Supplementary Materials

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(Dated: April 5, 2019)

I. WHY TEMPERATURE FREQUENCY OF ONE HEATER IS ALSO REFLECTED ON THE SPECTROGRAM OF THE OTHER?

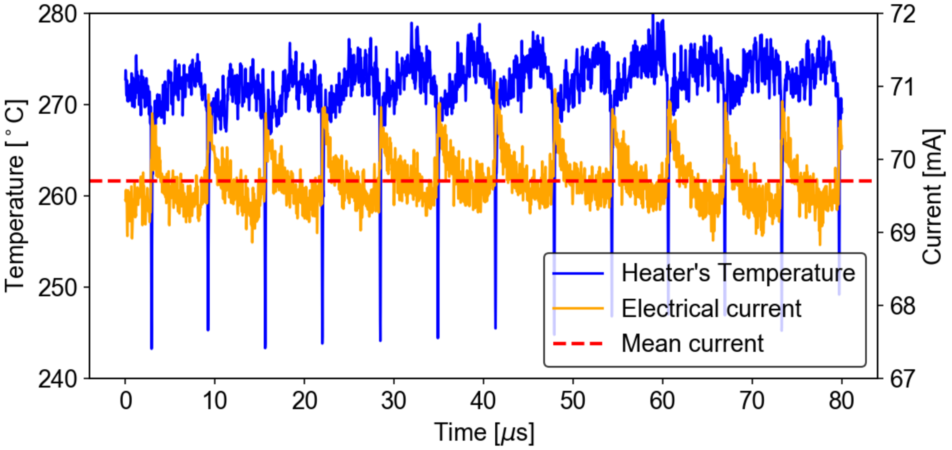

FIG. S 1: Instantaneous temperature and electrical current of a 15μm × 15μm heater that is producing stable oscillate boiling bubble. Here the average heating power is 86 mW.

Figure 8 in the main text shows a typical spectrogram of the emitted acoustic signal and the temperature of each heater in the regime where two bubbles oscillate independently. An interesting observation is that the temperature frequency of one heater is also reflected on that of the other heater. To understand this, we look back to the case of a single heater. Figure S1 shows the instantaneous temperature and electrical current of a 15μm × 15μm heater that is producing stable oscillate boiling bubble. During the bubble's collapse, the liquid jet impact induces a rapid cooling in the heater, resulting in a temperature oscillation matching the bubble's frequency. The electrical current, however, also oscillates even though the supplying device was set to operate in constant current mode. The reason is that in every cycle, the liquid jet cools down the heater and make its resistance drop. In order to keep the same current, the supplied voltage needs to be reduced accordingly. However, the bandwidth of the device is only up to 50 kHz, while the bubble frequency is about 200 kHz. The voltage therefore cannot keep up with the bubble and leads to the oscillation of the electrical current. When we have two oscillate boiling bubbles, this effect takes place at both heaters and results in two frequencies of the electrical current. Since two heaters are connected in series, they share the same current and are both affected. Therefore, we see two oscillations pattern in the temperature signal of each heater: A stronger one, created

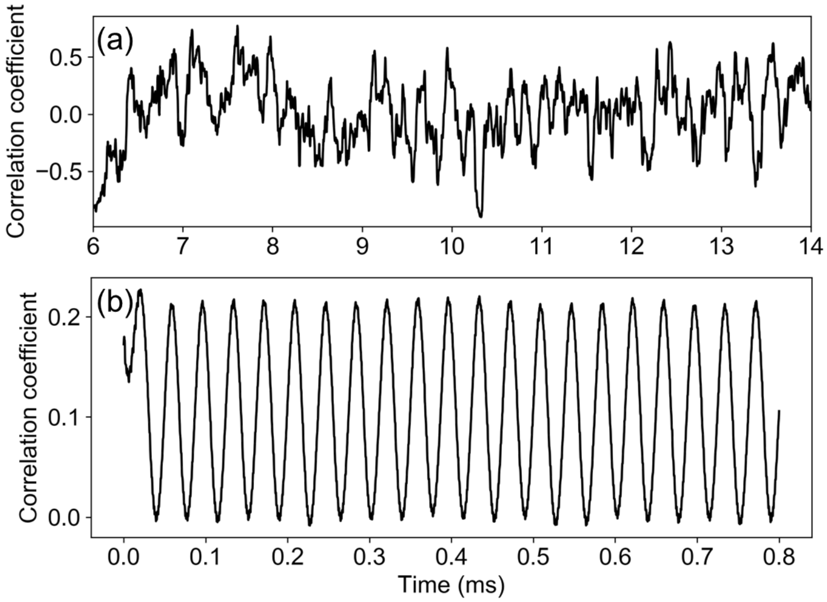

FIG. S 2: (a) Correlation coefficient between two bubble that corresponds to (a) Fig.3 of the main text and (b) Fig.6a of the main text. In both case the coefficient is calculated with a window time of 100 μs.

directly by its own bubble, and a weaker one, induced by the neighbor bubble through the electrical current. In another way, in stable oscillate boiling, the bubbles can communicate with each other both acoustically and electrically. This finding does not have a big impact on the current study of two bubbles. However, if one wants to scale-up oscillate boiling for application purpose, with many more heaters involved, clearly this effect should be considered.

