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Regulated state synchronization of homogeneous multiagent systems with partial-state coupling via low-gain adaptive protocol

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Summary

This paper studies regulated state synchronization for continuous-time homogeneous multiagent systems with weakly unstable agents where the reference trajectory is given by a so-called exosystem. The agents share part of their state over a communication network. We assume that the communication topology is completely unknown and directed. An algebraic Riccati equation–based low-gain adaptive nonlinear dynamic protocol design is presented to achieve the regulated state synchronizations. Utilizing the adaptive control, our nonlinear dynamic protocol is universal and does not depend on any information about the communication topology or the number of agents.

KEYWORDS

adaptive, low-gain, multiagent systems, regulated state synchronization

1 | INTRODUCTION

The problem of synchronization among agents in a multiagent system (MAS) has received substantial attention in recent years, because of its potential applications in cooperative control of autonomous vehicles, distributed sensor network, swarming and flocking, and others. The objective of synchronization is to secure an asymptotic agreement on a common state or output trajectory through decentralized control protocols (see other works[1-4] and references therein).

State synchronization basically requires a homogeneous MAS (ie, agents have identical dynamics),[5] which is therefore the focus of this paper. For state synchronization, two kinds of communication networks are considered, one pertaining to full-state coupling (full state is shared over the network) and the other to partial-state coupling (part of the state is shared over the network). Initially, most work has focused on state synchronization based on diffusive full-state coupling, where the agent dynamics progress from single- and double-integrator dynamics (eg, other works[6-10]) to more general higher-order dynamics (eg, other works[11-14]). State synchronization based on diffusive partial-state coupling has also been considered in other works.[11,15-20]

For partial-state coupling, state synchronization can be achieved via a dynamic protocol or a static protocol. For a MAS with partial-state coupling, some work has focused on conditions to achieve consensus by using static output feedback protocols like other works.[20-22] It is shown that only a restricted class of agents satisfy these solvability conditions. This class of agents basically consists of passive or passifiable agents.

Most of the literature considered dynamic protocols when considering partial state coupling. When using dynamic protocols, synchronization can be achieved for a larger class of agents. In the literature on MAS with partial-state coupling and purely decentralized protocols, two main classes have been identified for which this problem is solvable.[23]

- Agents are at most weakly unstable (ie, poles are in the closed left half plane).
- Agents are at most weakly nonminimum phase (ie, zeros are in the closed left half plane).

In this paper, we focus on the synchronization of a MAS with at most weakly unstable agents, which clearly implies that the agents can exhibit polynomial growth but not exponential instability. For this class of agents, most protocol designs are based on a low-gain design methodology. In the work of Seo et al,[18] general multi-input–multioutput agents with all eigenvalues in the closed-left half plane are considered and a low-gain based protocol is proposed. This work is extended in the work of Wang et al[24] to tolerate time delay in the input for each agent. In the work of Yang et al,[25] precompensators were developed to shape nonidentical agents to almost identical agents with all eigenvalues in the closed-left half plane.

Meanwhile, some researchers have tried to utilize the adaptive method in the protocol design for state synchronization of a MAS to avoid using the knowledge of the communication network.

In the work of Li et al,[26] the average state synchronization for a MAS with an undirected communication graph and with full-state coupling was considered and an adaptive protocol was designed. Furthermore, in the work of Li et al,[27] a fully adaptive protocol was developed to achieve leader-follower state synchronization of a MAS with a directed communication graph and full-state coupling.

This work is extended by the work of Li et al[28] for a MAS with partial-state coupling with additional information exchange. A higher-order adaptive protocol is designed in the work of Lv et al[29] for the leader-follower state synchronization of a MAS with partial-state coupling, which again requires additional information exchange.

In this paper, we study regulated state synchronization for homogeneous MAS when the agents are at most weakly unstable with partial state coupling but without additional information exchange. The main objective of this paper is to design low-gain adaptive protocol so that the state of each agents follow a reference trajectory generated by a so-called exosystem. We utilize an adaptive protocol to remove the need for any a priori knowledge about the communication network or even the number of agents.

