \boldsymbol{D}_e \cdot \dot{\boldsymbol{q}}(t) + \boldsymbol{K}_e \cdot \boldsymbol{q}(t) &= \boldsymbol{B}_e \cdot \boldsymbol{u}(t), \\ \boldsymbol{y}(t) &= \boldsymbol{C}_e \cdot \boldsymbol{q}(t) \end{aligned} \tag{1}$$

with the symmetrical sparse mass matrix $\boldsymbol{M}_e$, damping matrix $\boldsymbol{D}_e$, stiffness matrix $\boldsymbol{K}_e$, inputs $\boldsymbol{u}(t)$, outputs $\boldsymbol{y}(t)$ and states $\boldsymbol{q}(t)$.

In industrial applications large finite element models with millions degrees of freedom are generated to describe the elastic behavior. To enable the simulation of EMBS with large models, the degrees of freedom of the elastic body have to be reduced by approximating the nodal displacements with the help of ansatz functions. Modern reduction methods, like Krylov-subspace based moment matching or Gramian matrix based reduction, as described in [1], are used to find the optimal ansatz functions.

Main Calculation Step in Model Reduction

The main calculation step in modern reduction techniques is the solution of large sparse symmetric linear systems

$$\boldsymbol{A} \cdot \boldsymbol{X} = \boldsymbol{B} \tag{2}$$

with the large sparse matrix $\boldsymbol{A} \in \mathbb{C}^{N\times N}$, the right hand side $\boldsymbol{B} \in \mathbb{C}^{N\times r}$ and the solution $\boldsymbol{X} \in \mathbb{C}^{N\times r}$.

There are two possibilities to solve sparse linear systems, either to use a direct or an iterative solver. The iterative solver needs multiple steps to solve the system. This allows to store only one column of $\boldsymbol{X}$ and $\boldsymbol{B}$ in the solving process. In contrast, the direct method solves Equation (2) by a decomposition of $\boldsymbol{A}$, like LU-factorization, and a following forward elimination and backward substitution. For large right hand sides, which is common in using Krylov-subspace based model reduction, the LU-decomposition,

*Email: peter.eberhard@itm.uni-stuttgart.de

in contrast to the iterative method, is calculated only once. However, the LU-decomposition requires to store the lower triangular matrix $\boldsymbol{L} \in \mathbb{C}^{N\times N}$ and upper triangular matrix $\boldsymbol{U} \in \mathbb{C}^{N\times N}$. Because of the fill-ins these matrices have more nonzero entries than the corresponding part of $\boldsymbol{A}$. The large memory consumption of the LU-decomposition is the biggest numerical challenge in reducing large models.

Solving Process

At the Institute of Engineering and Computational Mechanics the software package Morembs [2] is developed to reduce the elastic degrees of freedom. This software is implemented in Matlab and C++. In the C++ Version different numerical libraries for solving large linear sparse systems are tested in this contribution. The different direct solvers are compared in [3]. In Morembs, freely available libraries are preferred. Therefore, well tested numerical libraries for the LU-decomposition, like Umfpack [4] or Mumps [5], are used.

Although using the most efficient direct solvers, the memory hardware limits the size of models which can be reduced in Morembs. One possibility to solve large systems with Morembs is using supercomputers. Therefore, a NEC SX-9 supercomputer at the High Performance Computing Center Stuttgart is used. The computation cluster allows a memory allocation of 512 GB. Morembs is a sequential program which runs slow on the vector supercomputer. On the newly installed faster supercomputer Cray XE6 Morembs runs faster but needs more time than the program needs on a serial standard computer (Intel-Xeon Quadcore, 2.4 GHz, 6 GB RAM). Furthermore, the usage of supercomputers is expensive and not all users have access to such supercomputers.

Some numerical direct solver packages, like Mumps, feature an Out-of-Core capability. This allows the solution of very large sparse linear systems with a standard computer by storing most parts of the lower and upper triangular matrices on the hard drive. With a solid-state-drive the reduction with the Out-of-Core solver Mumps is slower than the time reducing the model in-core but it is nearly four times faster than the serial reduction on the supercomputer. This allows the model reduction of large scaled industrial models with millions degrees of freedom on standard computers in a reasonable time.

References

[1] Lehner, M.: Modellreduktion in elastischen Mehrkörpersystemen (in German). Dissertation, Schriften aus dem Institut für Technische und Numerische Mechanik der Universität Stuttgart, Band 10. Aachen: Shaker Verlag, 2007.

[2] Fehr, J.; Eberhard, P.: Simulation Process of Flexible Multibody Systems with Non-modal Model Order Reduction Techniques. Multibody System Dynamics, Vol. 25, No. 3, pp. 313–334, 2011.

[3] Gould, N.I.M.; Scott, J.A.; Hu, Y.: A numerical evaluation of sparse direct solvers for the solution of large sparse symmetric linear systems of equations. ACM Transactions on Mathematical Software, Vol. 33, No. 2, p. 10, 2007.

[4] Davis, T.A.: Algorithm 832: UMFPACK, an unsymmetric-pattern multifrontal method. ACM Transactions on Mathematical Software, Vol. 30, No. 2, pp. 196–199, 2004.

[5] Amestoy, P.; Duff, I.; Robert, Y.; Rouet, F.; Ucar, B.: On computing inverse entries of a sparse matrix in an out-of-core environment. Technical report rt-apo-10-06, Institut national de recherche en informatique et en auotmatique, 2010.

EUROMECH Colloquium 524
Multibody system modelling, control
and simulation for engineering design

February 27–29, 2012
University of Twente
Enschede, Netherlands

Approximate Feedforward Control of Flexible Mechanical Systems

Thomas Gorius
University of Stuttgart
Pfaffenwaldring 9
70569 Stuttgart

Robert Seifried
University of Stuttgart
Pfaffenwaldring 9
70569 Stuttgart

Peter Eberhard*
University of Stuttgart
Pfaffenwaldring 9
70569 Stuttgart

Keywords: Flexible multibody systems, feedforward control, singular perturbation, integral manifolds

Introduction

The design of modern machines usually is focussed on increasing the speed of operation while reducing the energy consumption. Many machines, therefore, contain lightweight components. A drawback of these components is their structural f exibility which plays an important role in the design of an end-effector tracking control. Additionally, controller design techniques for rigid multibody systems, e.g. computed torque for robots, are not directly applicable in the case of f exible bodies. A main problem is the lack of appropriate passivity and minimum-phase properties.

In the early 80's many researchers further developed the theory of singular perturbed systems and singular perturbation based control [1]. These approaches make it possible to incorporate the elastic deformations of f exible multibody systems during the controller design. In recent publications the so called integral manifold control [2], which is based on a singular perturbed model, were applied to the end-effector tracking problem of a serial f exible manipulator [3]. Although the theoretical results were very good, they could not be experimentically verif ed. A main reason is the poor robustness property of the closed loop. However, singular perturbation modeling is in a certain way a natural approach to describe a f exible multibody system. Therefore in this presentation, ideas from the integral manifold control technique are used to reduce the feedback controller to a feedforward control that is based on a series expansion of the given mechanical system. Thus, this feedforward control is not an exact but an approximate inversion of the system. The variable to which the series expansion is applied corresponds to the stiffness of the involved f exible bodies. Thereby, the order of approximation needs to be increased when reducing the stiffness of the system to ensure adequate performance. In this presentation the ideas and use of this feedforward control will be demonstrated by simulation results.

Singular perturbations and approximate feedforward control

Roughly speaking, a singular perturbed system can be substituted into subsystems that signif cantly differ in their dynamical behaviour, i.e. the overall system contains different time scales. A simple example is a system described by the state x which is driven by an actuator with very fast dynamics denoted by z, i.e. $\dot{x} = -x + z$, $\epsilon\dot{z} = -z + u$ with the input signal u. If $\epsilon \ll 1$ a resonable simplif cation is achieved by setting $\epsilon = 0$ which leads to $z = u$. This step reduces the order of the dynamical system as the differential equation with respect to z degenerates to an algebraic equation, and this is why ϵ is called a singular perturbation. If in fact ϵ is not very small the simplif ed model cannot describe the exact model suff ciently precise. In this case integral manifolds are helpful. Instead of $z = h_0(x)$ the series expansion $z = h(x) = h_0(x) + \epsilon h_1(x) + \epsilon^2 h_2(x) + \ldots$ is used where the functions h_i must be calculated. Once $z = h(x)$ is fulf lled for one time this is fulf lled for all times afterwards which gives

*Email:peter.eberhard@itm.uni-stuttgart.de

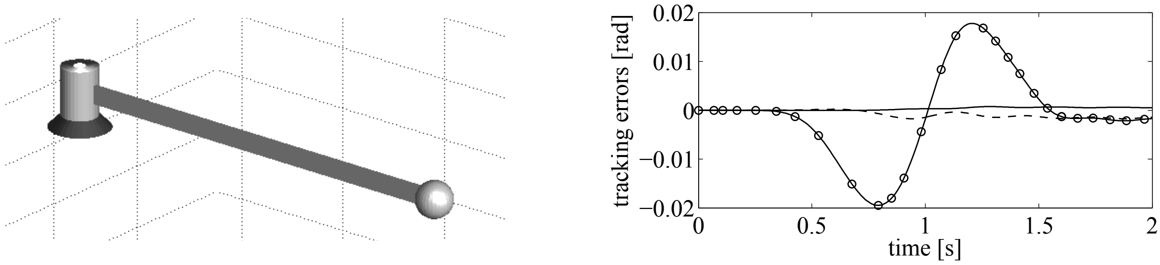


Figure 1: Left: Rotating f exible beam. Right: Tracking errors of different approximated feedforward controls. Circled line: $r = 0$, dashed line: $r = 2$, solid line: $r = 6$.

h the name integral manifold. Based on the series expansion a higher order approximation of the exact system can be derived. To translate this to mechanical systems, f rst it is noted that an important class of f exible multibody systems is singular perturbed. With $\mathbf{s}$ and $\boldsymbol{\delta}$ being the vectors of "rigid" and "f exible" degrees of freedom and $\mathbf{u}$ being the control, the equation of motion of the system

$$\mathbf{M}(\mathbf{s},\boldsymbol{\delta})\begin{bmatrix}\ddot{\mathbf{s}}\\ \ddot{\boldsymbol{\delta}}\end{bmatrix}+\begin{bmatrix}0\\ \mathbf{K}\end{bmatrix}\boldsymbol{\delta}+\mathbf{q}(\mathbf{s},\dot{\mathbf{s}},\boldsymbol{\delta},\dot{\boldsymbol{\delta}})=\mathbf{G}\mathbf{u} \tag{1}$$

is brought by $\epsilon = 1\backslash\sqrt{\lambda_{\min}(\mathbf{K})}$, $\mathbf{x}_1 = \mathbf{s}$, $\mathbf{x}_2 = \dot{\mathbf{s}}$, $\mathbf{z}_1 = \boldsymbol{\delta}\backslash\epsilon^2$, $\mathbf{z}_2 = \dot{\boldsymbol{\delta}}\backslash\epsilon$ to its singular perturbed form

$$\begin{aligned}\dot{\mathbf{x}}_1 &= \mathbf{x}_2\ , \qquad \dot{\mathbf{x}}_2 = \mathbf{a}_1(\mathbf{x}_1,\mathbf{x}_2,\epsilon^2\mathbf{z}_1,\epsilon\mathbf{z}_2)+\mathbf{A}_1(\mathbf{x}_1,\epsilon^2\mathbf{z}_1)\mathbf{z}_1+\mathbf{B}_1(\mathbf{x}_1,\epsilon^2\mathbf{z}_1)\mathbf{u}\ ,\\ \epsilon\dot{\mathbf{z}}_1 &= \mathbf{z}_2\ , \qquad \epsilon\dot{\mathbf{z}}_2 = \mathbf{a}_2(\mathbf{x}_1,\mathbf{x}_2,\epsilon^2\mathbf{z}_1,\epsilon\mathbf{z}_2)+\mathbf{A}_2(\mathbf{x}_1,\epsilon^2\mathbf{z}_1)\mathbf{z}_1+\mathbf{B}_2(\mathbf{x}_1,\epsilon^2\mathbf{z}_1)\mathbf{u}\ ,\end{aligned} \tag{2}$$

where $\mathbf{z}_1$ is a "generalized" spring force. Setting $\epsilon = 0$ will recover the rigid multibody system from its f exible formulation (2). First the integral manifold and its series expansion $\mathbf{z}_i = \mathbf{h}_i(\mathbf{x}_1,\mathbf{x}_2) = \mathbf{h}_{i0}(\mathbf{x}_1,\mathbf{x}_2)+\epsilon\mathbf{h}_{i1}(\mathbf{x}_1,\mathbf{x}_2)+\ldots$ are used. To do this, the control is written as $\mathbf{u} = \mathbf{u}_0(\mathbf{x}_1,\mathbf{x}_2)+\epsilon\mathbf{u}_1(\mathbf{x}_1,\mathbf{x}_2)+\ldots$ where $\mathbf{u}_i$ are calculated such that the output $\mathbf{y} = \mathbf{x}_1+\boldsymbol{\Psi}\mathbf{z}_1$, describing the end-effector, approximately tracks a given trajectory up to a certain order r. As a consequence $\mathbf{u}_0$ is the inversion of the rigid counterpart while the remaining $\mathbf{u}_i$ incorporate the structural f exibility. After expanding all terms in (2) the feedfoward control is given by

$$\mathbf{u} = \mathbf{u}_0(\mathbf{x}_1,\mathbf{x}_2)+\epsilon\mathbf{u}_1(\mathbf{x}_1,\mathbf{x}_2)+\ldots+\epsilon^r\mathbf{u}_r(\mathbf{x}_1,\mathbf{x}_2) \tag{3}$$

where $\mathbf{x}_1$ and $\mathbf{x}_2$ are calculated by the approximation

$$\dot{\mathbf{x}}_1 = \mathbf{x}_2\ , \qquad \dot{\mathbf{x}}_2 = \sum_{i=0}^{r}\left(\mathbf{a}_{1i}(\mathbf{x}_1,\mathbf{x}_2)+\sum_{j=0}^{i}\Big(\mathbf{A}_{1j}(\cdot)\mathbf{h}_{1(i-j)}(\cdot)+\mathbf{B}_{1j}(\cdot)\mathbf{u}_{i-j}(\cdot)\Big)\right)\epsilon^i\ . \tag{4}$$

As an example Figure 1 shows the end-effector tracking errors when different approximated feedforward controls are used to change the working point of a rotating f exible link.

References

[1] P.V. Kokotović, H.K. Khalil, *Singular perturbation methods in control*, Academic Press, London, 1986.

[2] F. Ghorbel, M.W. Spong, *Integral manifolds of singularly perturbed systems with application to rigid-link flexible-joint multibody systems*, International Journal of Non-Linear Mechanics 35, pp. 133–155, 2000.

[3] M. Vakil, R. Fotouhi, P.N. Nikiforuk, *Trajectory tracking for the end-effector of a class of flexible link manipulators*, Journal of Vibration and Control 17(1), pp. 55–68, 2011.

Joint Coordinate Subsystem Synthesis Method with Implicit Integrator in the Application to the Unmanned Military Robot

Junyoun Jo, Myoung-ho Kim*
Graduate school of Mechanical·Mechanical design·Mechantronics Engineering
Chungnam National University
220 Kung-dong, Yusong-Ku, Daejeon
305-764 Korea

Sung-Soo Kim†
Professor, Department of Mechatronics Engineering
Chungnam National University
220 Kung-dong, Yusong-Ku, Daejeon
305-764 Korea

Keywords: Subsystsem synthesis method, Implicit integrator.

Introduction

Efficient or real-time analysis of robotic vehicle systems is becoming important in order to realize Hardware-in-the loop simulation and active control of the suspension subsystems. Especially for the unmanned military robot system as shown in Fig. 1, real-time simulation is essential to judge whether it can move forward or not, based on the on-board simulation with the scanned terrain data in front of the robot.

A subsystem synthesis method has been developed for the vehicle system with several identical suspension systems in the real-time application. The joint coordinate has been used in the method with explicit integrator for the HILS application [1]. For the stiff suspension subsystem, the subsystem synthesis method based on the Cartesian coordinates has also been developed with the implicit integrator [2]. However, in this case, it is difficult to archive real-time simulation without efficient linear equation solvers.

In this paper, a subsystem synthesis method based on the joint coordinates has been developed with an implicit integrator, in order to achieve real-time simulation for stiff suspension subsystems. The expression of the system Jacobian matrix is very complicated in the formulation. Thus, the symbolic language MAPLE has been utilized to derive components of the system Jacobian matrix. A rough terrain run simulation of the unmanned military robot has been carried out to compare the efficiency from the joint coordinate based subsystem synthesis method with an explicit integrator to the one from the same subsystem synthesis method with an implicit integrator.

Figure 1: Unmanned military robot

Rough terrain simulations

The joint coordinate based subsystem synthesis method with the HHT-a implicit integrator [3] has been compared with the method with the Adams Bashforth 3rd order explicit integrator. Figure 2

† Email: sookim@cnu.ac.kr

shows the pitch angle time history of the half vehicle model which runs through the rough terrain with a speed of 10 km/h. Essentially the same results are obtained from both integrators. CPU time comparison has also been made as shown in Table 1. Slightly improved efficiency has been obtained with the implicit integrator.

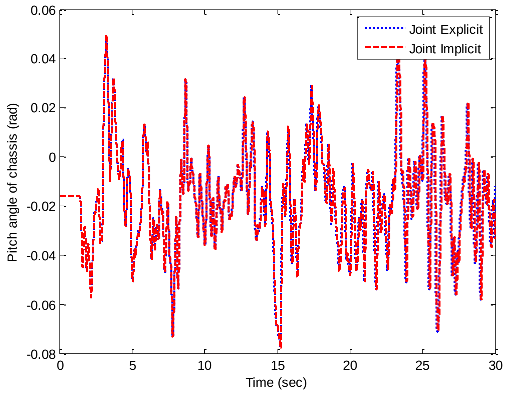


Figure 2: Pitch angle of chassis (1/2 robot vehicle).

Table 1: Simulation results comparison (1/2 robot vehicle).

Simulation : 30sec	**Explicit integrator Adams bashforth 3rd**	**Implicit integrator HHT-α**
Max. Step-size	0.7ms	6.3ms
CPU time (Matlab)	232.44s	210.08s

Conclusions and future works

The joint coordinate based subsystem synthesis method with the implicit integrator has been initially investigated with Matlab implementation. In order to achieve real-time simulation, implementation with C language must be considered. Fixed number of Newton Raphson iteration must also be considered in order to make the same amount of computation for each time step.

Acknowledgements

The Authors gratefully acknowledge the support from UTRC (Unmanned Technology Research Center) at KAIST (Korea Advanced Institute of Science and Technology), originally funded by DAPA, ADD.

References

[1] Kim, S.S., Jeong, W.H., Jung, D.H., Choi, H.J., *HIL-simulation for Evaluation of Intelligent Chassis Controller using Real-time Multibody Vehicle Dynamics Model*, The 1st Joint International Conference on Multibody System Dynamics, Lappeenranta, Finland, 2010.

[2] Han, J.B., Jo, J.Y., Kim, S.S., Wang, J.H., Kim, J.Y., *An Explicit-Implicit Integration Method for Multibody Dynamics Model based on a Subsystem Synthesis Method*, Multibody Dynamics 2011 ECCOMAS Thematic Conference, Brussels, Belgium, 2011.