II. CORRELATION COEFFICIENT IN INDEPENDENT OSCILLATION

Figure S2 shows the correlation coefficient between the bubble's radii when they are not in synchronization, both experimentally (Fig.3 of main text) and numerically (Fig.6a of main text). It can be seen that in both case the bubble's dynamics are uncorrelated with the correlation coefficient fluctuates between -0.5 and 0.5.

In the numerical results, if the calculation time is extended to milliseconds range, an interesting pattern is observed: within our parameter space with distance d up to 100 μm and power P up to 60 mW, for every X cycles of bubble 1, there are exactly Y cycles of bubble 2. Here X and Y are integer numbers, which depends on the value of the d and P. In an extreme case with $d = 500\mu$m, this is not observed. This suggest that the coupling term might still support some correlations between bubbles in the numerical system. However, we currently do not have a complete explanation for this observation.

III. PARAMETERS STUDY OF THE BUBBLE'S RADIUS AND DISTANCE

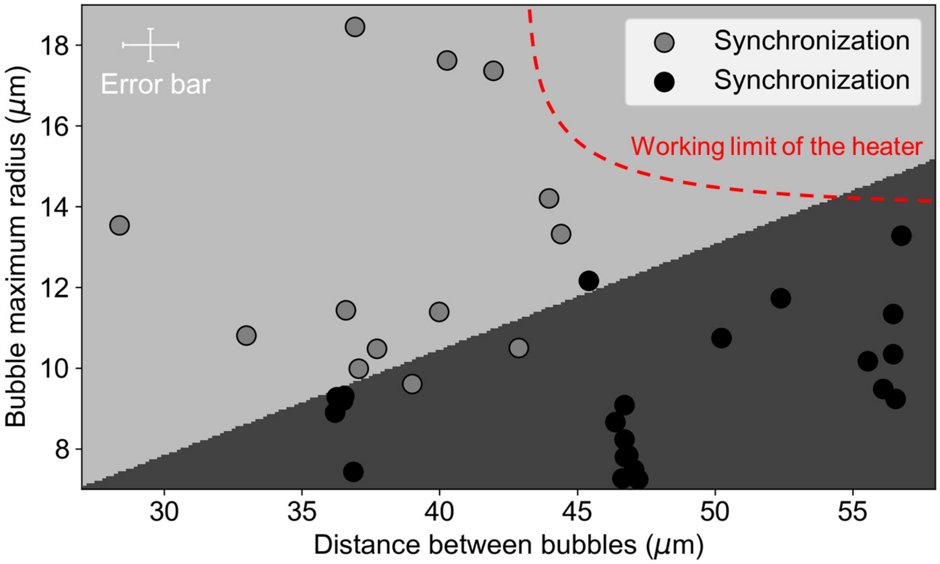

FIG. S 3: Distribution of the independence and synchronization regimes on the bubble's maximum radius and the distance between their centers of mass. Here the boundary line is calculated by fitting the data points to a logistic regression plane.

Figure S3 shows the distribution of the different regimes on the bubble's maximum radius and the distance between their centers of mass. In the synchronization regime, the bubble's distance and maximum radius are not constant. As shown in Fig.3 of the main text, the former keeps decreasing while the later keeps increasing. Therefore, here we choose a reference period of 0.1ms after the beginning of the synchronization to collect the data for this plot.

It can be seen that compared to the case of independent oscillation, synchronization tends to have larger radius and smaller distance. The first author initially came up with the hypothesis that these two parameters are determining the state of the system. One possible explanation is that small distance and large radius create stronger acoustical interaction and makes it easier for the bubbles to synchronize. However, this hypothesis has two problems. Firstly, it cannot cover the third regime where the bubbles just grow and merge. This case has no clear reference point like the synchronization case, thus the bubble's radius and distance are not well-defined. Secondly, these two parameters are in fact the direct results of the bubble's individual dynamics, i.e. if the bubble oscillates in unstable mode, it will accumulate gas and grow, thus having greater radius and smaller distance. Therefore, we believe that the oscillation mode of each bubble is the true origin and it is the better way to explain the interactive behaviors. The data presented here is for keenly interested readers to have a better view of our experimental results.