We achieve regulated state synchronization for a MAS with at most weakly unstable agents with an adaptive protocol, which does not require additional information exchange. A subset $\mathcal{C}$ of the nodes obtain information about the exosystem and our protocol does not require any a priori information as long as each agent is connected to an agent from this subset $\mathcal{C}$.

1.1 | Notations and definitions

Given a matrix $A \in \mathbb{R}^{m\times n}$, A^{T} denotes its conjugate transpose and $\|A\|$ is the induced 2-norm. A square matrix A is said to be Hurwitz stable if all its eigenvalues are in the open left half complex plane. We denote by $\operatorname{diag}\{A_1, \dots, A_N\}$, a block-diagonal matrix with $A_1, \dots, A_N$ as the diagonal elements. $A \otimes B$ depicts the Kronecker product between A and B. I_n denotes the n-dimensional identity matrix and 0_n denotes $n \times n$ zero matrix; sometimes, we drop the subscript if the dimension is clear from the context.

A *weighted graph* $\mathcal{G}$ is defined by a triple $(\mathcal{V}, \mathcal{E}, \mathcal{A})$, where $\mathcal{V} = \{1, \dots, N\}$ is a node set, $\mathcal{E}$ is a set of pairs of nodes indicating connections among nodes, and $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ is the weighting matrix. Each pair in $\mathcal{E}$ is called an *edge*, where $a_{ij} > 0$ denotes an edge $(j, i) \in \mathcal{E}$ from node j to node i with weight a_{ij}. Moreover, $a_{ij} = 0$ if there is no edge from node j to node i. We assume there are no self-loops, i.e. we have $a_{ii} = 0$. A *path* from node i_1 to i_k is a sequence of nodes $\{i_1, \dots, i_k\}$ such that $(i_j, i_{j+1}) \in \mathcal{E}$ for $j = 1, \dots, k-1$. A *directed tree* with root r is a subgraph of the graph $\mathcal{G}$ in which there exists a unique path from node r to each node in this subgraph. A *directed spanning tree* is a directed tree containing all the nodes of the graph.

For a weighted graph $\mathcal{G}$, the matrix $L = [\ell_{ij}]$ with

$$\ell_{ij} = \begin{cases} \sum_{k=1}^{N} a_{ik}, & i = j, \\ -a_{ij}, & i \neq j, \end{cases}$$

is called the *Laplacian matrix* associated with the graph $\mathcal{G}$. The Laplacian matrix L has all its eigenvalues in the closed-right half plane and at least one eigenvalue at zero associated with right eigenvector $\mathbf{1}$.

2 | PROBLEM FORMULATION

Consider a MAS consisting of N identical linear dynamic agents

$$\begin{cases} \dot{x}_i(t) &= Ax_i(t) + Bu_i(t), \\ y_i(t) &= Cx_i(t), \end{cases} \tag{1}$$

where $x_i \in \mathbb{R}^n$, $y_i \in \mathbb{R}^q$ and $u_i \in \mathbb{R}^m$ are the state, output, and the input of agent $i = 1, \dots, N$, respectively.

The network provides agent i with the following information:

$$\zeta_i(t) = \sum_{j=1}^{N} a_{ij}(y_i(t) - y_j(t)), \tag{2}$$

where $a_{ij} \geq 0$ and $a_{ii} = 0$. This communication topology of the network can be described by a weighted graph $\mathcal{G}$ associated with (2), with the a_{ij} being the coefficients of the weighting matrix $\mathcal{A}$. In terms of the coefficients of the associated Laplacian matrix L, ζ_i can be rewritten as

$$\zeta_i(t) = \sum_{j=1}^{N} \ell_{ij} y_j(t). \tag{3}$$

We refer to this as *partial-state coupling* since only part of the states are communicated over the network.