[3] Negrut. D., Rampalli. R., Ottarssom. G., *On the Use of the HHT Method in the Context of Index 3 Differential Algebraic Equations of Multibody Dynamics*, pp. 1–2 in Proceedings of the ASME DETC/CIE 2005, Long Beach, California, 2005.

Dynamic response of multibody systems with 3D contact-impact events: influence of the contact force model

M. Machado* P. Flores† D. Dopico‡ J. Cuadrado§
University of Minho
Guimarães, Portugal
Universidad de A Coruña
Ferrol, Spain

Keywords: Contact forces, Continuous models, Elastic and inelastic contacts

Introduction

Contact-impact events can frequently occur in the collision of two or more bodies that can be unconstrained or may belong to a multibody system. In many cases the behavior of the mechanical systems is based on them. As a result of an impact, the values of the system state variables change very fast, eventually looking like discontinuities in the system velocities. The knowledge of the peak forces developed in the impact process is very important for the dynamic analysis of multibody systems and has consequences in the design process. Therefore, the selection of the most adequate contact-impact method used to describe the process correctly is crucial for an accurate design and analysis of these types of systems. The constitutive contact force law utilized to assess contact-impact events plays a key role in predicting the dynamic response of multibody systems and simulation of the engineering applications. Thus, a study on the dynamic response of 3D multibody systems that experience contact-impact events is presented in this paper, where different contact force models are used in order to check how the contact force law affects the dynamic behavior of the whole system.

Contact-impact force models

In the present work, several compliant contact force models are considered to model the contact phenomena developed within the multibody systems, namely those proposed by Hunt and Crossley [1], Lankarani and Nikravesh [2] and Flores et al. [3]. In these models, the local deformations and normal contact forces are treated as continuous events and introduced into the equations of motion of the mechanical system as external generalized forces. The constitutive force laws mentioned above are based on the Hertz law and include a damping term to accommodate the energy loss during the impact. Thus, these three contact force laws can be divided into elastic and dissipative components as

$$F_N = K\delta^n + D\dot{\delta} \qquad (1)$$

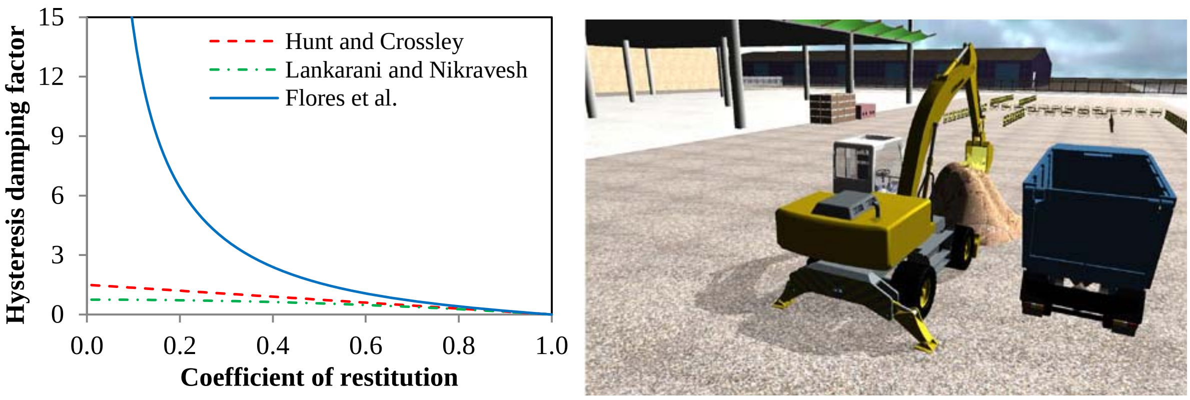
where the first term represents the elastic force and the second term accounts for the energy dissipation. In Eq. (1), K is the generalized stiffness parameter, δ is the relative penetration depth, D is the hysteresis damping coefficient and $\dot{\delta}$ is the relative impact velocity. The exponent n is equal to 3/2 for the case where there is a parabolic distribution of contact stresses. The generalized stiffness parameter K depends on the geometry and physical properties of the contacting surfaces. In turn, the damping term D has different expressions depending on the approach considered, which may be valid for very elastic and/or inelastic contacts. The similarities of and differences among the contact force models are investigated for elastic and inelastic contacts by means of the use of high and low values

* margarida@dem.uminho.pt
† pflores@dem.uminho.pt
‡ ddopico@udc.es
§ javicuad@cdf.udc.es

of the restitution coefficient for the contacting bodies. With the purpose to understand which are the main differences among the dissipative contact force models listed above, the evolution of the hysteresis damping factor for all range of the coefficient of restitution is investigated and shown in Figure 1-a. By observing the plots of Figure 1-a, it can be concluded that all the contact force models exhibit a similar behavior for high values of coefficient of restitution. In contrast, for low values of the coefficient of restitution, the Flores et al. approach is the one in which the hysteresis damping factor increases asymptotically with the decrease of the coefficient of restitution, which can be considered to a be superior model for inelastic contacts. These issues will be object of discussion within this study.

Demonstrative example of application

The demonstrative example is a multibody model of a Liebherr A924 Litronic, a medium-size wheeled excavator (see Figure 1-b). The model interacts with the environment by means of contact forces between its bodies and the surrounding objects and terrain. The normal force models implemented are those explained above, while the frictional model can be found in [4]. The excavator is placed in a working environment, standing on its wheels, and the operator performs two maneuvers: first, the machine is lifted on its legs and blade, and second, the bucket is actuated to hit the surface of a dump truck which is considered as a fixed rigid body. In order to compare the three normal contact models, the time history of the following magnitudes is obtained: position, velocity and acceleration of the chassis center of mass, contact forces, and energy dissipation due to the normal forces.



(a) (b)

Figure 1: (a) Evolution of the hysteresis damping factor as function of the coefficient of restitution for Hunt and Crossley, Lankarani and Nikravesh, and Flores et al. contact force models. (b) Excavator model in its working environment.

Acknowledgements

The support by the *Fundação para a Ciência e a Tecnologia* (FCT) through the project BIOJOINTS (PTDC/EME-PME/099764/2008) is acknowledged. The first author expresses her gratitude to FCT for the PhD grants SFRH/BD/40164/2007.

References

[1] Hunt, K.H., Crossley, F.R.E., *Coefficient of restitution interpreted as damping in vibroimpact*, Journal of Applied Mechanics, **7**, 440-445, 1975.

[2] Lankarani, H.M., Nikravesh, P.E., *A contact force model with hysteresis damping for impact analysis of multibody systems*, Journal of Mechanical Design, **112**, 369-376, 1990.

[3] Flores, P., Machado, M., Silva, M.T., Martins, J.M., *On the continuous contact force models for soft materials in multibody dynamics*, Multibody System Dynamics, **25**, 357-375, 2011.

[4] Dopico, D., Luaces, A., Gonzalez, M., Cuadrado, J., *Dealing with multiple contacts in a human-in-the-loop application*, Multibody System Dynamics, **25**, 167-183, 2011.

Challenges in using OpenSim as a multibody design tool to model, simulate, and analyze prosthetic devices: a knee joint case-study

M. Machado[*] P. Flores[†]
University of Minho
Guimarães, Portugal

J. P. Walter[‡] B. J. Fregly[§]
University of Florida
Gainesville (FL), USA

Keywords: Multibody modeling, Biomechanics, Knee joint, Contact loads, Muscle forces, OpenSim

Extended Abstract

The human knee is comprised of many elements including ligaments, articular cartilage, menisci and muscles, which are capable of bearing and transferring load during physical activities. Unfortunately, the knee joint is quite susceptible to injury and disease, because of the high loads to which it is subjected [1]. In view of this fact, identifying and quantifying the loads placed on the anatomical tissues that surround the human knee is critical for understanding and studying knee joint pathologies [1, 2]. Experiments cannot measure clinically important quantities such as muscle and articular contact forces [2]. Hence, muscle-actuated dynamic models are becoming a feasible approach for determining how musculoskeletal elements interact to generate movement [2, 3]. In general, these models combine a dynamic skeletal model with individual muscle models, but they rarely include articular contact models due to their high computational cost and complexity [2]. OpenSim is a freely available software program, developed within a multibody dynamics framework, that enables the construction and simulation of musculoskeletal models, the visualization of measured and simulated motion, and the extraction of useful information for simulations [3]. Given the capabilities of the OpenSim, the present work explored the use of this computational tool to study the dynamic response of a biomechanical model of a prosthetic knee. Thus, a conceptual framework for contact and muscle force modeling using OpenSim was described. The aim of this work was to investigate the potential for using this computational multibody tool to the design of medical devices, *i.e.*, to assess how modeling approaches could complement experiment studies, mainly for the prediction of contact and muscle forces [2].

The proposed framework involved four main tasks. The first task was the development of the skeletal model. In OpenSim, a skeleton is comprised of rigid bodies interconnected by joints that define how a child body can move with respect to its parent body. Based on this information, a knee model with a prosthetic device was built. This model was composed by ten rigid bodies that were connected by nine joints, as illustrated in Figure 1. Seven were weld joints and the remaining two were free joints corresponding to the tibiofemoral (TF) and patellofemoral (PF) articulations.

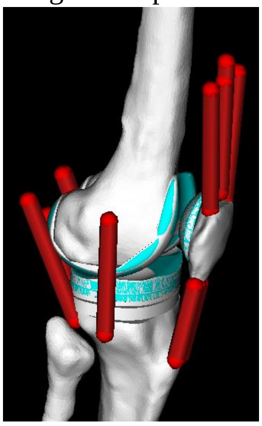


Figure 1: Knee model

The second step was inverse kinematics (IK), which was performed using experimental marker data acquired in a motion trial. Markers must be located on the body segments of the model consistent with

[*] margarida@dem.uminho.pt
[†] pflores@dem.uminho.pt
[‡] jonny8gt@ufl.edu
[§] fregly@ufl.edu

their physical locations during the motion trial. This step may also require a scaling procedure, which can be carried within OpenSim. The IK tool analyzes each time step and computes the generalized coordinates that places the model in a pose compatible with the experimental marker locations for that time step. At the end of an IK task, OpenSim generated a motion file containing the model's generalized coordinates over all time frames.

The next task dealt with contact modeling. In this step, a geometrical description of the contact surfaces has to be defined as well as an appropriate constitutive contact law to compute intra-joint contact forces. In OpenSim, simple geometries can be modeled as planes or spheres. In turn, if more accurate geometries are demanded, triangular mesh volumes can be utilized. Each mesh has to represent a closed manifold and must be stored as an .obj file. To model TF and PF contact, *i.e.*, two contact pairs, three triangular meshes were incorporated into the knee model, corresponding to the femoral component, tibial insert and patellar button. For the contact law, we utilized OpenSim's elastic foundation model (EFM). Material properties, namely the stiffness, damping and friction, were assigned to both contact pairs. To calculate contact loads, we ran an inverse dynamics analysis using input joint motions generated by a previously inverse kinematics analysis.

The fourth and final step consisted of including the principal muscles and associated tendons responsible for the desired kinematics of the model. Thus, fourteen musculotendinous actuators were added to the model using an origin, an insertion, and occasionally via-points. For each Hill-type muscle-tendon model, several parameters were set, including maximum isometric force, optimal fiber length, tendon slack length, and pennation angle. To estimate muscle activations, OpenSim offered two different methods: static optimization (SO) and computed muscle control (CMC). SO is an extension to inverse dynamics that resolves the net joint moments into individual muscle forces at each instant in time. In contrast, CMC computes a set of muscle excitation levels that will drive the generalized coordinates of the dynamic model toward the IK trajectory. Since CMC combines SO and forward dynamics (FD) in the same simulation, CMC was the method adopted in this study, though it is computationally costlier than SO.

The results obtained thus far suggest that combining dynamic musculoskeletal models with articular contact models is still a challenging problem requiring further research effort. Therefore, modeling capabilities of available computational multibody tools require investigation, improvement and validation, which was the main purpose of the present work.

Acknowledgements

Financial support provided by *Fundação para a Ciência e a Tecnologia* (FTC), under the projects DACHOR (MIT-Pt/BSHHMS/0042/2008) and BIOJOINTS (PTDC/EME-PME/099764/2008), and by the National Institutes of Health (R01 EB009351), is acknowledged. The first author also thanks FCT for her PhD grant SFRH/40164/2007.

References

[1] D.T. Felson, S.Z. Yanovski, G. Ateshian, L. Sharma, J.A. Buckwalter, K.D. Brandt. Local Bioechanical Factors. In: D.T. Felson (Conference Chair), Osteoarthritis: new insights. Part 1: The disease and its risk factors. Annals of Internal Medicine, 133(8), pp. 637-639, 2000.

[2] Y.C. Lin, J.P. Walter, S.A. Banks, M.G. Pandy, B.J. Fregly, *Simultaneous prediction of muscle and contact forces in the knee during gait*, Journal of Biomechanics, **43** (5), pp. 945-952, 2010.

[3] A. Seth, M. Sherman, J.A. Reinbolt and S.L. Delp, *OpenSim: a musculoskeletal modeling and simulation framework for in silico investigations and exchange*, Procedia IUTAM, **2** (22), pp. 212-232, 2011.

An Enhanced Cylindrical Contact Force Model for Multibody Dynamics Applications

Cândida Malça*
Polytechnic Institute of Coimbra
R. Pedro Nunes,
3030-199 Coimbra, Portugal

Jorge Ambrósio†
IDMEC-IST,
Tech. Univ. of Lisbon
Av. Rovisco Pais 1,
1049-001 Lisboa, Portugal

Amilcar Ramalho‡
Dept. of Mechanical Engng.,
University of Coimbra
R. Luís Reis dos Santos,
3030-788 Coimbra, Portugal

Keywords: Continuous contact force, Clearance joints, Roller-chain contact.

Introduction

The penalty formulations that describe the contact forces between different bodies of a mechanical system use the penetration as a representation of the local deformation. The dynamic analysis of the system is conducted assuming explicit or implicit relations between contact force and penetration, dependent on the geometries and material properties of the contacting points. Most of the cylindrical contact force models are based on the Hertz pressure distribution, exhibiting the same restrictions of the Hertz elastic contact theory, which prevent them from being used with conformal contact conditions often observed for low clearances. Furthermore, the existing cylindrical contact models represent the contact force as an implicit function of the penetration with logarithmic expressions, which pose some limitations in their use [1]. An alternative analytical cylindrical contact force model that describes the contact force as an explicit function of the penetration is proposed here. The new enhanced cylindrical contact force model is based on the Johnson contact model and complementary finite element analysis being valid for internal and external cylindrical contact. It is shown here that, within the domain of validity of the Johnson contact force model, the forces predicted with the proposed model are well correlated with reference models. The performance of the proposed model is demonstrated with the analysis of a multibody slider-crank mechanism, in which one of the joints exhibits mechanical clearances, and a roller-chain drive in which a generalized cylindrical clearance joint model is used to represent the contact between the roller and all regions of the sprocket [2].

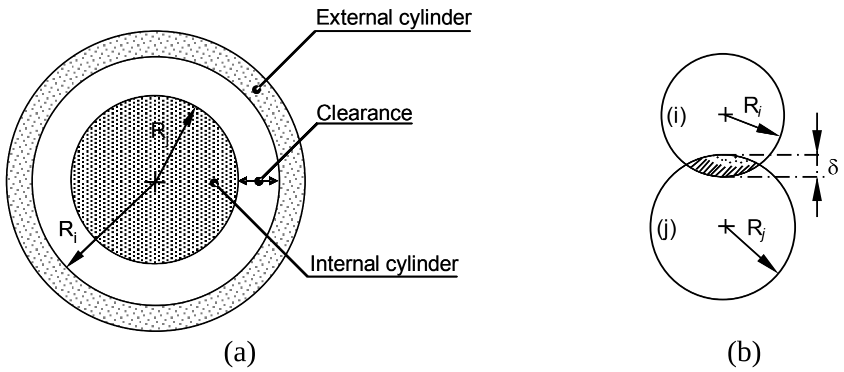


Figure 1: Dimensional characteristics and definitions for cylindrical contact: (a) internal contact; (b) external contact.

* Email: candida@isec.pt
† Email: jorge @dem.ist.utl.pt
‡ Email: amilcar.ramalho@dem.uc.pt

Enhanced Cylindrical Contact Law

The deformations δ are obtained with the Johnson cylindrical contact model for a wide number of combinations of loads, in the range of $1\text{N/mm} < f_n < 1000\text{N/mm}$, with material properties, in the range of $0.1 \le \nu \le 0.5$ and $20.7\text{GPa} < E < 10000\text{GPa}$, and geometric properties, in the range of $50\mu\text{m} < \Delta R < 10\text{mm}$ for internal contact and $5\text{mm} < \Delta R < 500\text{mm}$, for external contact. With the collection of results, the best correlation between the Johnson contact model and the new enhanced model is obtained in the form

$$f_n = \frac{(a\ \Delta R + b) L E^*}{\Delta R} \delta^n \left[1 + \frac{3(1-c_e^2)}{4} \frac{\dot{\delta}}{\dot{\delta}^{(-)}}\right] \tag{1}$$

in which δ is the indentation, accounting for the contribution of both cylinders, which is assumed to be measured at a point distant enough from the contact point, $E^* = E/2(1-\nu^2)$ is the composite modulus, assuming materials with similar elastic modulus, E, and Poisson coefficients, υ, and where for internal contact $\Delta R = R_i - R_j$ and for external contact $\Delta R = R_i + R_j$ are the radial clearance between the two contacting bodies. Furthermore,

$$a = \begin{cases} 0.49 & \text{internal} \\ 0.39 & \text{external} \end{cases}; \quad b = \begin{cases} 0.10 & \text{internal} \\ 0.85 & \text{external} \end{cases}; \quad n = \begin{cases} Y \Delta R^{-0.005} & \text{internal} \\ 1.094 & \text{external} \end{cases} \tag{2a}$$

In Equation (2a) the constant Y reflects the fact that for internal contact it is not possible to find a single expression to obtain a fit with good correlation between the Johnson and exponential fit function for the complete range of clearances ΔR. The best fit is obtained with the constant Y given by

$$Y = \begin{cases} 1.56\left[ln(1000\,\Delta R)\right]^{-0.192} & \text{if } \Delta R \in [0.005, 0.750[\ mm \\ 0.0028\,\Delta R + 1.083 & \text{if } \Delta R \in [0.750, 10.00[\ mm \end{cases} \tag{2b}$$

The relation between the Enhanced contact model proposed here and the Johnson contact model can be appraised in Figure 2 for particular cases of internal and external cylindrical contact.