IV. OBSERVATION OF "MIXED MODE"

The samples consist of two identical heaters connected in series. Ideally, they always produce the same heating power which makes two bubble having only one mode, either both stable or both unstable. Occasionally, however, one heater is damaged after several runs and has a higher resistance, which allows it to draw more power as compared to the other. The symmetry is therefore broken and a so-called "mixed mode" can be observed, where one bubble oscillates in the stable state while the other unstable.

Figure S4 presents one of the mixed mode observation. In this case the damaged, higher power left heater generates unstable oscillate boiling bubble while the normal right heater generates the stable one. It can be seen that in this mixed mode, even though there is some interactions, the two bubbles do not show any obvious correlation. The unstable bubble has greater radius and lower frequency, oscillates for a few milliseconds then stops and grows. The stable bubbles initially has its frequencies interfered by its neighbor. After the source of interference is gone, this bubble quickly adjusts to its stable size and frequency. The same qualitative behavior is also seen in the numerical calculation, where bubble 1 and 2 are put in the stable and unstable mode, respectively. This mix mode, however, is an uncommon event which takes place purely by accident and not very repeatable

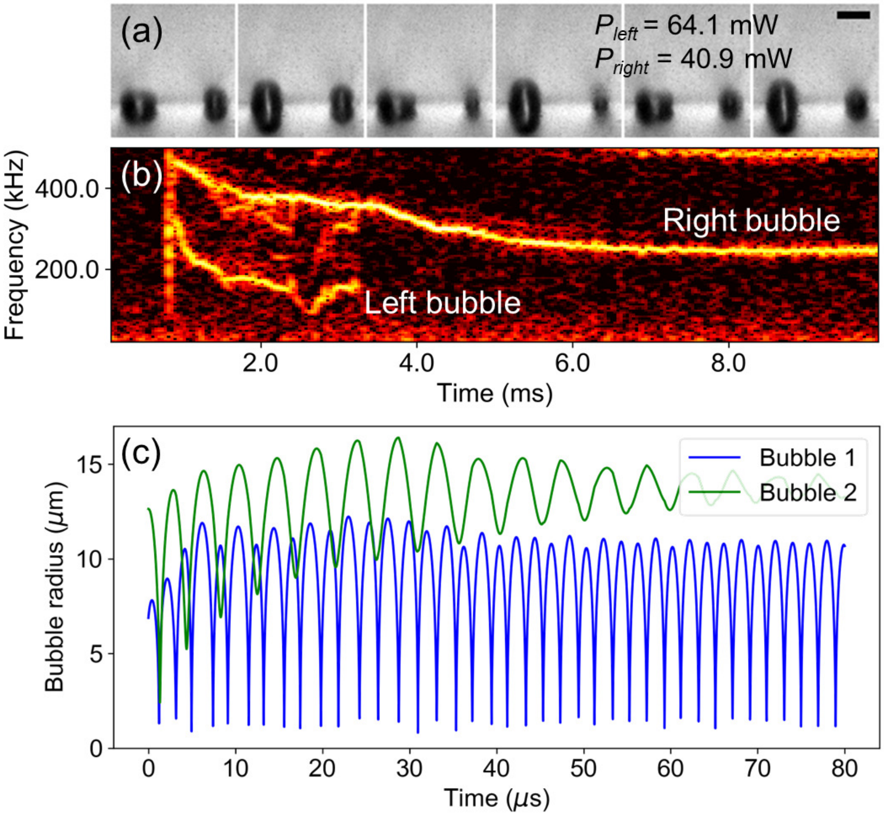

FIG. S 4: (a) Captured images when the left bubble shows unstable oscillate boiling and the right bubble shows stable oscillate boiling. The inter-frame duration is 3.33 μs and the scale bar represents 20 μm. (b) The spectrogram of the emitted acoustic signal. (c) Calculation of the evolution of bubble's dynamic in mixed mode with bubble 1 in stable mode and bubble 2 in unstable mode.

V. VIDEOS

Video 1: The first regime where two bubbles oscillate stably and independently. Here the power of the left heater is 25.4 mW, of the right heater is 25.1 mW. Images shown in Fig. 2 of the main text are taken from this video.

Video 2 a-d: All stages of the second regime where two bubbles oscillate in unstable mode then synchronize. Here the power of the left heater is 34.9 mW, of the right heater is 36.0 mW. **(a):** The initial explosion. **(b):** The initial independent oscillation. **(c):** Two bubbles synchronize. The images shown in Fig. 3 in the main text are taken from this video. **(d):** Two bubbles merge.

Video 3: The third regime where two bubbles grow and merges. Here the power of the left heater is 48.0 mW, of the right heater is 47.7 mW. Images shown in Fig. 4 of the main text are taken from this video.

In all videos, the scale bar represents 20 μm.