For homogeneous MAS, such as in this paper, almost all papers considered state synchronization without imposing requirements on the synchronized trajectory. However, for heterogenous agents, it has been shown in the works of Wieland et al[30] and Grip et al[31] that we basically need to consider regulated state synchronization where the objective of the agents is to ensure that their state asymptotically tracks a reference trajectory generated by a so-called exosystem. Although our paper considers homogeneous MAS, the adaptation required to handle the lack of information about the network introduces a certain level of heterogeneity. Therefore, we also consider regulated state synchronization in this paper.

The reference trajectory is generated by the following exosystem:

$$\begin{aligned} \dot{x}_r &= Ax_r \\ y_r &= Cx_r, \end{aligned} \tag{4}$$

with $x_r \in \mathbb{R}^n$. Our objective is that the agents achieve regulated state synchronization, ie,

$$\lim_{t\to\infty}(x_i - x_r) = 0, \tag{5}$$

for all $i \in \{1, \dots, N\}$. Clearly, we need some level of communication between the exosystem and the agents. We assume that a nonempty subset $\mathscr{C}$ of the agents have access to their own output relative to the output of the exosystem. Specially, each agent i has access to the quantity

$$\psi_i = \iota_i(y_i - y_r), \qquad \iota_i = \begin{cases} 1, & i \in \mathscr{C}, \\ 0, & i \notin \mathscr{C}. \end{cases} \tag{6}$$

Combining this with (3), we have the following information exchange:

$$\bar{\zeta}_i = \sum_{j=1}^{N} a_{ij}(y_i - y_j) + \iota_i(y_i - y_r). \tag{7}$$

To guarantee that each agent can achieve the required regulation, we need that there exists a path to each node starting with node from the set $\mathscr{C}$. In other words, we need the following assumption on the network graph

Assumption 1. Every node of the network graph $\mathcal{G}$ is a member of a directed tree, which has its root contained in the set $\mathscr{C}$.

Given the set C, we denote by $\mathbb{G}_C$ the set of all graphs, which contains all the nodes from the set C and satisfies Assumption 1. For any graph $\mathcal{G} \in \mathbb{G}_C$, with the Laplacian matrix L, we define the expanded Laplacian matrix as

$$\bar{L} = L + \text{diag}\{\iota_i\} = [\bar{\ell}_{ij}]_{N\times N}.$$

Note that $\bar{L}$ is not a regular Laplacian matrix associated to some graph since it does not have a zero row sum. From lemma 7 in the work of Grip et al,[32] we know that Assumption 1 guarantees that all eigenvalues of $\bar{L}$ are in the open right-half complex plane. In particular, the matrix $\bar{L}$ is invertible. Meanwhile, there exists a matrix $Z = \text{diag}(z_1, \dots, z_N) > 0$ such that

$$Z\bar{L} + \bar{L}^{\mathrm{T}}Z > 0. \tag{8}$$

This result can be found in Theorem 4.25 in the work of Qu.[33]

We formulate the problem for regulated state synchronization of a MAS with partial-state coupling.

Problem 1. Consider a MAS described by (1) and (2) and the associated exosystem (4). Let a set of nodes C be given, which defines the set $\mathbb{G}_C$. Let the asssociated network communication graph $\mathcal{G} \in \mathbb{G}_C$ be given by (7).

The **regulated state synchronization problem with partial-state coupling** of a MAS is to find a nonlinear dynamic protocol of the form

$$\begin{cases} \dot{x}_{c,i} &= \bar{f}(x_{c,i}, \bar{\zeta}_i(t)), \\ u_i(t) &= \bar{g}(x_{c,i}, \bar{\zeta}_i(t)), \end{cases} \tag{9}$$

where $x_{c,i} \in \mathbb{R}^{n_i}$, such that for any N and any graph $\mathcal{G} \in \mathbb{G}_C$, regulated state synchronization (5) is achieved for all agents.