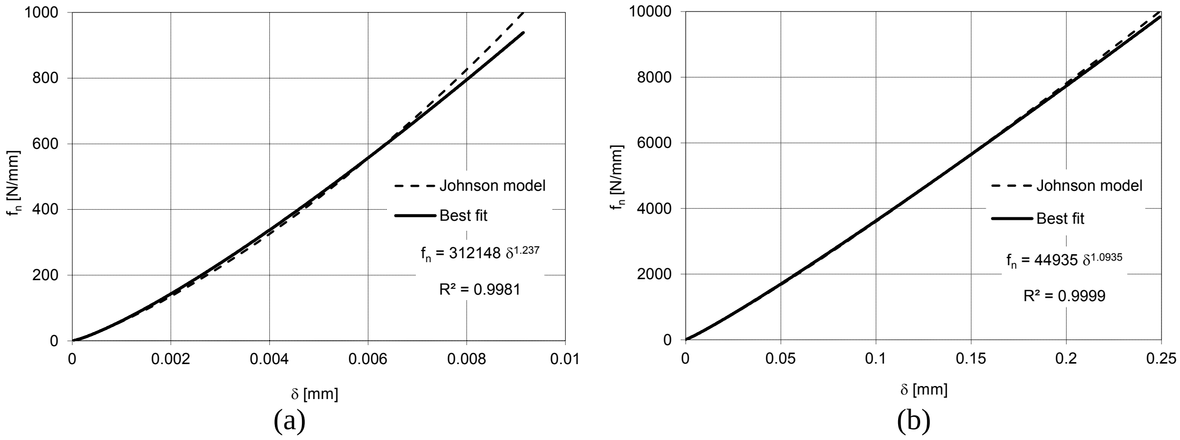


Figure 2: Best fit of the normal force as a function of the penetration with respect to the Johnson contact model for (a) internal contact with ΔR =50 μm; and (b) external contact with ΔR =140 mm, assuming E=207 GPa, υ=0.3 in both cases.

References

[1] C. Pereira, A. Ramalho, J. Ambrósio, A *Critical Overview of Internal and External Cylinder Contact Force Models*, Nonlinear Dynamics, **63**(4), pp. 681-697, 2011.

[2] P. Flores, J. Ambrósio, J. Pimenta Claro and H. Lankarani, Kinematics and Dynamics of Multibody Systems with Imperfect Joints, Springer, Dordrecht, The Netherlands, 2008.

Modelling and simulating the motion of a wire in a tube

J. P. Meijaard*
Olton Engineering Consultancy
Enschede, The Netherlands

Keywords: Guided wire, dynamics, finite-elemen modelling.

Introduction

This presentation reports on continuing research into the modelling and simulation of a fl xible wire that is guided by an enclosing tube. Both the wire and the tube have approximately a cross-section bounded by circles, but their centre lines can be sinuous curves in space. The input motion and forces are at one end of the wire, called the base or the proximal side, and the motion and forces at the other end, called the tip or the distal side, constitute the required output. Owing to the fl xibility and non-linear effects of contact and friction, the transmission of motion and force from one end to the other is rather indirect, which causes problems in the precise control. In many cases, tip feedback can be used, but in cases in which the tip is inaccessible, collocated control at the base has to be used. Examples of applications can be found in engineering as in the dynamics of drillstrings [1] and medical applications such as catheters [2] and glass micropipettes guided in PTFE tubes [3]. The purpose of the modelling and simulation of these systems is a prediction of their behaviour, a better understanding of this behaviour and improvements of their designs and ways of operation.

Model

The wire and the tube are both modelled as beams that can interact through contact. Both are discretized with two-node finit elements. The wire is highly fl xible and is retained by contact with the tube, which is stiffer, or even rigid, and can furthermore be supported by an elastic foundation. The contact in normal direction is described by a penalty approach, which assigns a finit stiffness and a damping to the contact. Friction according to a smoothed friction law can also be present. The contribution of the distributed contact force to the virtual work expression is integrated by a three-point Lobatto quadrature (Simpson's rule), which can integrate third-order polynomials exactly, where the end points of an element of the wire and a point in the middle of the element are taken as integration points. This choice also ensures that, except possibly at the ends due to imposed boundary conditions, there are as many integration points as there are degrees of freedom in a lateral direction of the wire, so for a large contact stiffness, all integration points can remain in contact with the tube wall. The normal force between the wire and the tube is a non-linear function of the indentation with C^1-continuity with a simple jump discontinuity in the second derivative, so a local quadratic convergence rate of a Newton–Raphson iteration to fin an equilibrium point can be expected.

Simulation

The equations of motion of the system are generated with the aid of the fl xible multibody code SPACAR [4] with a user routine added to it for the specificatio of the contact forces between the wire and the

*Email: j.p.meijaard@olton.nl

tube. The resulting equations are integrated with the standard explicit fourth/fift order Runge–Kutta–Fehlberg method [5] with a variable step size. With proper scaling of the problem and a judicious choice of the error bounds, this integration method proved to give reliable simulations.

Conclusion

Today's computing power makes it possible to simulate this kind of system with a general-purpose multibody code. The models can be more detailed than was possible in special-purpose programs in 1993 [1, 2]. Further improvements can be expected from the use of implicit integration methods, especially for cases in which the motion is almost quasistatic during large parts of the motion. Moreover, comparisons with experimental test results are planned.

References

[1] J. D. Jansen, *Nonlinear dynamics of oilwell drillstrings*, doctoral dissertation, Delft University Press, Delft, 1993.

[2] H. ten Hoff, *Scanning mechanisms for intravascular ultrasound imaging: a flexible approach*, doctoral dissertation, Universiteitsdrukkerij Erasmus Universiteit Rotterdam, Rotterdam, 1993.

[3] S. B. Kodandaramaiah, S. Malik, M. J. Dergance, E. S. Boyden, C. R. Forest, *Design and performance of telescoping micropipette arrays for high throughput* in vivo *patch clamping*, pp. 246–249 in 2010 Annual Meeting, Vol. 50, American Society for Precision Engineering, Raleigh NC, 2010.

[4] J. B. Jonker, J. P. Meijaard, *SPACAR—computer program for dynamic analysis of flexible spatial mechanisms and manipulators*, pp. 123–143 in *Multibody systems handbook*, ed. W. Schiehlen, Springer-Verlag, Heidelberg, 1990.

[5] E. Fehlberg, *Klassische Runge-Kutta-Formeln vierter und niedriger Ordnung mit Schrittweiten-Kontrolle und ihre Anwendung auf Wärmeleitungsprobleme*, Computing, **6**, pp. 61–71, 1970

A Planar Multibody Lumbar Spine Model for Dynamic Analysis

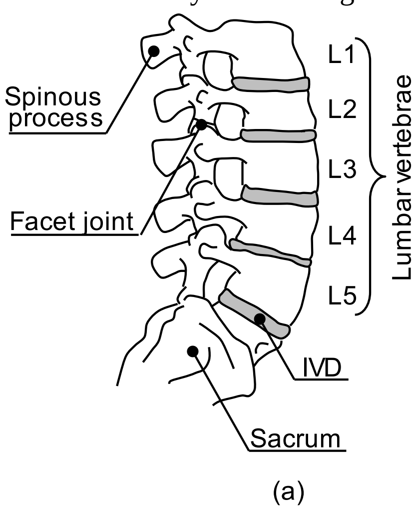
Sara Tribuzi Morais*, Paulo Flores†, J.C. Pimenta Claro‡
CT2M / Departamento de Engenharia Mecânica, Universidade do Minho
Campus de Azurém, 4800-058 Guimarães, Portugal

Keywords: Biomechanics, spine, low back pain, multibody lumbar model.

Extended Abstract

The human vertebral column is a complex system, which main functions are: (*i*) transferring weight and the resultant bending moments of the head, trunk, and any weights being lifted to the pelvis; (*ii*) allowing physiologic relative motions between the aforementioned anatomical elements; (*iii*) protecting the spinal cord from damaging actions produced by physiologic movements and/or trauma; and (*iv*) providing the attachment points for muscles and the ribcage [1]. Thus, the spine has been identified as one of the most susceptible human body parts to suffer traumatic and degenerative pathologies. The World Health Organization and the Portuguese Ministry of Health report chronic rheumatic diseases as the most frequent group of illnesses in developed countries, and low back pain (LBP) as the most common spinal disorder in this group. Lumbar disc degeneration (LDD) was identified as one of the main causes of LBP [2]. A frequently applied solution for LDD is intervertebral fusion, in which two vertebrae are fused together. However, this medical procedure has a consequence of limiting the spinal range of motion due to the elimination of the intervertebral disc (IVD) function.

The spine is composed by 33 vertebrae (24 articulating and 9 fused) and can be divided in five regions: cervical, thoracic, lumbar, sacral and coccygeal. However, this study will be focused on the lumbar spine region, where the structure has to support the highest network loads. The main objective is to build up a general two-dimensional multibody methodology able to model, analyze and simulate the dynamic behavior of the human lumbar spine system – in terms of vertebrae, ligaments and the IVD itself – as a basis for future applications in the study of pathological and non-pathological situations, as well as in the evaluation of load and displacement performance under work of the IVD, suitable for the analysis and design of substitution implants.



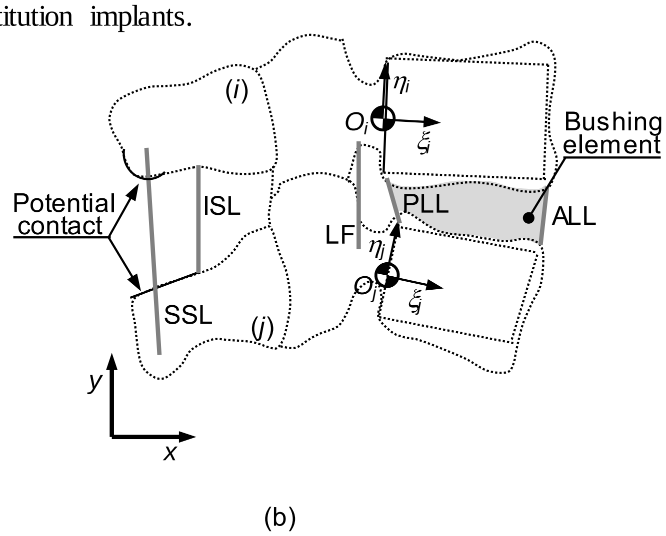


Figure 1: (a) Schematic representation of the lumbosacral spine; (b) Multibody model of the consecutive vertebrae with their fundamental elements (bodies, ligaments, bushing elements, potential contact areas).

* Email: a49993@alunos.uminho.pt
† Email: pflores@dem.uminho.pt
‡ Email: jcpclaro@dem.uminho.pt

The multibody spine model considered in the present work includes six rigid bodies, representing the sacral structure and five lumbar vertebrae, as it is illustrated in Figure 1(a). The fundamental geometric data used to build this multibody model is based on the material available in the literature, namely, in the works by Panjabi [3] and Monteiro [4]. For the definition of the vertebra it is important to define a body-local coordinate system of each element that is located at the body center of mass, as Figure 1(b) shows. The bodies of the system are considered to be rigid and kinematically unconstrained, which allows the relative flexion-extension motion and some compression-tension in the IVDs. Bushing elements of linear and rotational types are used to model the IVDs, which act as a spring-damper components. For this purpose, translational and rotational stiffness and damping properties are included in a penalty approach to model the bushing elements [4]. In turn, the ligaments are modeled as simple nonlinear spring, which work in two different regimens according to the ligament strain during relative motion. The main ligaments considered in the spine model are shown in Figure 1(b).

Furthermore, possible contact events can take place between adjacent spinous processes. This contact scenario is modeled as a contact between a spherical surface and a planar surface, as it is illustrated the Figure 1(b). The contact-impact forces are computed by employing a continuous contact force model, based on the elastic Hertz theory, together with a dissipative term associated with the internal damping [5]. The selection of an appropriate contact force model is of paramount importance in the measure that the contacting bodies are characterized by high damping properties.

The formulation of multibody system dynamics adopted in this work uses the generalized Cartesian coordinates and the Newton-Euler approach to derive the equations of motion. This formulation results in the establishment of a mixed set of partial differential and algebraic equations, which are solved in order to simulate the dynamic behavior of multibody systems. The Newton-Euler approach is very straightforward in terms of assembling the equations of motion and providing all joint reaction forces. Additionally, the equations of motion are solved by using the Baumgarte stabilization technique with the intent of keeping the constraint violations under control [6].

The forces developed at the potential contact areas located at the spinous processes, the resistive forces and moments associated with the bushing elements and the force produced in the ligaments are introduced into the system's equations of motion as external generalized forces. Finally, results in terms of the dynamic simulations of the multibody spine model described above are used to demonstrate the accuracy and efficiency of the presented approach, to discuss the main assumptions and procedures adopted, and compare the outputs with those available in the literature for similar conditions.

Acknowledgement

This work is part of the 'NPmimetic – Biomimetic nano-fiber-based nucleus pulposus regeneration for the treatment of degenerative disc disease' research project, funded by the European Commission under the Seventh Framework Programme (FP7).

References

[1] A.A. White, M.M. Panjabi, *Clinical biomechanics of the spine*, Lippincott, Philadelphia, 1990.

[2] K. Luoma, H. Riihimäki, R. Luukkonen, R. Raininko, E. Viikari-Juntura, A. Lamminen, *Low Back Pain in Relation to Lumbar Disc Degeneration*, Spine, **25**(4), pp. 487-492, 2000.

[3] M.M. Panjabi, V. Goel, T. Oxland, K. Takata, J. Duranceau, M. Krag, M. Price, *Human lumbar vertebrae. Quantitative three-dimensional anatomy*, Spine, **17**(3), pp. 299-306, 1992.

[4] N. Monteiro, *Analysis of the Intervertebral Discs Adjacent to Interbody Fusion using a Multibody and Finite Element Co-Simulation*, MSc Thesis, Mechanical Engineering Department, Technical University of Lisbon, 2009.

[5] P. Flores, M. Machado, M.T. Silva, J.M. Martins, *On the continuous contact force models for soft materials in multibody dynamics*, Multibody System Dynamics, **25**, 357-375, 2011.

[6] P.E. Nikravesh, *Computer-aided analysis of mechanical systems*, Prentice-Hall, 1988.

EUROMECH Colloquium 524
Multibody system modelling, control
and simulation for engineering design

February 27–29, 2012
University of Twente
Enschede, Netherlands

Sub-System Global Modal Parameterization for efficient inclusion of highly nonlinear components in Multibody Simulation

Frank Naets*
KULeuven

Wim Desmet†
KULeuven

Celestijnenlaan 300, 3001 Heverlee, Belgium

Keywords: Sub-System, Global Modal Parameterization, nonlinear model reduction.

Introduction

In recent years, multibody models have become increasingly complex. In order to accurately capture the behaviour of the system, the inclusion of nonlinear effects is essential. In practice there are two techniques in use to include nonlinear components:

- use approximate lumped force-elements [1]
- use fully nonlinear finite element model (possibly in co-simulation).

Unfortunately, using approximate lumped components might lead to an over-simplification of the nonlinearities and usually doesn't allow inclusion of internal dynamic effects of the components because the component is included as a force-element. On the other hand, a co-simulation with a nonlinear finite element model leads to accurate results, but greatly increases the computational load.

In order to meet these two drawbacks, the authors propose a novel reduction technique for nonlinear components, namely the *Sub-System Global Modal Parameterization (SS-GMP)*. This techniques allows the reduction of a complex nonlinear FE model into a compact dynamical model which can be included in a MBD-model.

Sub-System Global Modal Parameterization for components

Sub-System Global Modal Parameterization has been previously introduced as a technique for system-level reduction of flexible multibody systems [2, 3]. In this approach, a minimal set of rigid degrees-of-freedom (DOFs) is used to dynamically parameterize the configuration of the system.

In this work, the formalism is extended to include general degrees-of-freedom, which don't have to be rigid motion DOFs. By, for example, also including flexible DOFs to parameterize the motion of a component, nonlinear stiffening phenomena can be simulated accurately.

To facilitate the use of a SS-GMP reduced component in a general multibody model, a new set of generalized DOFs is proposed for the SS-GMP model. Previously, the configuration of the floating reference frame, the parameterization DOF and flexible deformation participation factors served as reduced DOFs. Unfortunately, these DOFs lead to a very complex description of the connections with other components. In this work, the SS-GMP is formulated such that the interface DOFs of the reduced component are used for the description of the configuration of the SS-GMP model, augmented by flexible deformation participation factors.

*Email: frank.naets@mech.kuleuven.be
†Email: wim.desmet@mech.kuleuven.be

Numerical Validation

In order to demonstrate the proposed approach, a highly flexible slider-crank mechanism is considered. During the motion of this system, the crank exhibits large deformation. A comparison is made between a full nonlinear model, a FFR-CMS model and a model in which the crank is reduced with the SS-GMP approach. The model with the SS-GMP component shows good accuracy whereas the FFR-CMS model fails.

Acknowledgements

The research of Frank Naets is funded by a Ph.D. grant from the Institute for the Promotion of Innovation through Science and Technology in Flanders (IWT-Vlaanderen).

References

[1] J. Ambrosio, P. Verissimo, *Improved bushing models for general multibody systems and vehicle dynamics*, Multibody Syst. Dyn., **22**(4), pp. 341–365, 2011.

[2] F. Naets, G.H.K. Heirman, W. Desmet, *Sub-System global Modal Parameterization for efficient simulation of flexible multibody systems*, International Journal for Numerical Methods in Engineering, ACCEPTED, 2011.

[3] Naets, F., Heirman, G.H.K., Desmet, W. *A novel approach to real-time flexible multibody simulation: Sub-System Global Modal Parameterization*, in Proceedings of the ASME 2011 International Design Engineering Technical Conferences & Computers and Information in Engineering Conference, Washington DC, USA, 2011.

State Estimation Using Multibody Models and Unscented Kalman Filters

Roland Pastorino*
Javier Cuadrado
University of La Coruña
Mendizábal, s/n 15403 Ferrol, Spain

Dario Richiedei†
Alberto Trevisani
DTG, Universitá degli Studi di Padova
Stradella S. Nicola 3 - 36100 Vicenza, Italy

Keywords: nonlinear state observers, multibody models, unscented Kalman filters.

1 Introduction

Over the last years, state estimation in mechanical systems has gained interest with the recent development of real-time state estimation using MBSs (Multibody Systems). In practice, the knowledge of the system's states allows to improve the closed-loop performances of it, reducing the use of expensive sensors by replacing them with virtual sensors, and finally improving reliability by making the system fault tolerant.

On the one hand, numerous works address the synthesis of optimal observers for linear mechanical systems through the KF (linear Kalman Filter). On the other hand, when nonlinear mechanical systems are considered, such as MBSs, only sub-optimal approaches based on the LKF (Linearized KF) have usually been adopted to ensure high-frequency and hard real-time estimation. Indeed, up to now, the use of other types of nonlinear observers using MBSs has only been investigated marginally. The lack of such observers in this field of engineering is mainly due to the difficulty in performing fast integration of the nonlinear equations of motion for MBSs, which usually involve high frequency dynamics and severe nonlinearities. In [1], it is shown how the improvements in multibody dynamics raise the possibility to employ complex models in real-time state observers. The estimation was performed through the EKF (Extended KF) in its continuous form (also known as EKBF (Extended Kalman-Bucy Filter)). Generally speaking, the EKF is the most widely used algorithm for nonlinear estimation. It makes use of the nonlinear system model to perform the time update, but propagates the mean and covariance of the states through the linearized model. As long as the system remains linear on the timescale of the updates, the linearization error and consequently the estimation error stay small and accuracy is guaranteed. However, when nonlinearities are severe, EKF often gives unreliable or divergent estimates. On top of that, the linearization requires a Jacobian matrix which could either be difficult to calculate or not exist. Implementation difficulties are particularly relevant if the system model is modeled by DAE (Differential Algebraic Equations) as it is common in MBSs.

Recent developments in Kalman filtering algorithms make possible to overcome part of the EKF shortcomings. The SPKFs (Sigma-Point Kalman Filters), also called LRKFs (Linear Regression Kalman Filters), use a set of deterministically calculated weighted samples, also named sigma-points or even regression points [2]. This set has to capture at least the first and second order moments of the actual state probability distribution. Each sigma-point is then individually propagated through the nonlinear system equations. The posterior statistics are approximated using simple functions involving the transformed sigma-points. Thereby, this approximation, that does not require the calculation of a Jacobian matrix, is more accurate than the EKF linearization. Different sigma-point set definitions lead to different filter characteristics that allow to give priority to estimation accuracy or to computational efficiency. The

*Email: rpastorino@udc.es
†Email: dario.richiedei@unipd.it

most famous variants are the UKF (Unscented KF), the CDKF (Central Difference KF) [2], the SSUKF (Spherical Simplex UKF) [3] and their respective square-root forms which allow to improve the numerical stability. Therefore, a natural approach to overcome the EKF problems when employing multibody models would be to use SPKFs. To the best of the authors' knowledge, UKFs have never been applied to the estimation of multibody models.

2 Using multibody models in Unscented Kalman filters

The aim of this work is to present the first implementation of UKFs using multibody models and to discuss their performances. The objective is not to define all the possible ways of using multibody models in UKFs but to present the most relevant. As the multibody formulation is involved, the state-space reduction method known as matrix-R method is employed to convert the DAE of the multibody model into an ODE with a dimension equal to the number of degrees of freedom of the system, as in [1]. Both implicit and explicit integration schemes have been used. The filters' state vector used in this work contains the independent coordinates (in order to minimize the computational cost) and its first time derivative. For the sake of simplicity, additive white Gaussian noise has been considered. The selection of such a multibody formulation and such a state vector leads to a straight-forward implementation of the observer with a direct physical significance that is valid for all the SPKFs. Performance comparisons between UKFs, SSUKFs, their square-root forms and EKBFs have been carried out in simulation on a 5-bar linkage. The mechanism's parameters have been obtained from an experimental 5-bar linkage and the sensor's characteristics from off-the-shelf sensors to reproduce a realistic simulation.

3 Conclusion

This work presents the development of nonlinear state observers based on the UKF family, that use multibody models. Although these filters outperform the accuracy of the EKBF by using better approximations of system nonlinearities, the computational cost related to the sigma-points is high. As a consequence, the choice of the most suitable filter depends on the application requirements and is a trade off between estimation accuracy and computational efficiency. Future works will be devoted to deeper investigations on a wider class of multibody formulations and filters, and to the experimental validation of the proposed observers.

Acknowledgements

The authors would like to thank the Spanish Ministry of Science and Innovation and ERDF funds through the grant TRA2009-09314 for its support in this research.

References

[1] J. Cuadrado, D. Dopico, A. Barreiro, E. Delgado, *Real-time state observers based on multibody models and the extended Kalman filter*, Journal of Mechanical Science and Technology, **23**(4), pp. 894–900, 2009.

[2] Julier, S.J., Uhlmann, J.K., *Unscented filtering and nonlinear estimation*, in Proceedings of the IEEE, **92**(3), pp. 401–422, 2004.

[3] Julier, S.J., *The spherical simplex unscented transformation*, in Proceedings of the American Control Conference, (3), pp. 2430–2434, 2003.

Application of the Non-Smooth Contact Dynamics method to the analysis of historical masonries subjected to seismic loads

Quintilio Piattoni*
Polytechnic University of Marche
Via Brecce Bianche
60131, Ancona, Italy

Giovanni Lancioni†
Polytechnic University of Marche
Via Brecce Bianche
60131, Ancona, Italy

Stefano Lenci‡
Polytechnic University of Marche
Via Brecce Bianche
60131, Ancona, Italy

Enrico Quagliarini§
Polytechnic University of Marche
Via Brecce Bianche
60131, Ancona, Italy

Keywords: NSCD method, historical masonries.

Introduction

Within the framework of the multi-body systems, the Non-Smooth Contact Dynamics (NSCD) method [1] has been applied to various research fields like tensegrity structures [2], fracture [3], granular materials [4] and masonries [5, 6]. The latter works, in particular, used the LMGC90 software [7], that is dedicated to the application of NSCD method to systems with a large number of bodies with various contact interactions. This software has been further used in [8, 9] to investigate the dynamical behaviour of *historical* masonries, which constitute an important part of the cultural heritage. The knowledge of their dynamical behaviour under seismic loads is useful to assess their safety level and to choose the most appropriate retrofitting method. The monuments made of stone blocks not bonded by mortar (e.g. the columns of the ancient temples) and the masonries with mortar of poor quality may be considered discontinuous structures, and naturally fall within the realm of NSCD, as they are multi-body systems made of rigid bodies subjected to sticking, slipping, impacts and free-fly.

Object and aims of the research

In this research the dynamical behaviour of the masonries of the S. Maria in Portuno's Church [10] has been investigated by means of LMGC90 software. At the present time the considered church has a unique nave but ruins of ancient aisles have been found, confirming the hypothesis of a pre-existing medieval church constituted by a nave and two aisles. A traumatic event may have been the cause of the collapse of the original aisles of the church; subsequently the church has been rebuilt with a unique nave. Between the various hypotheses, an earthquake dated in 1269 A.D. seems to be consistent with the chronological events and with the collapse of the original aisles of the church. Starting from the geometrical model that represents the original configuration of the church, several seismic analyses have been performed with the following aims: (i) to analyse the influence of the contact parameters, masonry connections and block sizes on the dynamic response under seismic loadings; (ii) to obtain numerical results which confirm the past damage of the church due to an earthquake.

* Email: quintiliopiattoni@libero.it
† Email: g.lancioni@univpm.it
‡ Email: lenci@univpm.it
§ Email: equaglia@mta01.univpm.it

Modelling and first results

The church has been modelled as a collection of rigid bodies with complex geometries. Since the joints between blocks are constituted by poor mortar, their strength and cohesion has been neglected. Thus the contact between blocks has been modelled by considering only the dry friction law.
An approach based on macro-elements (nave, façade, aisles, apses) has been used to reduce the system degree of freedom: a refined mesh has been used only for singular macro-elements, and a coarse mesh has been used for the rest of the church.
Parametric analyses have been conducted, considering a real earthquake accelerogram applied to the supporting base of the three-dimensional church. The performed analyses have allowed to assess (i) the sensitivity of the dynamical behaviour of the investigated masonries to the changes of the contact parameters, (ii) the effectiveness of the masonry connections, and (iii) the effect of the block sizes. The on-going developments, still in progress, concern the analysis of the dynamical behaviour of the macro-elements of the church with different contact laws, different contact parameters and with different real seismic accelerograms. These analyses will allow to highlight the non-smooth dynamical behaviours of the considered masonries and to investigate the hypothesis of a past damage due to an earthquake. Comparisons between the simulated damages and the actual ones [10] have been performed.

References

[1] M. Jean, *The non-smooth contact dynamics method*, Computer Methods in Applied Mechanics and Engineering, **177**(3-4), pp. 235–257, 1999.

[2] S. Nineb, P. Alart, D. Dureisseix, *Approache multi-échelle des systèmes de tenségrité*, Revue Européenne de Mécanique Numérique, **15**(1-3), pp. 319–328, 2006.

[3] F. Dubois, *Fracturation as a non smooth contact dynamic problem*, in Proceedings of the VIII International Conference on Computational Plasticity, COMPLAS VIII, CIMNE, International Center for Numerical Methods in Engineering, Ed. Onate, E., Owen, D.R.J., Barcelona, Spain, 2005.

[4] M. Renouf, F. Dubois, P. Alart, *A parallel version of the non-smooth contact dynamics algorithm applied to the simulation of granular media*, Journal of Computational and Applied Mathematics, **168**(1-2), pp. 375–382, 2004.

[5] B. Chetouane, F. Dubois, M. Vinches, C. Bohatier, *NSCD discrete element method for modeling masonry structures*, International Journal for Numerical Methods in Engineering, **64**(1), pp. 65–94, 2005.

[6] V. Acary, M. Jean, *Numerical modeling of three dimensional divided structures by the non smooth contact dynamics method: application to masonry structure*, pp. 211–222 in Proceedings of the Fifth International Conference on Computational Structures Technology 2000, Civil-Comp Press, Ed. Topping, B.H.V., Edinburgh, UK, 2000.

[7] F. Dubois, M. Jean, *The non smooth contact dynamic method: recent LMGC90 software developments and application*, Lecture Notes in Applied and Computational Mechanics, **27**, pp. 375–378, 2006.

[8] A. Rafiee, M. Vinches, C. Bohatier, *Application of the NSCD method to analyse the dynamic behaviour of stone arched structures*, International Journal of Solids and Structures, **45**(25-26), pp. 6269–6283, 2008.

[9] A. Rafiee, M. Vinches, C. Bohatier, *Modelling and analysis of the Nîmes arena and the Arles aqueduct subjected to a seismic loading using the Non-Smooth Contact Dynamics method*, Engineering Structures, **30**(12), pp. 3457–3467, 2008.

[10] S. Lenci, E. Quagliarini, Q. Piattoni, *Ingegneria e archeologia. Dalla planimetria di scavo alla terza dimensione dell'elevato. La chiesa paleocristiana di S. Maria in Portuno di Corinaldo (AN)*, Aracne Editrice, Roma, 2011 [in Italian].

Investigation of dynamic behaviour of inverted pendulum attached using fibres at non-symmetric harmonic excitation

Pavel Polach*
VZÚ Plzeň s.r.o.
Tylova 46, Plzeň, Czech Republic

Michal Hajžman†
VZÚ Plzeň s.r.o.
Tylova 46, Plzeň, Czech Rep.

Keywords: Inverted pendulum, fibres, vibration.

Introduction

Replacing rigid elements of manipulators or mechanisms by flexible fibres can be advantageous in the achievement of a lower moving inertia, which leads to a higher machine speed. The possible cable modelling approaches should be tested and their suitability verified in order to create efficient mathematical models of cable-based mechanisms. In this paper the motion of the inverted pendulum driven by two fibres attached to a frame (see Figure 1), which is the simple representation of a typical cable manipulator, is investigated using an in-house software created in the MATLAB system and using the **alaska** simulation tool. The influence of the fibres' motion (in case of their simultaneous harmonic excitation) on the pendulum motion was investigated in [1], a case of non-symmetric harmonic excitation is investigated in this paper.

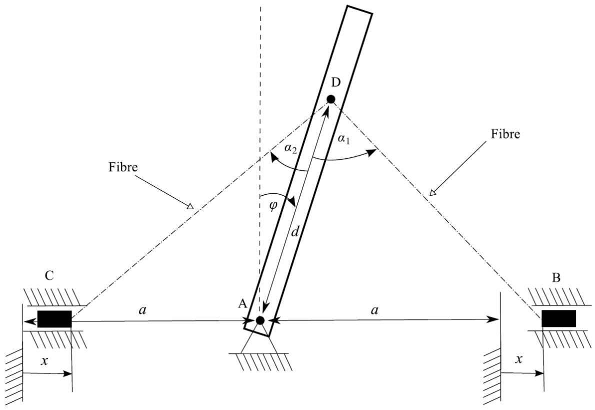


Figure 1: Inverted pendulum actuated by the fibres.

Inverted pendulum and simulations results

The cable (fibre, wire etc.) modelling [2] should be based on considering the cable flexibility and the suitable approaches can be based on the flexible multibody dynamics (see [3]). As an example for the investigation of fibres' behaviour an inverted pendulum, which is supported and driven by two fibres (see Figure 1) [1], was chosen. It was supposed that the nonlinear system of the inverted pendulum fixed with two fibres will show an unstable behaviour under specific excitation conditions. In order to create the fibres' model in the inverted pendulum models two approaches (a massless model and a point-mass model) were used. The model of the fibre based on the point-mass model with lumped point masses (corresponding to the mass of the fibre) is geometrically identical to the massless fibre model. Contrary to the massless fibre model each of the fibres has been discretized using 10 point

* Email: polach@vzuplzen.cz
† Email: hajzman@vzuplzen.cz

masses. The stiffness and the damping between the masses are determined in order to keep the global properties of the massless fibre model.
The kinematic excitation was given by function $x(t) = x_0 \sin(2\pi f t + \varphi)$, where x_0 is the chosen amplitude of motion, f is the excitation frequency and φ is the phase shift. The influence of the excitation frequency shift on the pendulum motion is investigated. Excitation in spots designated B and C is considered to be different (with chosen mutual phase shift φ) and of the same amplitude x_0 = 0.02 m. Excitation frequency f was considered in the range from 0.1 Hz to 200 Hz. Time histories and extreme values of pendulum angle and of the force in the fibres are the monitored quantities.
From obtained results it is evident that the pendulum performs motion influenced by the excitation frequency of the moving fibres (see Figure 2). When the pendulum is displaced from the equilibrium position (i.e. “upper” position) it is returned back to the equilibrium position by the tightened fibre. In reality and in the case of the point-mass models the situation may not be as clear as in the case of the massless models. The maximum pendulum angle appears at excitation frequency 5 Hz. The phase shift does not manifest itself too much in the change of the character of pendulum vibration at the given excitation frequency, but, especially at excitation frequency 10 Hz, in the maximum pendulum angle (maximum pendulum angle increases with increasing phase shift).

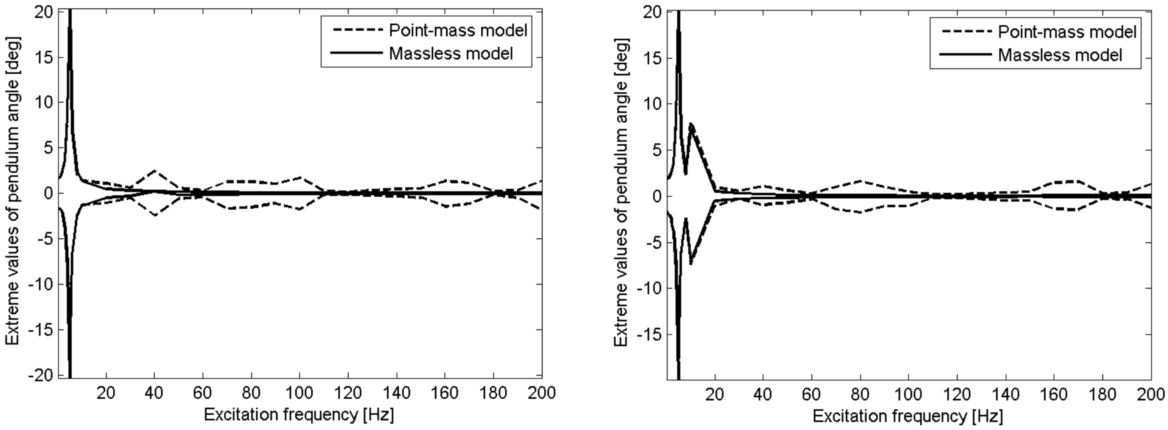


Figure 2: Extreme values of time histories of pendulum angle in dependence on the excitation frequencies – simultaneous harmonic excitation (taken from [1]) and non-symmetric harmonic excitation with 30° phase shift.

Conclusions

Two approaches to the cable modelling are presented in the paper (based on the force and on the lumped point-mass representations) and studied by means of the inverted pendulum system. These modelling approaches are advantageous mainly due to a relatively simple modelling and fundamental getting acquainted with a dynamic behaviour of mechanisms based on fibre drives. It has been demonstrated that the non-symmetric excitation of fibres does not influence the pendulum vibration to the extent that was originally supposed (unstable behaviour etc.). Experimental verification of the cable dynamics within the manipulator systems and research aimed at measuring material properties of selected fibres are considered important steps in further research.

References

[1] P. Polach, M. Hajžman, *Investigation of Dynamic Behaviour of Inverted Pendulum Attached Using of Fibres*, pp. 403-408 in Proceedings of 11th Conference on Dynamical Systems – Theory and Applications, Nonlinear Dynamics and Control, Eds. Awrejcewicz, J., Kaźmierczak, M., Olejnik, P., Mrozowski, J., Łódź, Poland, 2011.

[2] M. Hajžman, P. Polach, *Modelling of Cables for Application in Cable-Based Manipulators Design*, CD-ROM Proceedings of ECCOMAS Thematic Conference Multibody Dynamics 2011, Eds. Samin, J.-C., Fisette, P., Brussels, Belgium, 2011.

[3] A.A. Shabana, *Flexible Multibody Dynamics: Review of Past and Recent Developments*, Multibody System Dynamics, **1**(2), pp. 189-222, 1997.

Influence of external damping on phase difference measurement of a Coriolis mass-flow meter

L. van de Ridder*[1], J. van Dijk[1], W.B.J. Hakvoort[1,2], and J.C. Lötters[1,3]

[1]University of Twente, P.O. Box 217, 7500AE Enschede, The Netherlands
[2]DEMCON, Oldenzaal, The Netherlands
[3]Bronkhorst High-Tech B.V., Ruurlo, The Netherlands

Keywords: Flexible Multibody Dynamics, Coriolis Mass-Flow Meter, Modal Analysis, Damping, Mounting uncertainties.

Introduction

A Coriolis Mass-Flow Meter (CMFM) is an active device based on the Coriolis force principle for direct mass-fl w measurements independent of flui properties with an high accuracy, rangeability and repeatability [1]. The basic working principle of a CMFM is simple, a flui conveying tube is excited by an actuator to oscillate, with a low amplitude, whereby resonance frequencies are used to minimize the amount of supplied energy. A flui fl w in the vibrating tube induces Coriolis forces which affect the tube motion, changing the vibration shape. The forces and the consequent motion are proportional to the mass-fl w. Measuring the tube displacement, such that the change of its vibration shape can be measured, allows measuring the mass-fl w.

Experience shows that a problem can arise when a CMFM is placed on a nonrigid surface, resulting in a drift in the measurement. The accuracy of the sensor is thereby influence by the place and type of support. Therefore a CMFM with its connection to a rigid surface is modelled to analyse the effect of external damping and stiffness on the mass-fl w measurement.

Modelling

A fl xible multibody model [2, 3] is made based on the dimensions of an example CMFM with the modelling package SPACAR [4]. A structural representation is given in figur 1, showing the fl xible tube-window, the casing, trusses for applying forces and reading out displacements and the connection between the casing and the fi ed world.

Considering only small displacements the mass matrix M, the damping matrix C and the stiffness matrix K of the linear system can be determined. Analysing the complex eigenvalue problem $(-\omega^2 M + i\omega C + K)\vec{v} = \vec{0}$, the natural frequencies and modeshapes can be determined. The modeshapes of the excited mode and the fl w-induced mode, which occur 90 degrees out of phase, are depicted in figur 2.

A second result of the eigenvalue problem are the displacements of the tube at the location of the sensors $s1$ and $s2$, which become complex valued for a system with non-proportional damping or nonzero flui fl w. The phase difference between those two displacements is a measure of the mass-fl w [2, 3]:

$$\Delta\theta_{s1,s2} = \frac{\Im(s1)}{\Re(s1)} - \frac{\Im(s2)}{\Re(s2)} = 2\frac{\Im(s1+s2)}{\Re(s1-s2)} = S_0\dot{m} + f(\vec{d},\vec{k}) \quad (1)$$

where S_0 the measurement sensitivity, $\dot{m}$ the mass-fl w and a offset function f due to the mounting damping $\vec{d} = \{d_A, d_B\}^T$ and stiffness $\vec{k} = \{k_A, k_B\}^T$. This offset results in a mass-fl w measurement error $\dot{m}_{error} = \frac{f(\vec{d},\vec{k})}{S_0}$, where $S_0 \approx \frac{0.1}{NominalFlow}[rad]$ for the analysed CMFM for low fl ws.