We show by explicit design of a protocol that regulated synchronization, as defined in Problem 1, can be achieved for the class of agents which are at most weakly unstable.

3 | REGULATED STATE SYNCHRONIZATION FOR A MAS WITH WEAKLY UNSTABLE AGENTS

In this section, we will consider the regulated state synchronization problem for a MAS with weakly unstable agents via partial-state coupling.

We will design an adaptive nonlinear protocol for agent $i \in \{1, \dots, N\}$ as follows:

$$\begin{cases} \dot{\chi}_i &= (A + K_{\varepsilon_i}C)\chi_i - K_{\varepsilon_i}\bar{\zeta}_i \\ u_i &= -B^{\mathrm{T}}P_{\varepsilon_i}\chi_i, \\ \dot{\varepsilon}_i &= -\varepsilon_i^2 u_i^{\mathrm{T}} u_i, \end{cases} \tag{10}$$

for $i = 1, \dots, N$, where ε_i is an adaptive parameter satisfying $\varepsilon_i(0) \in (0, 1]$,

We have

$$K_{\varepsilon_i} = -\frac{1}{\varepsilon_i}QC^{\mathrm{T}},$$

and $Q > 0$ is the unique solution of the Algebraic Riccati Equation (ARE)

$$AQ + QA^{\mathrm{T}} - QC^{\mathrm{T}}CQ + I = 0. \tag{11}$$

Moreover, P_{ε_i} satisfies

$$A^{\mathrm{T}}P_{\varepsilon_i} + P_{\varepsilon_i}A - \varepsilon_i^{\nu}P_{\varepsilon_i}BB^{\mathrm{T}}P_{\varepsilon_i} + \varepsilon_i P_{\varepsilon_i} = 0 \tag{12}$$

for $\nu > 0$. Note that there exist unique solutions for (11) and (12), for any $\varepsilon_i > 0$ and $\nu > 0$.

Then, the synchronization result based on adaptation is stated in Theorem 1. To obtain this result, we need the following lemma.

Lemma 1. *There exists ν small enough such that P_ε satisfying*

$$A^{\mathrm{T}}P_{\varepsilon} + P_{\varepsilon}A - \varepsilon^{\nu}P_{\varepsilon}BB^{\mathrm{T}}P_{\varepsilon} + \varepsilon P_{\varepsilon} = 0$$

is increasing in ε. Moreover P_ε is of order $\varepsilon^{1-\nu}$.

Proof. Note that

$$P_\varepsilon = \varepsilon^{-\nu}\bar{P}_\varepsilon$$

with $\bar{P}_\varepsilon$ the solution of the standard Riccati equation

$$A^{\mathrm{T}}\bar{P}_\varepsilon + \bar{P}_\varepsilon A - \bar{P}_\varepsilon BB^{\mathrm{T}}\bar{P}_\varepsilon + \varepsilon\bar{P}_\varepsilon = 0.$$

Note that the derivative of P_ε with respect to ε is

$$\dot{P}_\varepsilon = -\nu\varepsilon^{-\nu-1}\bar{P}_\varepsilon + \varepsilon^{-\nu}\dot{\bar{P}}_\varepsilon = \varepsilon^{-\nu-1}\left[\varepsilon\dot{\bar{P}}_\varepsilon - \nu\bar{P}_\varepsilon\right],$$

and therefore, it is sufficient to show that

$$X_\varepsilon = \varepsilon\dot{\bar{P}}_\varepsilon - \nu\bar{P}_\varepsilon \geq 0.$$

By differentiating the Riccati equation with respect to ε, we obtain

$$\dot{\bar{P}}_\varepsilon(A - BB^{\mathrm{T}}\bar{P}_\varepsilon) + (A - BB^{\mathrm{T}}\bar{P}_\varepsilon)^{\mathrm{T}}\dot{\bar{P}}_\varepsilon + \bar{P}_\varepsilon + \varepsilon\dot{\bar{P}}_\varepsilon = 0.$$