*Email: L.vanderidder@utwente.nl

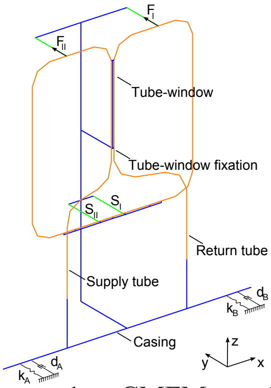


Figure 1: CMFM model with two connections to the fi ed world

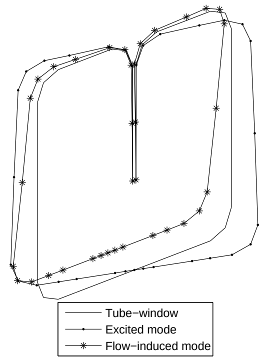


Figure 2: Modeshapes of tube-window

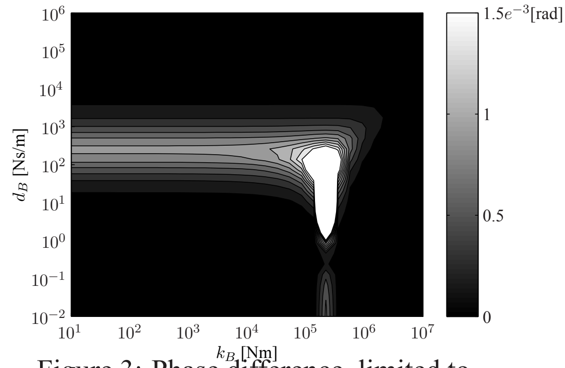


Figure 3: Phase difference, limited to $1.5e^{-3}$ rad, by changing d_B and k_B with a constant d_A, k_A and zero fl w.

Analysis

To answer the question if the mounting properties can influenc the measurement, the mounting properties of the model depicted in figur 1 are variated. For the asymmetric case, side A of the CMFM is connected to fi ed world with low damping $d_A = 0.3[\frac{Ns}{m}]$ and low stiffness $k_A = 400[\frac{N}{m}]$, but the values are varied at side B of the CMFM (figur 1) forcing asymmetry.

The calculated phase differences, equation 1, for the asymmetric variation of the mounting stiffness $k_B = [10, \ldots, 10^7]$ and damping $d_B = [0.01, \ldots, 10^6]$ are depicted in figur 3. An artifact is seen around $k_B \approx 2 \cdot 10^5$, this stiffness parameter results in a suspension frequency approximating the natural frequency of the CMFM excited mode, therefore a mounting stiffness around this value should be avoided. Below $k_B < 10^5$ the phase difference is approximately zero except around a peak at $d_B = 250$, indicating an optimum in the cross-modal damping between external damping and the tube-window vibration mode damping. Above $k_B > 10^6$ the phase is approximately zero independent of damping, so high mounting stiffness or high damping is recommended to avoid a phase difference measurement.

Conclusions

The influenc of the CMFM support on the measurement value is considered. The analysis shows that the suspension indeed affects the measurement. A suspension frequency around the actuation frequency or asymmetric damping of the CMFM casing can affect the measurement value. By applying a low frequent suspension, the unwanted sensitivity due to external damping creates a CMFM design challenge.

Besides influenc of external damping, also asymmetric tube-window damping is expected to introduce a measurement error. More research is needed to understand the fundamentals of this phenomenon.

References

[1] M. Anklin, W. Drahm and A. Rieder, *Coriolis mass flowmeters: Overview of the current state of the art and latest research*, Flow Measurement and Instrumentation, **17**(6), pp. 317–323, 2006.

[2] W.B.J. Hakvoort, J.P. Meijaard, R.G.K.M. Aarts, J.B Jonker and J.M. Zwikker, *Modeling a Coriolis Mass Flow Meter for Shape Optimization*, In procedings of the 1st Joint International Conference on Multibody System Dynamics, 2010.

[3] L. van de Ridder, *Analysis of a Coriolis Mass Flow Meter to reduce influence of external disturbances*, MSc Thesis, University of Twente, 2011.

[4] J.B. Jonker and J.P. Meijaard, *SPACAR - computer program for dynamic analysis of flexible spatial mechanisms and manipulators*, in Multibody Systems Handbook, Springer-Verlag, Berlin, pp. 123–143, 1990.

Dynamic loads in kinematically determined multibody systems

Werner Schiehlen*
Institute of Engineering and Computational Mechanics
University of Stuttgart
Pfaffenwaldring 9, 70550 Stuttgart, Germany

Keywords: Rheonomic constraints, inverse dynamics, dynamic loads

Abstract

Multibody systems with scleronomic constraints are usually characterized by one or more degrees of freedom. If rheonomic constraints are included during the design process even multibody systems without any degree of freedom may perform a prescribed motion. Rheonomic constraints are today easily available by mechatronic actuators like precisely controlled electric drives or stepping motors, respectively, providing high positioning accuracy independent on the load required. Nevertheless, the mechanical parts of a kinematically determined motion system have to be designed carefully often subject to the requirement of light weight structures.

The dynamic analysis of kinematically determined multibody systems is reduced to inverse dynamics. In the design process the dynamic loads have to be evaluated for the system elements involved.

The paper presents a systematic approach for the computation of dynamics loads and some examples will be discussed. The approach is based on the Newton-Euler equations and the generalized constraint forces related to the implicit representation of the scleronomic and rheonomic constraints. First the rheonomic constraints are used to get the f actuator loads from the equations of motion via inverse dynamics and then the q remaining generalized constraint forces due to the scleronomic constraints follow from the equations of reaction. These two sets of equations read as

$$\boldsymbol{M}(\boldsymbol{y},t)\cdot\ddot{\boldsymbol{y}}(t)+\boldsymbol{k}(\boldsymbol{y},\dot{\boldsymbol{y}},t)=\boldsymbol{q}(\boldsymbol{y},\dot{\boldsymbol{y}},\boldsymbol{g},t),$$
$$\boldsymbol{N}(\boldsymbol{y},t)\cdot\boldsymbol{g}(t)+\hat{\boldsymbol{q}}(\boldsymbol{y},\dot{\boldsymbol{y}},\boldsymbol{g},t)=\hat{\boldsymbol{k}}(\boldsymbol{y},\dot{\boldsymbol{y}},t)$$

where the inertia matrix $\boldsymbol{M}$ and the reaction matrix $\boldsymbol{N}$ appear. For more details see Ref. [1,2]. The evaluation of constraint forces was also considered by Wojtyra [3].

For a multibody system with $6p = f + q$ bodies there does not remain any degree of freedom, the system performs a motion prescribed by the rheonomic constraints.

The generalized constraint forces allow the computation of stress and strain states, e.g. by finite element programs, in a first but very efficient approximation neglecting the eigendynamics of the bodies.

References

[1] K. Popp, W. Schiehlen, *Ground Vehicle Dynamics, Springer, Berlin, 2010.*
[2] W.Schiehlen, P. Eberhard, *Technische Dynamik* (in German), 3rd edition, Teubner, Wiesbaden, 2012.
[3] M. Wojtyra, Joint reaction forces in multibody systems with redundant constraints. *Multibody System Dynaimcs,* Vol. 14, No.1, pp. 23–46, 2005

*Email: werner.schiehlen@itm.uni-stuttgart.de

Trajectory Planning Optimization of Mechanisms with Redundant Kinematics for Manufacturing Processes with Constant Tool Speed

Andreas Scholz* Francisco Geu Flores† Andrés Kecskeméthy‡

Chair of Mechanics and Robotics, University of Duisburg-Essen,
Lotharstr. 1, 47057 Duisburg, Germany

Keywords: Trajectory planning, cooperating mechanisms, redundant kinematics

Introduction

Optimal path planning techniques for kinematically redundant mechanisms have been throughly studied in the past ([1]). A common practice to find possible solutions with commercial software is to formulate the problem as a nonlinear optimization problem ([2]) and solve it using standard SQP-methods. A major task is hereby to formulate the optimization problem such that convergence is guaranteed.

The present paper describes a three-step optimization method to compute the optimal control *and* design of mechanisms with redundant kinematics for tasks requiring constant tool velocities along given spatial trajectories defined with respect to one of the moving links. The solution approach consists in describing the kinematics of the system as a one-parametric function of the motion of the tool along the workpiece and parametrizing the motion of the redundant joints as a spline function of this one parameter. The optimal control problem is therefore transformed into a nonlinear optimization problem with the spline coefficients as optimization parameters, which can be solved by SQP routines.

Three-stage optimization method

The method is presented by means of an application example (see figure below), comprising a planar serial robot with three degrees of freedom $\underline{\theta} \in \mathbb{R}^3$, as well as an auxiliary mechanism with one rotational degree of freedom φ around an axis normal to the robot working plane. The robot carries the tool at its end effector whilst the auxiliary mechanism carries the workpiece. The goal is to find the optimal controls $\underline{\theta}(t)$ and $\varphi(t)$ of the robot and the auxiliary mechanism, respectively, as well as the optimal relative pose of the mechanism with respect to the robot inertial coordinate system K_0, which allow for a maximal constant tool speed $\dot{s}$ along a given path S on the workpiece, subject to the maximally allowed joint angular displacements, velocities, and accelerations.

The kinematics of both mechanisms is described as a one-parametric function of the path parameter s, by (1) interpolating the relative motion of the tool with respect to the given trajectory S using a DARBOUX-frame parametrization scheme, and (2) by parametrizing the motion of the redundant degree of freedom φ as a spline function $\varphi(s)$ of third order. To this end, the problem is formulated as a nonlinear optimization problem with the relative pose of the auxiliary mechanism with respect to the robot, the constant speed $\dot{s}$ of the tool along the given trajectory, and the spline coefficients of $\varphi(s)$ as optimization parameters. The optimal parameters are sought by means of a three-stage optimization method. Each stage uses the optimal solution of the previous stage as an initial guess.

*Email: andreas.scholz@stud.uni-due.de
†Email: francisco.geu@uni-due.de
‡Email: andres.kecskemethy@uni-due.de

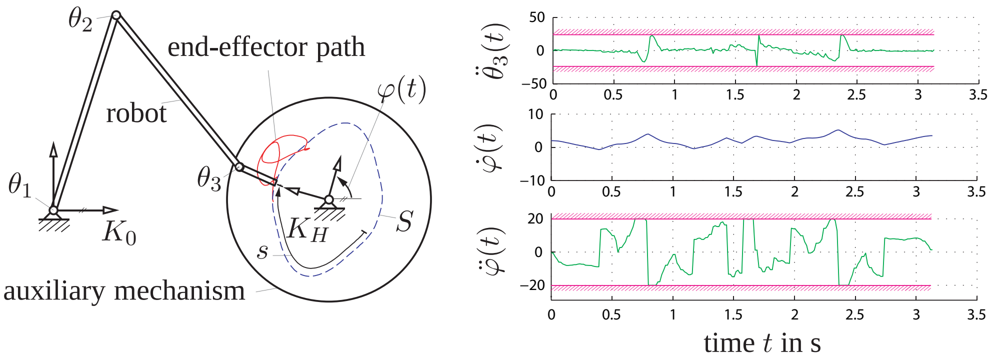


Figure 1: Optimal motion of the robot and the auxiliary mechanism

The first stage consists in searching for a feasible initial guess by minimizing the standard deviation of the joint angles during the motion. In this stage, a low constant speed $\dot{s}$ along the path as well as a low number of spline segments of $\varphi(s)$ are prescribed, and the velocity and acceleration constraints at the robot joints are ignored. In the second stage, the number of spline segments is refined (in our case to 150 segments) and the sum of squared values of the angular velocities and accelerations is minimized. Furtheremore, the maximally allowed angular displacements, velocities and accelerations at the robot actuators, as well as the determinant of the transmission JACOBIAN are regarded as constraints. The third stage consists of a sequence of optimization runs with the same cost and constraint functions of stage two, but with increasing tool speed $\dot{s}$ at each new step. Hereby, the tool speed is increased manually in small increments such that the optimizer can correct the respective constraint violations and the tool speed $\dot{s}$ approaches asymptotically its maximal value.

The figure shows the optimal solution for the presented example. The blue dashed curve represents the prescribed path S on the workpiece, whereas the red curve depicts the motion of the end effector with respect to K_0. It can be seen directly that both $\ddot{\theta}_3$ and $\ddot{\varphi}$ reach their limits during the trajectory, hence the mechanism cannot be driven with a higher tool speed $\dot{s}$. The auxiliary mechanism supports the robot such that the motion takes place widely inside the robot working space, allowing for a higher constant tool speed $\dot{s}$ along the given trajectory.

Conclusions

This paper addresses the optimal control of mechanisms with redundant kinematics for tasks requiring constant tool velocities along given spatial trajectories. A three-stage optimization method has been presented by means of an application example. The paper shows that the method yields good convergence properties, allowing for the determination of the minimal process cycle time. For future research it is planned to automate stage 3 of the procedure and to compare the current approach with collocation methods such as [2].

References

[1] D. P. Martin, J. Baillieul, and J. M. Hollerbach. Resolution of Kinematic Redundancy Using Optimization Techniques. In *IEEE Transactions on Robotics and Automation*, volume 5, no. 4, pages 529–533, August 1989.

[2] Oskar von Stryk. Numerical Solution of Optimal Control Problems by Direct Collocation. In *Optimal Control, International Series in Numerical Mathematics 111*, pages 129–143, 1993.

EUROMECH Colloquium 524
Multibody system modelling, control
and simulation for engineering design

February 27–29, 2012
University of Twente
Enschede, Netherlands

A necessary condition for bicycle self-stability: steer toward the fall

A. L. Schwab *
Delft University of Technology
Delft, The Netherlands

J. P. Meijaard
Olton Engineering Consultancy
Enschede, The Netherlands

Keywords: bicycle, dynamics, rolling, non-holonomic constraints, stability, design.

Long known, but still amazing, is that a bicycle can balance itself[1]. In 1897, French mathematician Carvallo [1] and then, more generally, Cambridge undergraduate Whipple [2] used rigid body equations to show this self-stability [3]. After that, there has been a quest for determining the necessary conditions for this self-stability in terms of bicycle design parameters. Unfortunately, this has not been very successful to this date [4, 5]. In this paper we present one necessary, but not sufficient, condition for bicycle self-stability, viz that *a self-stable bicycle must steer toward the fall*.

To investigate the necessary conditions for bicycle self-stability, we use the Whipple model of the bicycle. This model has recently been benchmarked [3]. Throughout this paper we will use the results and notation from that paper. The model comprises four rigid bodies: the rear wheel, the rear frame with a rigid rider attached, the front frame or steering assembly and the front wheel. These are interconnected by hinges and the wheels contact a flat level road by idealized holonomic and non-holonomic constraints for pure rolling. The resulting model has three velocity degrees of freedom parametrized by the forward speed v, the rear frame lean angle rate $\dot{\varphi}$ and the steering angle rate $\dot{\delta}$; the latter two describe the lateral motion. We investigate the stability of the upright straight-ahead motion, for which the forward and lateral motion are decoupled. The lateral motion of the bicycle is described by the linearized equations of motion which take the form of a set of two coupled second order differential equations, $\mathbf{M}\ddot{\mathbf{q}}+v\mathbf{C}_1\dot{\mathbf{q}}+[g\mathbf{K}_0+v^2\mathbf{K}_2]\mathbf{q}=\mathbf{f}$, expressed in terms of the lateral degrees of freedom $\mathbf{q}=[\varphi,\delta]^{\mathrm{T}}$. Here the forward speed v is a parameter. With the general assumption of exponential motions, $\mathbf{q}=\mathbf{q}_0\exp(\lambda_i t)$, the characteristic equation, $\det\left(\mathbf{M}\lambda^2+v\mathbf{C}_1\lambda+g\mathbf{K}_0+v^2\mathbf{K}_2\right)=0$, can be formed and the eigenvalues λ_i $(i=1,\ldots,4)$ can be calculated. For asymptotically stable bicycle motions, all real parts of the eigenvalues must be negative, $\mathrm{Re}(\lambda_i)<0$. The characteristic equation is a fourth order polynomial in λ of the form $A\lambda^4+B\lambda^3+C\lambda^2+D\lambda+E=0$. The Routh stability criteria [6] now state that for all eigenvalues λ satisfying the quartic characteristic equation to have a negative real part, all polynomial coefficients A,B,C,D,E and the Routh determinant $X=BCD-ADD-EBB$ must have the same sign. However, the coefficients A to E and X are lengthy expressions in 25 bicycle design parameters, such as wheelbase, trail, head angle and mass parameters, and the forward speed, which makes the expressions for these necessary conditions for self-stability in terms of the bicycle design parameters unwieldy. Of course, these 25 parameters are not a minimal set for the description of the equations of motion. A possibility would be to combine the 25 parameters into groups and non-dimensionalize the problem by proper scaling, finally resulting in a minimal set of 5 parameters. However, changes in these 5 parameters are hard to understand in terms of the 25 bicycle design parameters, so we are stuck with the larger parameter set.

Fortunately, we have a clear interpretation of the two characteristic equation coefficients A and E. The first is the determinant of the mass matrix, $A=\det(\mathbf{M})$, which must be positive for a non-degenerate mechanical system. This implies that the signs of the other coefficients must be positive for stability. The second is the determinant of the total stiffness matrix, $E=\det(g\mathbf{K}_0+v^2\mathbf{K}_2)$, and this determinant shows up in the analysis of steady turning.

In a steady turning analysis, we assume the bicycle is running in a steady circle where the applied forces are a steer torque applied by the rider, T_δ, and a zero lean torque. All time derivatives besides

*a.l.schwab@tudelft.nl

[1] See for instance http://bicycle.tudelft.nl/yellowbicycle/

the forward speed are zero, which reduces the linearized equations of motion to the static equations, $[g\mathbf{K}_0 + v^2\mathbf{K}_2]\mathbf{q} = \mathbf{f}$, with $\mathbf{f} = [0, T_\delta]^{\mathrm{T}}$. These static equations yield a relation between the steady turn lean angle and the steer angle at a given v and an expression for the steady turning torque in terms of the bicycle parameters and the steer angle, the latter being

$$T_\delta = \frac{\det(g\mathbf{K}_0 + v^2\mathbf{K}_2)}{g m_{\mathrm{T}} z_{\mathrm{T}}}\delta. \tag{1}$$

The numerator in this expression is the determinant of the total stiffness matrix, which must be positive for stability. The denominator has gravity g, total mass, m_{T}, and the height, $-z_{\mathrm{T}}$, of the centre of mass of the total system. With the positive z-axis pointing down the denominator is always negative. We conclude that *a necessary condition for a bicycle to have self-stability is that the steady turn torque applied by the rider has the opposite sign of the handlebar angle*. Thus, to keep a self-stable bicycle in a rightward (clockwise looking down) circular path, the rider must apply a leftward (counter-clockwise) torque to the handlebars; in other words, the rider is restraining the handlebars from turning even further. Then, if the rider would let go the handlebars, the bicycle would steer toward the fall. Note that this is a necessary, but not a sufficient, condition, since the remaining coefficients, B, C, D and X, have to be possitive, too, for stability.

Finally, we endeaver to examine the steady turning expressions, and consequently self-stability conditions, in terms of the bicycle design parameters. Solving the equations, where again we us the notation of the benchmark paper [3], results in the steady turn lean angle and steer torque at given v and δ,

$$\varphi = \left(\frac{v^2\cos\lambda}{gw}\left(1 - \frac{S_{\mathrm{T}}}{m_{\mathrm{T}} z_{\mathrm{T}}}\right) + \frac{S_{\mathrm{A}}}{m_{\mathrm{T}} z_{\mathrm{T}}}\right)\delta, \tag{2}$$

$$T_\delta = \left(\frac{v^2\cos\lambda}{gw}\left(\frac{S_{\mathrm{F}}}{S_{\mathrm{A}}}\sin\lambda + \frac{S_{\mathrm{T}}}{m_{\mathrm{T}} z_{\mathrm{T}}}\right) - \left(\frac{S_{\mathrm{A}}}{m_{\mathrm{T}} z_{\mathrm{T}}} + \sin\lambda\right)\right) g S_{\mathrm{A}}\delta, \tag{3}$$

with headangle λ, wheelbase w, a static moment term about the z-axis S_{A}, and the gyroscopic coefficients S_{F} (front wheel) and $S_{\mathrm{T}} = S_{\mathrm{F}} + S_{\mathrm{R}}$ (front plus rear wheel), which can be further expressed in the bicycle parameters. For a point mass bicycle with non-rotating wheels, the steady turning lean angle is clearly $v^2/(gR)$, with the turning radius $R = w/(\delta\cos\lambda)$. The border of stability is at the speed where the steady turning torque is zero. The forward speed at which this occurs, the capsize speed, can be determined from (3). Note that there is no specific capsize speed when there is no gyroscopic effect of the wheels. To get an impression of the order of magnitude of the individual terms, we substitute the values of the benchmark bicycle parameters in the dimensionless terms of (2–3), and with $S_{\mathrm{T}}/(m_{\mathrm{T}} z_{\mathrm{T}}) = -0.0148$, $S_{\mathrm{A}}/(m_{\mathrm{T}} z_{\mathrm{T}}) = -0.0321$, $S_{\mathrm{F}}/S_{\mathrm{A}}\sin\lambda = 0.0951$, and $\sin\lambda = 0.3090$, resulting in,

$$\varphi = \Big((v^2\cos\lambda)/(gw)\,(1 + 0.0148) - 0.0321\Big)\,\delta,$$

$$T_\delta = \Big((v^2\cos\lambda)/(gw)\,(0.0951 - 0.0148) - (-0.0321 + 0.3090)\Big)\,g S_{\mathrm{A}}\delta.$$

References

[1] E. Carvallo. *Théorie du mouvement du monocycle et de la bicyclette.* Gauthier-Villars, Paris, France, 1899. (Submitted in 1897 for the Prix Fourneyron, awarded shared second place in 1898.).

[2] F. J. W. Whipple. The stability of the motion of a bicycle. *Quarterly Journal of Pure and Applied Mathematics*, 30:312–348, 1899.

[3] J. P. Meijaard, Jim M. Papadopoulos, Andy Ruina, and A. L. Schwab. Linearized dynamics equations for the balance and steer of a bicycle: a benchmark and review. *Proceedings of the Royal Society A*, 463:1955–1982, 2007.

[4] J. P. Meijaard, Jim P. Papadopoulos, Andy Ruina, and A. L. Schwab. History of thoughts about bicycle self-stability. *eCommons@Cornell http://hdl.handle.net/1813/22497*, 2011.

[5] J. D. G. Kooijman, J. P. Meijaard, Jim P. Papadopoulos, Andy Ruina, and A. L. Schwab. A bicycle can be self-stable without gyroscopic or caster effects. *Science*, 332:339–342, April 15, 2011.

[6] E. J. Routh. Stability of a dynamical system with two independent motions. *Proceedings of the London Mathematical Society*, 1(1):97, 1873.

Efficient flexible contact simulation by means of a consistent LCP\generalized-α scheme and static modes switching

Tommaso Tamarozzi* Wim Desmet
Katholieke Universiteit Leuven
Celestijnenlaan 300b - bus 2420 3001 Heverlee

Keywords: Reduction techniques, Contact Modeling, LCP, generalized-α, Static Modes Switching.

General framework

Contact modeling is a very active research area in the field of multibody (MB) dynamics. Despite a large number of milestone papers (e.g. [1, 2]) describing in details different contact methodologies for continuous contact and impact (e.g. time-stepping schemes, penalty formulation, LCPs, augmented Lagrangian approaches, etc.), two of the main challenging issues, namely accuracy and speed, are far from being jointly solved. Industrial needs run towards lighter and more powerful machines, in which the flexibility of the components have to be addressed, especially when contact and impact are involved as for e.g. gear meshing in drivelines. In this context fast simulation for preliminary and\or design evaluation and improvement is necessary not only in order to obtain the correct kinematics and reaction load disribution but also for detailed information related to the stress field to have correct estimates of e.g. durability related quantities.

This work describes an ongoing research activity performed at KUL focusing on efficient numerical strategies for flexible MB applied to contact problems and sliding components. The main outcomes of this research brought to the implementation of a flexible contact approach based on a consistent LCP\generalized-α scheme with post-stabilization. This scheme allows the natural implementation of the novel efficient model reduction approach published in [3] by G.H.K Heirman and the authors, named Static Modes Switching (SMS). This approach will be described in details together with several interpolation techniques that are needed in this context, for an accurate local deformation prediction avoiding numerically dangerous instabilities.

Proposed methodology

The goal of the proposed approach is to solve a Mixed Linear Complementarity index-3 DAE (MLCP-DAE) problem by combining standard LCP approaches based on Lemke's algorithm (for which the well know Path [4] solver is used) together with the generalized-α solver. As in common MLCP implementations, the MLCP problem is first transformed into an ordinary LCP but the DAE structure of the problem is preserved, without transforming the problem in ODE form (for the LCP-ODE formulation see [5] and [6]).

This particular scheme, solves an LCP for each iteration step to find an approximate value of the contact forces, while the generalized coordinates and derivatives are solved by using the generalized-α scheme. The last iteration, when convergence is reached, is performed in an "*explicit like*" fashion. This allows to keep consistency between the kinematic quantities related to the LCP contact problem and the equations of motion. Moreover, as suggested by [7],different post stabilization techniques has been

*Email: tommaso.tamarozzi@mech.kuleuven.be

tried in order to reduce drift problems related to the acceleration level complementarity conditions. Approaches based on LCPs and on position level post stabilization are analyzed and compared for accuracy and efficiency.

The work also explores different interpolation techniques for the local normal and tangent vector computation and their spatial and time derivatives, necessary for the contact kinematics definition. These quantities are of fundamental importance for a stable numerical scheme. For this reasons an approach based on Overhauser interpolation [8] has proven to be very beneficial for flexible curves interpolation, maintaining the necessary continuity conditions. Different schemes has been compared and proved to give worse results in terms of numerical discontinuities.

Finally the efficiency issue is treated; the novel SMS approach is briefly discussed and applied in the context of contact modeling. SMS is an efficient model reduction scheme that is based on the judicious choice of the modal base used to represent a flexible component, updated online during simulation purely based on the instantaneous boundary conditions. This technique already proved to be efficient and to give good results for problems including sliding components and stress recovery and it is tested in this work for contact problems.

Numerical results are compared against detailed benchmark simulations, adopting different techniques and issues like efficiency and accuracy will deeply discussed, together with some limitations and possible improvements to the proposed scheme.

Acknowledgments

The European Commission is gratefully acknowledged for the support of the Marie Curie ITN ”VECOM” FP7-213543, from which Tommaso Tamarozzi holds an ”Initial Training Grant” (http://www.vecom.org/)

References

[1] B. Brogliato, AA Ten Dam, L. Paoli, F. Genot, and M. Abadie. Numerical simulation of finite dimensional multibody nonsmooth mechanical systems. *Applied Mechanics Reviews*, 55:107, 2002.

[2] P. Alart and A. Curnier. A mixed formulation for frictional contact problems prone to newton like solution methods. *Computer methods in applied mechanics and engineering*, 92(3):353–375, 1991.

[3] G.H.K. Heirman, T. Tamarozzi, and W. Desmet. Static Modes Switching for more efficient flexible multibody simulation. *International Journal for Numerical Methods in Engineering, in press*, 2011.

[4] S.P.D.M.C. Ferris. The path solver: A non-monotone stabilization scheme for mixed complementarity problems. *Optimization Methods and Software*, 5:123–156, 1995.

[5] F. Pfeiffer and C. Glocker. *Multibody dynamics with unilateral contacts*. Wiley Online Library, 1996.

[6] S. Ebrahimi. *A contribution to computational contact procedures in flexible multibody systems*. PhD thesis, University of Stuttgart, 2007.

[7] M.B. Cline and D.K. Pai. Post-stabilization for rigid body simulation with contact and constraints. In *Robotics and Automation, 2003. Proceedings. ICRA'03. IEEE International Conference on*, volume 3, pages 3744–3751. IEEE, 2003.

[8] TZ. Sinokrot, J. Laughlin, and W.C. Prescott. A new flexible point curve joint for the dynamic simulation of flexible multibody systems. In *Proc. The first Joint International Conference on Multibody System Dynamics*, Lappeenranta, May 2010.

EUROMECH Colloquium 524
Multibody system modelling, control
and simulation for engineering design

February 27–29, 2012
University of Twente
Enschede, Netherlands

A level set approach for the optimal design of flexible components in multibody systems

Emmanuel Tromme[*], Olivier Brüls[†], Laurent Van Miegroet, Geoffrey Virlez, Pierre Duysinx[‡]
University of Liège
Aerospace and Mechanical Department (LTAS)
Chemin des chevreuils 1, Building B52,
4000 Liège (Belgium)

Keywords: Optimal design, Flexible multibody system dynamics, Level set method.

During the development process, one of the designer goals is to find the optimal design of the product. An automatic procedure based on optimization methods would offer important advantages compared to trial-errors methods. Over the last 30 years, many efforts have been devoted to structural optimization. Nowadays, structural optimization has reached a certain level of maturity and is currently used for industrial applications, not only in optimal sizing, but also in shape and topology optimization.

Recently, structural optimization has been extended from a component level approach towards a system level approach, which means that a multibody system simulation is used to analyze the global response of the whole mechanical system. This evolution aims at better capturing the real loading conditions accounting for the component interactions and couplings in the system. As a result, the optimal design is much more accurate because the optimal structural design of the component can be very sensitive to the boundary and loading conditions. Moreover, as the component shape is changing during the optimization process, the mass repartition is also changing, thus modifying the interactions between the components and the loading conditions at each iteration.

Other authors focused on the structural optimization of components considering loading conditions coming from the multibody system dynamic analysis. Nevertheless most of these studies consider that the component is isolated from the rest of the mechanism and use quasi-static loads cases to mimic the complex dynamic loading [1, 2].

In our previous work [3, 4, 5], a more integrated approach has been proposed with an optimization loop directly based on the dynamic response of the flexible MBS [6]. The first study validated this approach by realizing the topology optimization of a robot arm composed of two truss linkages [3]. In Ref. [4], the "fully integrated" optimization problem of flexible components under dynamics loading conditions was investigated and was illustrated on the mass optimization of robot arms subject to trajectory tracking constraints. Sizing and shape optimization were performed. In these two studies, it has been shown that the MBS optimization is not a simple extension of structural optimization. The coupled problem between vibrations and interactions within the components generally results in complex design problems and convergence difficulties. The design problem is complicated and naive implementations lead to fragile and unstable results. In Ref. [5], the formulation of the MBS optimization problem was investigated because it had turned out that it is a key point to obtain convergence in a stable and robust way. The study was performed on the shape optimization of a connecting rod under cyclic dynamic loading.

[*]Email: emmanuel.tromme@ulg.ac.be
[†]Email: o.bruls@ulg.ac.be
[‡]Email: p.duysinx@ulg.ac.be

The present study continues along the investigation of optimization of flexible components included in MBS and extends the shape optimization presented in reference [5] to an intermediate level between shape and topology optimization thanks to the introduction of the level set method which is a numerical technique for tracking interfaces and shapes [7]. This approach combines the advantages of shape optimization with some possibilities of changing the topology. The advantages of topology optimization are that it allows getting the best topology for the structure and suggests innovative design since no initial shape of the component is required. However, the number of design variables is very large which leads to complex optimization problems. Furthermore, the optimal design can not be immediately manufactured. Shape optimization is not able to modify the component topology but can be useful after topology optimization to improve the current design and to introduce stress or manufacturing constraints which are difficult to deal with in topology optimization.

This extension allows optimizing any boundaries of the structure defined by a compound level set. With this approach, there is less mesh problems since a fixed mesh grid is considered. The elements inside the domain defined by the component geometry are full density while the elements outside are void. Concerning the elements intersected by the boundaries, an intermediate density defined by the SIMP law is used. Design variables are the parameters of basic geometric primitives described by a level set representation. The number of design variables remains small. The bottleneck is that with our level set approach, the topology is not completely defined by the optimization process. The process can suppress holes but can not create them. The considered application will be the optimization of a connecting rod (slider-crank mechanism) under cyclic dynamic loading.

Acknowledgments

The author Emmanuel Tromme would like to acknowledge the Lightcar Project sponsored by the Walloon Region of Belgium for its financial support.

References

[1] B. S. Kang and G. J. Park. Optimization of flexible multibody dynamic systems using the equivalent static load method. *AIAA Journal*, **43**(4), 846–852, 2005.

[2] P. Häussler, J. Minx, D. Emmrich, P. Allinger. Topology optimization of dynamically loaded parts in mechanical systems: Coupling of MBS, FEM and structural optimization. In *NAFEMS Seminar - Analysis of Multibody systems using FEM and MBS*, Wiesbaden, Germany, October 2004.

[3] O. Brüls, E. Lemaire, P. Eberhard and P. Duysinx. Topology optimization of structural components included in flexible multibody systems. In *Proceedings of the 7th World Congress on Structural and Multidisciplinary Optimization*, COEX Seoul, Korea, May 2007.

[4] P. Duysinx, J. Emonds-Alt, G. Virlez, O. Brüls and M. Bruyneel. Advances in optimization of flexible components in multibody systems: Application to robot-arms design. In *Proceedings of the 5th Asian Conference on Multibody Dynamics*, Kyoto, Japan, August 2010.

[5] E. Tromme, O. Brüls, P. Duysinx. Optimization of flexible components in reciprocating engines with cyclic dynamic loading. In *Proceedings of the Multibody dynamics 2011, Eccomas Thematic Conference*, Brussels, Belgium, July 2011.

[6] M. Géradin and A. Cardona. *Flexible Multibody Dynamics: A Finite Element Approach.* John Wiley & Sons, New York, 2001.

[7] P. Duysinx , L. Van Miegroet, T. Jacobs and C. Fleury. Generalized shape optimization using X-FEM and Level Set methods. In *M.-P. Bendsøe, N. Olhoff and O. Sigmund eds. IUTAM Symposium on Topological Design Optimization of Structures, Machines, and Materials. Status and Perspectives.*, Vol 137, Solids Mechanics and Its Application, Springer Verlag, 2006.

A static balancer for a large-deflection compliant finger mechanism

Jan Vaandrager*, Just L. Herder†
University of Twente
Enschede, The Netherlands

Keywords: Compliant mechanism, self-adaptive finger, underactuation, static balancing.

Introduction

Compliant mechanisms accomplish their motion due to the deformation of slender segments [1]. Advantages include the absence of backlash and friction, and no need for assembly. One disadvantage is elastic energy storage in the compliant segments, affecting the input-output relationship.

Static balancing can be used to overcome the drawback of energy storage in compliant mechanisms. In statically balanced mechanisms, the potential energy is constant throughout a certain range of motion. As a result, the mechanism has no preferred position. Static balancing of linkages was studied in [2] while extension to compliant mechanisms was presented in [3,4]. Case studies are presented in [5,6], showing that synthesis of large deflection static balancers is still a challenge.

Grippers are often used as benchmark applications to study compliant mechanisms. This case is interesting because in gripping devices, poor mechanical efficiency and poor force transmission are problematic. In particular the recent development of a large-deflection compliant finger [7] raises the challenge of extending synthesis methods for statically balanced compliant mechanisms towards large range of motion. This paper will present a step towards such a method.

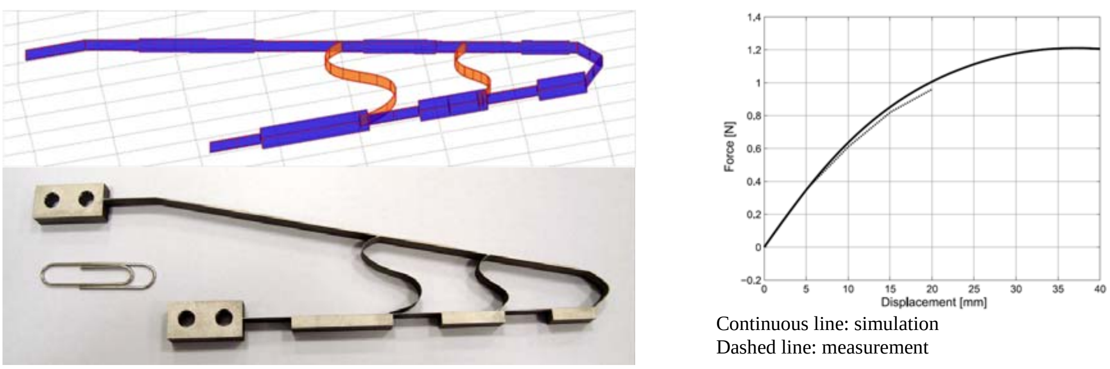


Figure 1: Compliant finger (based on [7]): (a) Spacar model, (b) Photograph, (c) Characteristic.

Method

Assuming the gripper part as given, a balancing part needs to be added such that the total potential energy is constant. The balancing potential therefore needs to be the opposite of the gripper potential, plus a constant to avoid negative balancing potentials. In practice this means that the mechanism needs to be preloaded. As an example case, the compliant finger in [7] was selected. A Spacar nonlinear finite element model [9] was created to obtain the target potential energy function, see Fig. 1a and 1b, respectively. The elastic energy in the system was determined by taking their inner product of the generalized stress and generalized strain vectors [10].

As a building block for the balancer, a mechanism with an interesting elastic potential was selected, namely an S-shaped compliant mechanism, see Fig. 2a [8]. The elastic potential was

* Email: j.vaandrager@gmail.com
† Email: j.l.herder@utwente.nl

determined by modelling the mechanism in Spacar [2] similar to the finger. The shape was modeled using pre-curved beam elements. When the upper tip is allowed to translate, but not rotate, the elastic potential is as in Fig. 2a. The dashed arrow shows an end point path with progressively decreasing elastic potential. This path is proposed to serve as starting point for the design of the balancing mechanism. The overall size of the balancing mechanism is determined a stroke of 40mm at the actuation point, resulting in a rotation of around 90 degrees of the distal phalanx. The start position and direction of the end point path were used to determine an optimal balancing solution.