Moreover, this yields

$$X_\varepsilon\left(A - BB^{\mathrm{T}}\bar{P}_\varepsilon\right) + \left(A - BB^{\mathrm{T}}\bar{P}_\varepsilon\right)^{\mathrm{T}}X_\varepsilon = -\varepsilon\bar{P}_\varepsilon - \varepsilon^2\dot{\bar{P}}_\varepsilon + \nu\bar{P}_\varepsilon BB^{\mathrm{T}}\bar{P}_\varepsilon + \nu\varepsilon\bar{P}_\varepsilon.$$

Following the property of parametric low-gain Ricatti equation, (see Theorems 1 and 4 in the work of Zhou et al[34]) $\dot{\bar{P}}_\varepsilon > 0$, then we get

$$X_\varepsilon\left(A - BB^{\mathrm{T}}\bar{P}_\varepsilon\right) + \left(A - BB^{\mathrm{T}}\bar{P}_\varepsilon\right)^{\mathrm{T}}X_\varepsilon \leq 0$$

provided

$$\nu\bar{P}_\varepsilon BB^{\mathrm{T}}\bar{P}_\varepsilon \leq \varepsilon(1-\nu)\bar{P}_\varepsilon. \tag{13}$$

Since $A - BB^{\mathrm{T}}\bar{P}_\varepsilon$ is asymptotically stable, this implies $X_\varepsilon \geq 0$ as required.

To verify (13), we note that, according to the work of Zhou et al,[34] the matrix $\bar{P}_\varepsilon$ is rational in ε with $\bar{P}_0 = 0$. Hence,

$$\bar{P}_\varepsilon = \varepsilon\bar{P}_{1,\varepsilon}$$

with $\bar{P}_{1,\varepsilon}$ rational in ε. Since $\bar{P}_\varepsilon$ is well-defined for all $\varepsilon \in [0,1]$, we find that the rational matrix $\bar{P}_{1,\varepsilon}$ has no poles in the interval $[0,1]$, which yields that $\bar{P}_{1,\varepsilon}$ is bounded. We needed to show

$$\nu\bar{P}_{1,\varepsilon}BB^{\mathrm{T}}\bar{P}_{1,\varepsilon} \leq (1-\nu)\bar{P}_{1,\varepsilon}$$

or, equivalently,

$$\nu\bar{P}_\varepsilon^{1/2}BB^{\mathrm{T}}\bar{P}_\varepsilon^{1/2} \leq (1-\nu)I,$$

which is obviously satisfied for all $\varepsilon \in [0,1]$ for $\nu > 0$ sufficiently small since $\bar{P}_{1,\varepsilon}$ is bounded. From the work of Zhou et al,[34] $\bar{P}_\varepsilon$, is of order ε; hence, P_ε is of order $\varepsilon^{1-\nu}$. □

Our formal result is stated in the following theorem.

Theorem 1. *Consider a MAS described by (1) and the associated exosystem (4). Let a set of nodes C be given, which defines the set $\mathbb{G}_C$. Let the asssociated network communication be given by (7).*

Assume (A, B) is stabilizable and (A, C) detectable; then, the regulated state synchronization problem as stated in Problem 1 is solvable. In particular, the adaptive nonlinear dynamic protocol (10) solves the regulated state synchronization problem for any N and any graph $\mathcal{G} \in \mathbb{G}_C$, provided $\nu \in (0, \frac{1}{4})$, satisfies Lemma 1 and $\varepsilon_i(0) < 1$ is such that

$$\varepsilon_i^{-3/2}P_{\varepsilon_i}BB^{\mathrm{T}}P_{\varepsilon_i} \leq \frac{1}{6}Q^{-2} \tag{14}$$

for all $\varepsilon_i < \varepsilon_i(0)$, which is always possible.