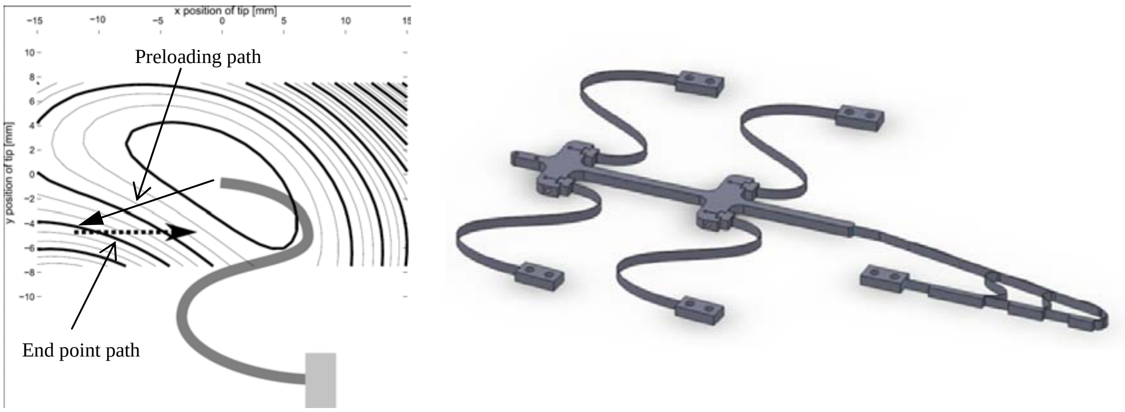


Figure 2: S-shaped balancer: (a) Potential energy field with end point path, (b) Drawing of assembly.

Result

An optimal balancer path was found and verified by using a polynomial approximation and a Spacar simulation. In addition, a prototype was made of Ti6AI4V using wire EDM, consisting of four S-shaped balancing building blocks connected to a central beam driving the finger. Operating force and displacement of the central beam were recorded. It was found that the actuation force was reduced from 1.2N for the unbalanced finger to 0.17N at the maximal deflection of 40mm. In the range of 15mm to 32mm the force reduction is above 95%. The simulation results and the polynomial approximation are in fair agreement with the measurements where it is noted that the error increases for the larger excursions.

Discussion

This paper proposed a method to synthesize static balancers based on a potential energy field. While promising results were achieved, obtaining the field is computationally demanding. Once obtained, fast optimization based on polynomial approximation of the field was possible.

References

[1] L.L. Howell, *Compliant Mechanisms*, Wiley and Sons, New York, 2001.
[2] Herder, J.L. *Energy free systems, theory, conception and design of statically balanced spring mechanisms*, PhD thesis, Delft University of Technology, Delft, 2001.
[3] J.A. Gallego Sanchez, J.L. Herder, *Criteria for the static balancing of compliant mechanisms*, ASME IDETC, Paper number DETC2010-28469, Montreal, Canada, 2010.
[4] J.A. Gallego Sanchez, J.L. Herder, S*ynthesis methods in compliant mechanisms: an overview*, ASME IDETC, Paper number DETC2009-86845, San Diego, California, 2009.
[5] K. Hoetmer, G. Woo, C. Kim, J.L. Herder, *Negative stiffness building blocks for statically balanced compliant mechanisms: design and testing*, ASME Journal of Mechanisms and Robotics, 2(4)041007, 2010.
[6] N. Tolou, V.A. Henneken, J.L. Herder, *Statically balanced compliant micro mechanisms (SB-MEMS): concepts and simulation*, ASME IDETC, DETC2010-28406, Montreal, Canada, 2010.
[7] P. Steutel, G.A. Kragten, J.L. Herder, *Design of an underactuated finger with a monolithic structure and distributed compliance*, ASME IDETC, Paper number DETC2010-28127, Montreal, Canada, 2010.
[8] J. Vaandrager, A. Zagaris, J.L. Herder, *Synthesis method for large deflection building blocks in compliant mechanisms, based on elastic potential*, MSc Thesis, University of Twente, 2011.
[9] *Spacar - flexible multibody modelling for mechatronic design* (http://www.spacar.nl).
[10] J.P. Meijaard, J.B. Jonker, *Definition of deformation parameters for beam elements and their use in flexible multibody system analysis*, Multibody Dynamics 2009, ECCOMAS Thematic Conference, 2009.

Modelling of flexible dynamic links in Nano-Positioning Motion Systems

Rob Waiboer*
Mechatronic Systems Development
ASML Netherlands B.V
P.O.Box 324, 5500 AH Veldhoven
The Netherlands

Keywords: Mechatronics, Flexible Multibody Systems, Input-Output Equations, Dynamic modelling.

Introduction

In high-speed and nano-scale positioning systems such as the stages encountered in the wafer scanning industry, high-speed motion is combined with nano-scale tracking precision. In terms of achieving servo performance, the combination of both speed and accuracy puts heavy demands on the control systems and design. As the amount of disturbance rejection of the control system is limited due to the fact the servo bandwidth is restricted by elastic modes of the wafer or reticle positioning system, it is required to minimise the disturbance forces acting on the stage. Sources of disturbance forces are the so-called dynamic links and these are for instance:

- hoses for transportation of coolant and gas, and
- wires and flexible PCB's for electrical power and sensor signals.

These flexible links cross between two moving bodies in the stage, see **Figure 1**. The relative motion between these bodies are small, with movements in the order of 1 [mm], for the short-stroke and the long-stroke. Large movements (in the order of 1 [m]) are typical for the long-stroke moves with respect to the base-frame.

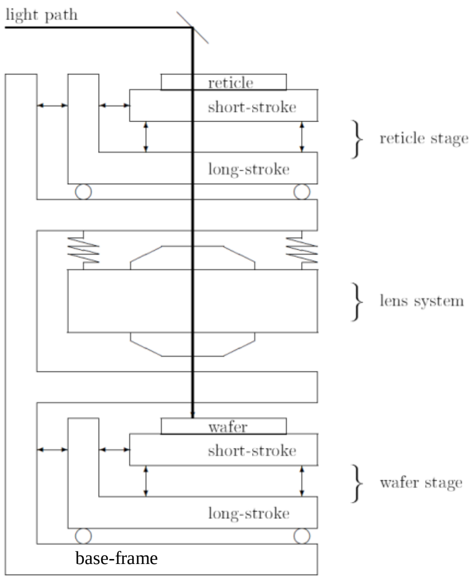


Figure 1: Schematic overview of the wafer stage and the reticle stage in a wafer scanner.

* Email: rob.waiboer@asml.com

Modelling

Two typical dynamic links have been modelled using the SPACAR simulation software package [1]; a coolant hose that resides between the short-stroke and the long-stroke and a cable slab which enables large movements between the base-frame and the long-stroke, see **Figure 2**. For the coolant hose dynamics, it is of interest in what way small movements between the short-stroke and the long-stroke give rise to disturbance forces on the short-stroke. This effect is analysed by means of the so-called transmissibility of the hose, which is the transfer between small movements at the side of the long-stroke to reaction forces at the side of the short-stroke. The transmissibility is obtained by means of a linearised input-output representation of the model [2]. For the cable slab, the analysis focuses on the non-linear behaviour.

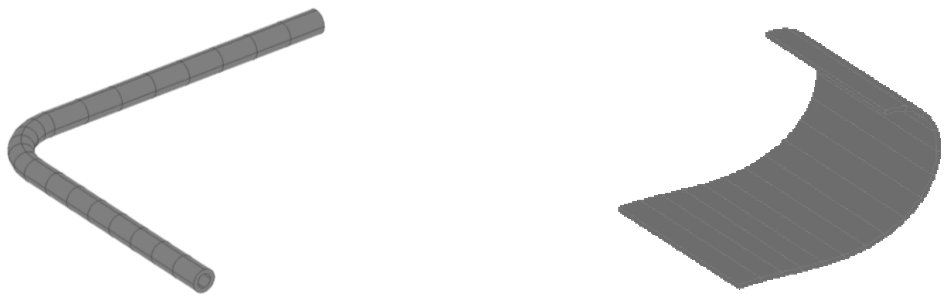


Figure 2: a) Model of a coolant hose and b) a cable slab.

Results

The linearised input-output representation of transmissibility of the hose model has been compared with direct measurements of the transmissibility on a test set-up and they show good agreement. This confirms that the models can be used for designing dynamically improved dynamic links. The modelling of the cable slab has given important insights of the non-linear behaviour associated with the large deformation it undergoes during operation, resulting in a bifurcation point along its move path.

Acknowledgements

The author acknowledges the work carried out by Willem Wijma on the modelling of the cable slab.

References

[1] J.B. Jonker and J.P. Meijaard. SPACAR-computer program for dynamic analysis of flexible spational mechanisms and manipulators. *Multibody Systems Handbook*, W. Schiehlen (ed.), 123-143. Springer-Verlag, Berlin, 1990.

[2] Jonker, J., Aarts, R., and Van Dijk, J. A linearized input-output representation of flexible multibody systems for control synthesis. *Multibody System Dynamics 21* (2009), 99–122.

Experimental results of a high speed dynamically balanced redundant planar 4-RRR parallel manipulator

V. van der Wijk*
University of Twente
Enschede, The Netherlands

S. Krut, F. Pierrot
LIRMM
Montpellier, France

J.L. Herder
University of Twente
Enschede, The Netherlands

Keywords: Shaking force balance, shaking moment balance, parallel manipulator.

When manipulators (i.e. robots, mechanisms, machines) run at high speeds, forces and moments for the acceleration of mass and inertia lead to considerable vibrations of the base. These vibrations are transmitted to, for example, measuring and processing equipment that is mounted on the same base, affecting the manipulator performance. Base vibrations also lead to, among others, increased noise, wear, and fatigue, and to reduced comfort for hand-held tools.

To reduce the influence of base vibrations, (active) damping can be applied or waiting times can be included in the motion cycle to wait until vibrations have died out. Another solution is to design the manipulator to be dynamically balanced. Of a dynamically balanced manipulator the net dynamic forces (the shaking forces) and the net dynamic moments (the shaking moments) the manipulator exerts to the base are zero for all motion. Motion of the manipulator then does not cause the base to vibrate.

A disadvantage of dynamic balancing is that often a considerable amount of mass and inertia is added [1]. This may be the reason that up to now little research has been done on the balancing of parallel manipulators. Various studies, however, have led to guidelines for low mass and low inertia dynamic balancing [2]. Based on these guidelines a high speed balanced redundant planar 4-RRR parallel manipulator was designed [3] and a prototype was developed and tested.

In this paper the experimental results of the balanced 4-RRR manipulator will be presented. Since no experiments for high speed dynamically balanced manipulators are known, these are exclusive results. In addition to the shaking forces and the shaking moment, also other issues will be addressed such as the driving torques, the bearing forces, the sensitivity for balance inaccuracies, and the influence of payload. The experimental results are compared with the results of the same manipulator being unbalanced.

References

[1] V. Van der Wijk, J.L. Herder, B. Demeulenaere, *Comparison of Various Dynamic Balancing Principles Regarding Additional Mass and Additional Inertia*, J. of Mechanisms and Robotics, **1**(4), 04 1006, 2009.

[2] Van der Wijk, V., Herder, J. L., *Guidelines for Low Mass and Low Inertia Dynamic Balancing of Mechanisms and Robotics*, pp. 21–30 in Kröger and Wahl, Advances in Robotics Research, Proc. of the German Workshop on Robotics, Springer, ISBN 978-3-642-01212-9, 2009.

[3] Van der Wijk, V., Krut, S., Pierrot, F., Herder, J.L., *Generic Method for Deriving the General Shaking Force Balance Conditions of Parallel Manipulators with Application to a Redundant Planar 4-RRR Parallel Manipulator*, A12-523 in Proceedings of the 13th IFToMM World Congress on Mechanism and Machnine Science, Guanajuato, Mexico, 2011

*Email:v.vanderwijk@utwente.nl

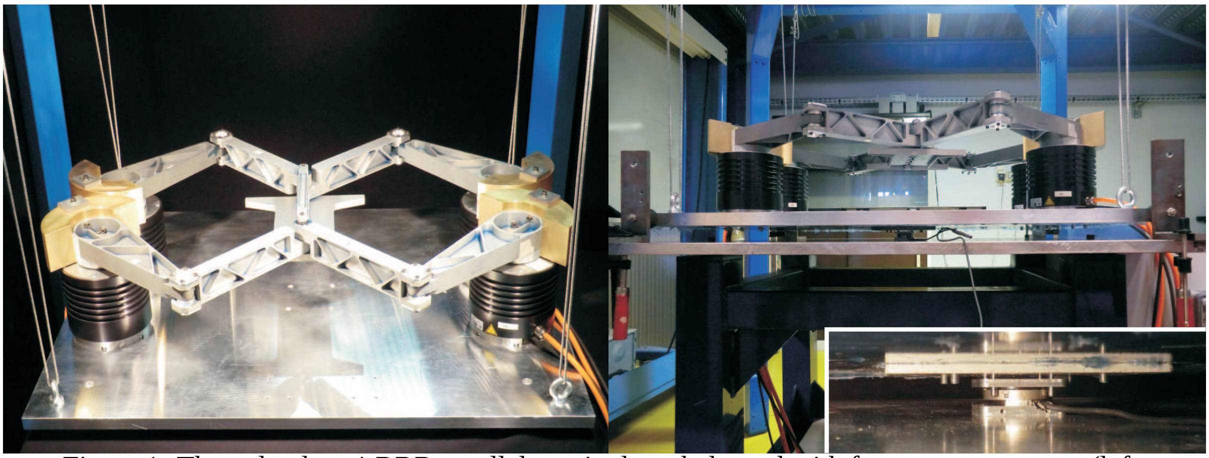


Figure 1: The redundant 4-RRR parallel manipulator balanced with four counter-masses (left, patented) is supported by four cables to float freely within the horizontal plane. A 6-DoF force-torque sensor is mounted in between the manipulator base and the fixed base (right) to measure the resulting in-plane shaking forces and shaking moment.

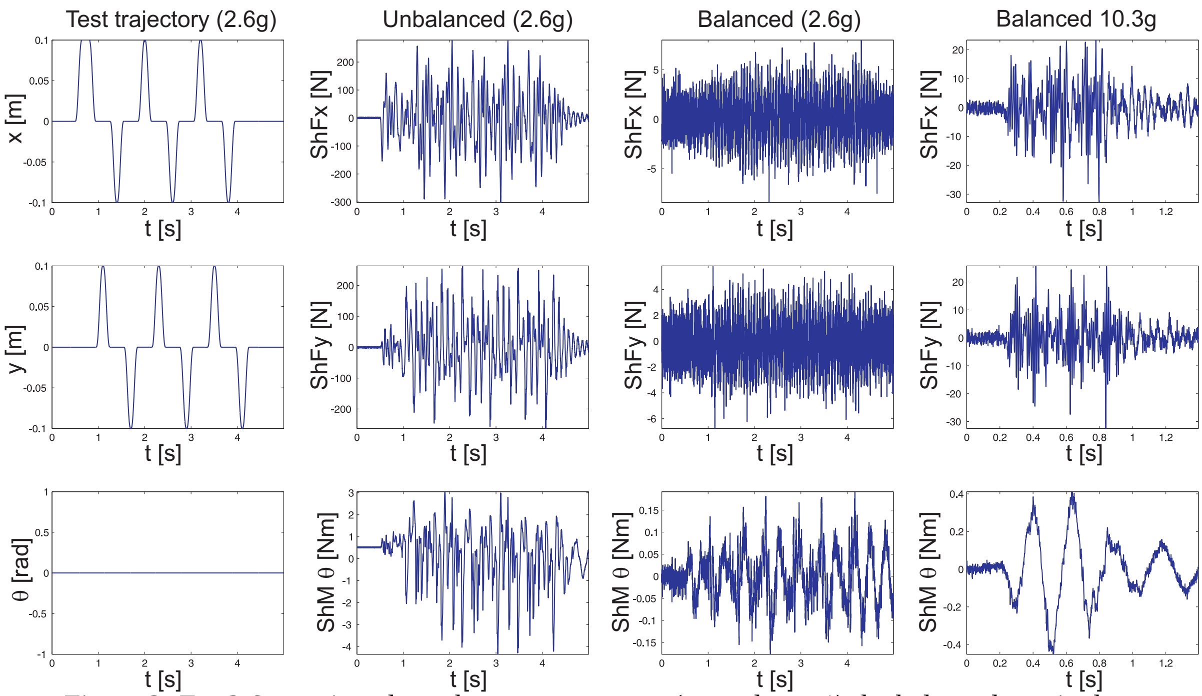


Figure 2: For 2.6g motion along the symmetry axes (see column 1) the balanced manipulator is compared with the same manipulator without counter-masses which then is unbalanced. The measured shaking forces and shaking moment of the unbalanced and of the balanced manipulator are shown in columns 2 and 3, respectively. While for the unbalanced manipulator shaking forces up to 300 N are measured, the shaking forces of the balanced manipulator are just above the sensor noise at about 5-6 N. Also the shaking moment of the balanced manipulator is considerably lower than that of the unbalanced manipulator. Column 4 shows for the balanced manipulator for 10.3g motion the measured shaking forces being up to 30 N and the measured shaking moment being up to 0.44 Nm. This means that for four times higher accelerations, the shaking forces and shaking moment of the balanced manipulator are about ten times lower than those of the unbalanced manipulator.

Overconstrained multibody systems – known and emerging issues

Marek Wojtyra* and Janusz Frączek
Warsaw University of Technology, Institute of Aeronautics and Applied Mechanics
Nowowiejska 24, 00-665 Warsaw, Poland

Keywords: redundant constraints, joint reactions, flexible bodies.

Redundant constraints are defined as the constraints that can be removed from the system without changing its kinematics. Real-world mechanisms quite often are purposely overconstrained, mainly in order to strengthen or simplify their construction. Redundant constraints are defined on the basis of kinematics, however, their influence on static and dynamic properties of the system should not be neglected. For example, in overconstrained systems assembly or thermal stresses result in additional, usually unwanted, joint loads, which may contribute to joint friction effects and hence may affect the mechanisms motion.

Multibody modelling of overconstrained systems is troublesome, since the *constraint equations are dependent* [1], and thus the constraint Jacobian matrix is rank-deficient. As a result, some or all *constraint reactions cannot be uniquely determined using a rigid body model* [2]. Usually, purely mathematical operations, not supported by laws of physics, are performed to find the reaction solution [3]. Hence, the obtained solution reflects the properties of redundant constraints handling method rather than the physical properties of the investigated multibody system, however, contribution [4] indicates that, under certain restrictive assumptions, the minimal norm reaction solution may be privileged over other solutions. It was proven that in the case of an overconstrained rigid body mechanism, despite the fact that <u>all</u> constraint reactions cannot be uniquely determined, *reactions in <u>selected</u> joints can be specified uniquely*, and methods allowing detection of these joints were formulated [5].

Special attention should be paid to rigid body models of overconstrained systems with Coulomb friction in joints. Since the joint reactions cannot be uniquely determined, the joint fiction forces cannot be uniquely determined as well. As a result, *<u>simulated</u> motion of an overconstrained system with joint friction is non-unique* [6], hence, the direct dynamic problem is unsolvable. It was proven that – in a special case – *when friction forces appear only in joints with unique reaction solutions, the simulated motion of the mechanism is unique*, moreover, methods of checking whether the direct dynamic problem is solvable were derived [6].

The methods proposed in [5] and [6] allow checking whether joint reactions as well as motion predicted by a rigid body model of a given overconstrained mechanism are unique. In a general case, however, it is impossible to obtain the unique solution using a rigid body model. *In the case of overconstrained mechanisms, the rigid body hypothesis is a too-far reaching assumption*.

The known modelling issues, summarized above, lead to the following question: *which factors determine joint reactions distribution in a real-world overconstrained mechanism and should be considered in its multibody model?* Engineering experiences suggest that flexibility of bodies, elasticity of bearings, assembly stresses, thermal loads, joint clearances as well as geometric imperfections (in [7] it was shown that, in case of redundantly constrained mechanisms, rigid body assumption must be accompanied by a postulate that links are geometrically perfect) should be taken into account. The problem is that in various mechanisms various factors dominate.

In our attempt to propose a more realistic model of a redundantly constrained mechanism, we can cautiously state that *elasticity of bodies is not the only factor determining reaction forces but is a very important one*. Let's consider an example of a simple overconstrained parallelogram mechanism presented in three versions in Figure 1. Assume that all links of the mechanism have identical mass properties, and that one link, indicated by a dashed line, is significantly less stiff than the other two. It is reasonable to expect different reaction solutions in all three cases, thus stiffness properties of the

* Email: mwojtyra@meil.pw.edu.pl

links must be included in the model (when rigid body assumption is held, the model in all three cases is exactly the same).

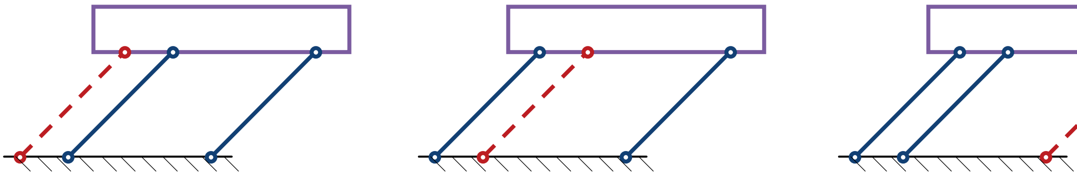


Figure 1: Three versions of an overconstrained parallelogram mechanism.

In many models elasticity of bodies is taken into account, however, some effects associated with constraint redundancy are still observed, even when mechanisms are modelled as entirely flexible systems [8]. Quite often in a multibody model only selected rigid bodies are substituted by flexible ones [9]. It can be shown that unrealistic joint reactions can be found when this approach is applied. In many cases *problems with finding realistic reactions, typical for overconstrained systems, are observed when analyzing partially flexible models without redundant constraints* [10].

In many cases of partially flexible models of mechanisms with redundant constraints, the decision on which parts should be treated as flexible is difficult or, at least, is not straightforward. Thus, an interesting question emerges: *which bodies may remain rigid in a model of an overconstrained mechanism when realistic joint reactions are sought?*

The following rule of rigid bodies selection can be deducted: if all joint reactions acting on the selected part can be uniquely determined using a rigid body model (methods described in [5] can be applied to check this), then – when solving for joint reactions – there is no point in taking into account flexibility of this part [10]. This will neither (significantly) change the calculated reactions acting on the selected part nor the reactions acting on the other parts.

Our recent numerical experiments show that in some cases the above mentioned rule is too restrictive. It is possible to model more parts as rigid bodies and still obtain realistic reaction solution. It should be mentioned that, when some "symmetries" of mechanical properties within the analysed mechanism are broken, the number of bodies that must be modelled as flexible ones increases.

Formulating more subtle or precise rules, telling which bodies may remain rigid in a multibody model of an overconstrained mechanism, is an open and challenging problem.

References

[1] E.J. Haug, *Computer Aided Kinematics and Dynamics of Mechanical Systems*, Allyn and Bacon, 1989.

[2] M. Wojtyra, *Joint reaction forces in multibody systems with redundant constraints*, Multibody System Dynamics, **14**(1), pp. 23–46, 2005.

[3] M. Wojtyra, J. Frączek, *A comparison of various methods of redundant constraints handling in multibody system simulations*, Proc. of the ASME 2011 IDETC/CIE, Washington, USA, 2011.

[4] J. Garcia de Jalon, *Multibody Systems Made Simple and Efficient*, ASME 2011 IDETC/CIE D'Alembert Award keynote, http://mat21.etsii.upm.es/mbs/ASME2011/Washington2011.pdf.

[5] M. Wojtyra, *Joint reactions in rigid body mechanisms with dependent constraints*, Mechanism and Machine Theory, **44**(12), pp. 2265–2278, 2009.

[6] J. Frączek, M. Wojtyra, *On the unique solvability of a direct dynamics problem for mechanisms with redundant constraints and Coulomb friction in joints*, Mechanism and Machine Theory, **46**(3), pp. 312–334, 2011.

[7] A. Müller, *Generic Mobility of Rigid Body Mechanisms: On the Existence of Overconstrained Mechanisms*, Proc. of the ASME 2007 IDETC/CIE, Las Vegas, USA, 2007.

[8] R.G.K.M. AArts, S.B. Boer, J.P. Meijaard, D.M. Brouwer, J.B. Jonker. *Analyzing overconstrained design of compliant mechanisms*, Proc. of the ASME 2011 IDETC/CIE, Washington, USA, 2011.

[9] E. Zahariev, J. Cuadrado, *Dynamics of over-constrained rigid and flexible multibody systems*, Proc. of the 12th IFToMM World Congress, Besançon, France, 2007.

[10] M. Wojtyra, J. Frączek, *Joint reactions in overconstrained rigid or flexible body mechanisms*, Proc. of the ECCOMAS Thematic Conference Multibody Dynamics, Brussels, Belgium, 2011.

Simulation of non-stationary vibration of large multibody systems

Evtim Zahariev
Institute of Mechanics, Bulgarian Academy of Science
Acad. G. Bonchev St., bl. 4, 1113 Sofia, BULGARIA, evtimvz@bas.bg

Keywords: multibody systems, nonlinear dynamics, vibration, reonomic constraints.

Introduction

Non-stationary vibration of a mechanical system is that characterized by varying and time-dependent parameters of the dynamic model. In practice typical examples include earthquake-excited vibration, vibration of vehicles and sea vessels [1]. Actually, non-stationary vibration is due to configuration and time-dependent non-linear dynamics. Two typical examples of non-stationary mechanical systems are presented in Fig. 1 (a) and (b). Fig 1 (a) presents a beam with a moving mass along it (velocity *v*) subject to force excitation $\mathbf{f}(t)$ in point *M*. Due to the motion of the mass the resulting vertical vibration is non-stationary. The second one (Fig. 1 b) presents a flexible structure compiled of two slender beams and two rigid bodies subject to basement displacement excitation $\ddot{\rho}(t)$. Due to large flexible deflections of the beams their stiffness matrices are time dependent. For both cases the excitations ($\mathbf{f}(t)$ or $\ddot{\rho}(t)$ are stationary and as a result of the time varying dynamic parameters the vibration is non-stationary. In the first case the system mass matrix is varying, while for the second these are the mass matrix and the system stiffness.

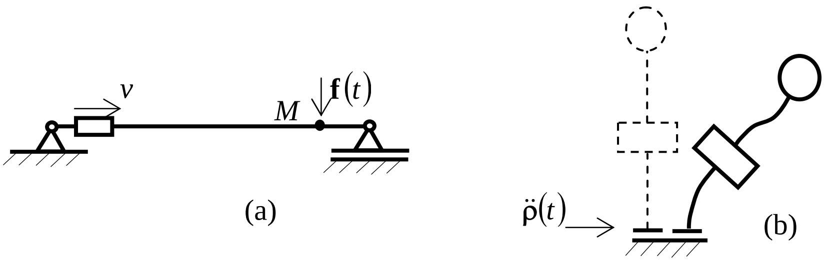


Figure 1: Examples of structures subject to non-stationary vibrations

The examples of Fig.1 is typical for cranes, floating platforms, space manipulators, flexible structures. The present paper deals with the second example of large flexible structures subject to seismic excitations. Earthquake shaking affects compulsory motion of the structure basement. The ground motion is registered by the so called strong-motion accelerographs and can be analyzed to obtain a direct estimation of the peak ground motion, amplitude and frequency. The information about the specific seismic excitation for a region is the input data of the computational procedure for vibration and health monitoring of a structure.

Dynamics of non-stationary vibration

For the general case of force and displacement excitation the dynamic equations are described by the ordinary differential equations (ODE) with respect to the generalized coordinates, i.e.:

$$\mathbf{M}(t)\cdot\ddot{\mathbf{q}}+\mathbf{B}(\mathbf{q},\dot{\mathbf{q}},t)+\mathbf{K}(t)\cdot\mathbf{q}=\mathbf{S}-\left(\frac{\partial\rho_M}{\partial\mathbf{q}}\right)^{\backslash}\cdot\mathbf{f}(t),, \tag{1}$$

subject to the *m* reonomic constraints

$$\ddot{\rho}=\ddot{\rho}(t)\rightarrow\ddot{q}_i=\ddot{q}_i(t),=1,2,\ldots,m\,. \tag{2}$$

M and **K** are configuration/time dependent mass and stiffness matrices; **S** is matrix-vector of the generalized forces. If the system is imposed on displacement excitation the last term of Eq. 1 vanishes. The external displacement excitation pointed out as $\rho = \rho(t)$ describe the coordinates of a vector that are part of the generalized coordinates, i.e., Eq. 2 presents reonomic constraints of number *m* imposed on *m* generalized coordinates. The ODE, Eq. 1, could be transformed as follows:

$$\mathbf{M}(t)\cdot\left[\ddot{q}_1 \;\cdots\; \ddot{q}_{k+1} \;\cdots\; \ddot{q}_{k+m} \;\cdots\; \ddot{q}_n\right]^{\backslash} = \left[S_1 \;\cdots\; S_{k+1} \;\cdots\; S_{k+m} \;\cdots\; S_n\right]^{\backslash} - \mathbf{B}(\mathbf{q},\dot{\mathbf{q}},t) - \mathbf{K}(t)\cdot\mathbf{q}\,. \qquad (3)$$

In Eq. 3 $\ddot{q}_i = \ddot{q}_i(t), i = k+1, k+2,.., k+m$ are known, while $S_i = S_i(t), i = k+1, k+2..., k+m$ are unknown. The solution of Eq. 1 subject to the constraints, Eq. 2, results in solution of mixed direct and inverse dynamic problem to find the unknown values of $\ddot{q}_i = \ddot{q}_i(t), i = 1,..., k, k+m+1,..., n$ and the generalized forces $S_i = S_i(t), i = k+1, k+2..., k+m$. After transformation of Eq. 3 one obtains the linear equation system with respect to the unknown, as follows:

$$\underline{\mathbf{M}}\cdot\left[\ddot{q}_1 \;\cdots\; S_{k+1} \;\cdots\; S_{k+m} \;\cdots\; \ddot{q}_n\right]^{\backslash} = -\underline{\underline{\mathbf{M}}}\cdot\left[\ddot{q}_{k+1}(t) \;\cdots\; \ddot{q}_{k+m}(t)\right]^{\backslash} + \underline{\mathbf{S}} - \mathbf{B}(\mathbf{q},\dot{\mathbf{q}}) - \mathbf{K}\cdot\mathbf{q}\,, \qquad (4)$$

where $\underline{\mathbf{M}} = \begin{bmatrix} m_{1,1} & \cdots & 0 & \cdots & 0 & \cdots & m_{1,n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ m_{k+1,1} & \cdots & -1 & \cdots & 0 & \cdots & m_{k+1,n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ m_{k+m,1} & \cdots & 0 & \cdots & -1 & \cdots & m_{k+m,n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ m_{n,1} & \cdots & 0 & \cdots & 0 & \cdots & m_{n,n} \end{bmatrix}; \underline{\underline{\mathbf{M}}} = \begin{bmatrix} m_{1,k+1} & m_{1,k+2} & \cdots & m_{1,k+m} \\ m_{2,k+1} & m_{2,k+2} & \cdots & m_{2,k+m} \\ \cdots & \cdots & \cdots & \cdots \\ m_{n,k+1} & m_{n,k+2} & \cdots & m_{n,k+m} \end{bmatrix}; \underline{\mathbf{S}} = \begin{bmatrix} S_1 \\ \cdots \\ S_k \\ 0 \\ \cdots \\ 0 \\ S_{k+m+1} \\ \cdots \\ S_n \end{bmatrix}.$

With known function of the displacement excitation, the dynamic equations, Eq. 4, are solved for long time period of operating time to analyze the non-stationary solutions. For large flexible structures the system configuration changes significantly which requires on every step of the integration the mass and stiffness matrices to be calculated. For numerical integration of the nonlinear dynamic equations the program IVPAC of the package IMSL is applied with parameters pointing out that these matrices depend on time. Examples of test structures (Fig. 2) imposed on 3-D displacement excitation are presented in the paper.

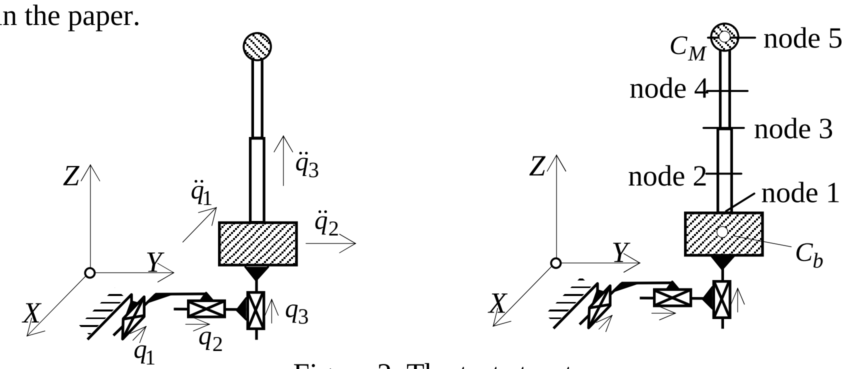


Figure 2. The test structure

Acknowledgements

The authors acknowledge the financial support of Ministry of Education, Youth and Science, contract No. DUNC-01/3, 12.03.2009.

References

[1] A.G. Poulimenos, S.D. Fassois, *Parametric time-domain methods for non-stationary random vibration modeling and analysis — A critical survey and comparison*, Mechanical Systems and Signal Processing, **20**, pp. 763–81, 2006.

FEA based modelling of rolling bearings for high fidelity multibody system modelling

Johann Zeischka*
MSC.Software GmbH
Am Moosfeld 13
DE-81829 München

Keywords: finite element analysis, multibody dynamics, rollings bearings

Introduction

Over the last decade, finite element engineering and multi body dynamics were grown to companions for the kinematic and dynamic analysis of mechanical systems, as flexible bodies are now in common use for the numerical analysis of mechanical systems in all industries. One of reasons for this success is the correct representation of flexibility and its associated structural dynamic effects, as the concept of ‘rigid’ bodies is reaching limits with the increasing design optimization.

The dynamic response of a mechanical system is not only governed by the flexibility of components. Compliances between the different components do not only influence the system response. They may have complex transfer functions and their proper function may be vital to the system. Rolling bearings are surely a representative of complex transfer functions.

It is a common practice to replace rolling bearings by simplified linear or nonlinear force functions. More complex modelling utilizes the theory from Hertz, which is widely described in literature. This paper describes a complete modelling process based on finite element analysis, where accurate contact stiffness replaces the application of the theory of Hertz. Although complex modelling is applied, high user friendliness and high performance make this method more than competitive to simplified modelling.

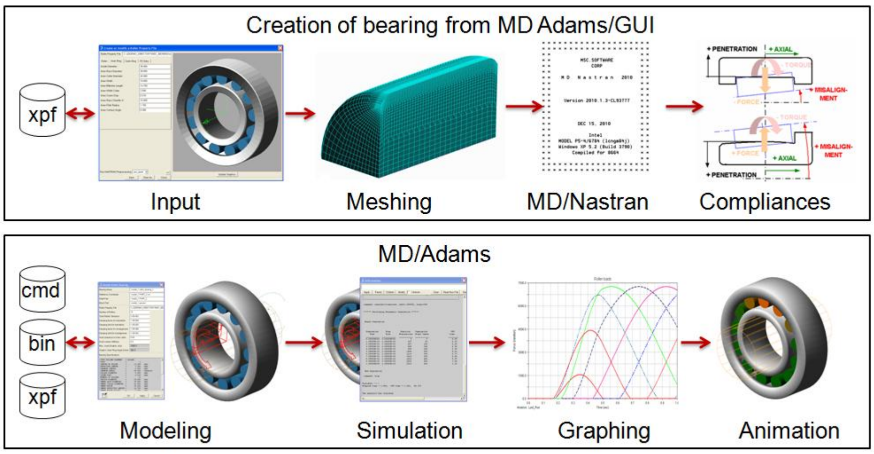


Figure 1: modelling process rolling bearings

* Email: Johann.Zeischka@mscsoftware.com

Modelling process

The modelling process is depicted on figure 1. A rolling bearing is defined by a limited number of geometric dimensions. Upon completion of all input, fine finite element meshes for rolling elements and rings are automatically generated and submitted to finite element analysis. The results of the finite element analysis are utilized by a fast contact processor, which determines the compliances of the rolling elements against the ring. The accurate compliances are stored and utilized by a force element in the multi body code Adams, which considers the relative position, orientation and rotation of the shaft against the housing and the clearance.

Simulation results

Some characteristic results of the modelling process are shown and discussed. Case studies confirm the high performance and the ability of the new development to improve system simulation and to provide simultaneously some insight into the bearing design.

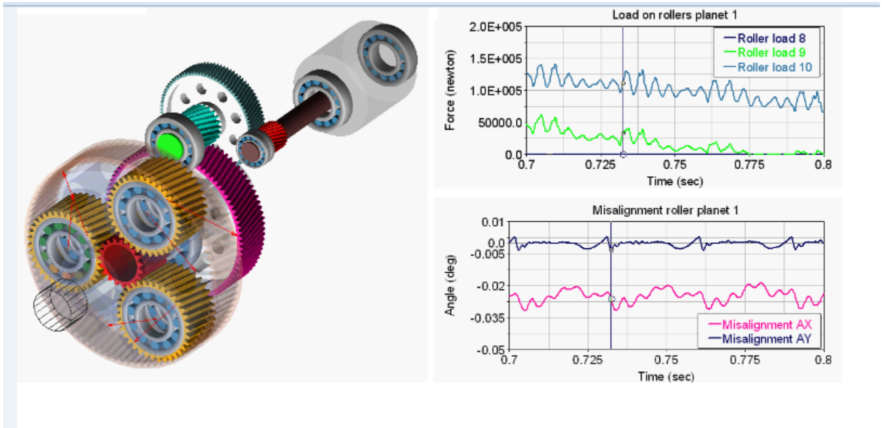


Figure 2: results of case study

Acknowledgements

The author is very grateful to all the customers, who contributed in one or the other way to this technology. Further I want to thank Tarik El-Dsoki for his support at MSC.Software.

References

[1] J. Zeischka, K. Osodo, J. Saito, T. Iwamoto, Ball bearing modeling based on accurate contact stiffness for efficient mechanical system simulation, IUTAM 2006

[2] To appear; H. Ott, J. Zeischka, Dynamic analysis of a refiner-rotating-unit employing Adams/Bearing AT, NAFEMS, Wiesbaden, 8.-9.November 2011