Proof of Theorem 1. From Lemma 1, we know that P_ε is of order $\varepsilon^{1-\nu}$. Therefore, for $\nu < 1/4$, we find that the left-hand side of (14) converges to zero as $\varepsilon \to 0$. Therefore, the required $\varepsilon_i(0)$ always exist. Next, let

$$\tilde{x}_i = x_i - x_r,$$
$$\tilde{y}_i = y_i - y_r.$$

We have

$$\begin{aligned}
\dot{\tilde{x}}_i &= A\tilde{x}_i + Bu_i \\
\tilde{y}_i &= C\tilde{x}_i \\
\bar{\zeta}_i &= \sum_{j=1}^{N} \bar{\ell}_{ij}\tilde{y}_j \\
\dot{\chi}_i &= (A + K_{\varepsilon_i}C)\chi_i - K_{\varepsilon_i}\bar{\zeta}_i \\
u_i &= -B^{\mathrm{T}}P_{\varepsilon_i}\chi_i.
\end{aligned} \tag{15}$$

Then, we have

$$\tilde{x} = \begin{pmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \vdots \\ \tilde{x}_N \end{pmatrix}, u = \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_N \end{pmatrix}, \tilde{y} = \begin{pmatrix} \tilde{y}_1 \\ \tilde{y}_2 \\ \vdots \\ \tilde{y}_N \end{pmatrix}, \chi = \begin{pmatrix} \chi_1 \\ \chi_2 \\ \vdots \\ \chi_N \end{pmatrix}$$

and

$$\begin{aligned}
p &= (\bar{L} \otimes I)\tilde{x} \\
e &= (\bar{L} \otimes I)\tilde{x} - \chi.
\end{aligned}$$

We get

$$\dot{e} = \left[(I \otimes A) + \bar{K}_\varepsilon(I \otimes C)\right] e + (\bar{L} \otimes B)u \tag{16}$$

$$\dot{p} = (I \otimes A)p + (\bar{L} \otimes B)u \tag{17}$$

$$u = -(I \otimes B^{\mathrm{T}})\bar{P}_\varepsilon(p - e), \tag{18}$$

where

$$\bar{P}_\varepsilon = \begin{pmatrix} P_{\varepsilon_1} & & & \\ & P_{\varepsilon_2} & & \\ & & \ddots & \\ & & & P_{\varepsilon_N} \end{pmatrix}, \ \varepsilon = \begin{pmatrix} \varepsilon_1 & & & \\ & \varepsilon_2 & & \\ & & \ddots & \\ & & & \varepsilon_N \end{pmatrix},$$

and

$$\bar{K}_\varepsilon = \begin{pmatrix} K_{\varepsilon_1} & & & \\ & K_{\varepsilon_2} & & \\ & & \ddots & \\ & & & K_{\varepsilon_N} \end{pmatrix}.$$

If ε_i for $i = 1, \ldots N$ are bounded away from zero, then we have that $u_i \in \mathcal{L}_2$ for all i, ie, $u \in \mathcal{L}_2$.

Since $(I \otimes A) + \bar{K}_\varepsilon(I \otimes C)$ converges to an asymptotically stable matrix while $u \in \mathcal{L}_2$, based on the classical input-output property of linear systems,[35] we find from (16) that $e \in \mathcal{L}_2$. This implies that

$$v = (I \otimes B^{\mathrm{T}})\bar{P}_\varepsilon p = (I \otimes B^{\mathrm{T}})\bar{P}_\varepsilon e - (I \otimes B^{\mathrm{T}})u \in \mathcal{L}_2.$$

However, then (17) yields that

$$\dot{p} = \left[(I \otimes A) - (I \otimes BB^{\mathrm{T}})\bar{P}_\varepsilon\right] p + (I \otimes B)v + (\bar{L} \otimes B)u,$$

and therefore, $p \in \mathcal{L}_2$ since $(I \otimes A) - (I \otimes BB^{\mathrm{T}})\bar{P}_\varepsilon$ converges to an asymptotically stable matrix while $v \in \mathcal{L}_2$ and $u \in \mathcal{L}_2$. This immediately implies that synchronization is achieved.

We will show by contradiction that all ε_i for $i = 1, \ldots, N$ are bounded away from zero. Assuming that this is not true, we find that there exist some agents, for which $\varepsilon_i(t) \to 0$ as $t \to \infty$. Without loss of generality, we assume that, for agents $i = 1, \ldots, k$, we have $\varepsilon_i(t) \to 0$ (ie, $u_i \notin \mathcal{L}_2$), while for agents $i = k+1, \ldots, N$, we have that $\varepsilon_i(t)$ is bounded away from zero (ie, $u_i \in \mathcal{L}_2$). We define

$$\tilde{x}_I = \begin{pmatrix} \tilde{x}_1 \\ \vdots \\ \tilde{x}_k \end{pmatrix}, \quad u_I = \begin{pmatrix} u_1 \\ \vdots \\ u_k \end{pmatrix}, \quad \tilde{y}_I = \begin{pmatrix} \tilde{y}_1 \\ \vdots \\ \tilde{y}_k \end{pmatrix}, \quad \chi_I = \begin{pmatrix} \chi_1 \\ \vdots \\ \chi_k \end{pmatrix},$$

and

$$\tilde{x}_{II} = \begin{pmatrix} \tilde{x}_{k+1} \\ \vdots \\ \tilde{x}_N \end{pmatrix}, \quad u_{II} = \begin{pmatrix} u_{k+1} \\ \vdots \\ u_N \end{pmatrix}, \quad \tilde{y}_{II} = \begin{pmatrix} \tilde{y}_{k+1} \\ \vdots \\ \tilde{y}_N \end{pmatrix}, \quad \chi_{II} = \begin{pmatrix} \chi_{k+1} \\ \vdots \\ \chi_N \end{pmatrix}.$$

Note that, in the special case where $k = N$ (ie, all ε_i converge to zero), the aforementioned decomposition is not needed. The following proof is then still valid, except all terms involving $\tilde{x}_{II}, u_{II}, \tilde{y}_{II}$, and χ_{II} are no longer present.

The subsystem consisting of agents 1, …, k can then be written as

$$\begin{aligned}\dot{\tilde{x}}_I &= (I \otimes A)\tilde{x}_I + (I \otimes B)u_I \\ \tilde{y}_I &= (I \otimes C)\tilde{x}_I\end{aligned} \tag{19}$$

$$\begin{aligned}\dot{\chi}_I &= (I \otimes A)\chi_I + \bar{K}_{I,\varepsilon}\left[(I \otimes C)\chi_I - (\bar{L}_1 \otimes C)\tilde{x}_I - (\bar{L}_2 \otimes C)\tilde{x}_{II}\right] \\ u_I &= -(I \otimes B^{\mathrm{T}})\bar{P}_{I,\varepsilon}\chi_I,\end{aligned} \tag{20}$$

where

$$\bar{P}_{I,\varepsilon} = \begin{pmatrix} P_{\varepsilon_1} & & & \\ & P_{\varepsilon_2} & & \\ & & \ddots & \\ & & & P_{\varepsilon_k} \end{pmatrix}, \quad \varepsilon_I = \begin{pmatrix} \varepsilon_1 & & & \\ & \varepsilon_2 & & \\ & & \ddots & \\ & & & \varepsilon_k \end{pmatrix},$$

and

$$\bar{K}_{I,\varepsilon} = \begin{pmatrix} K_{\varepsilon_1} & & & \\ & K_{\varepsilon_2} & & \\ & & \ddots & \\ & & & K_{\varepsilon_k} \end{pmatrix},$$

while

$$\bar{L} = \begin{pmatrix} \bar{L}_1 & \bar{L}_2 \\ \bar{L}_3 & \bar{L}_4. \end{pmatrix}$$

Meanwhile, based on (8), we have a diagonal matrix Z such that

$$Z\bar{L} + \bar{L}^{\mathrm{T}}Z > 0$$

with

$$Z = \begin{pmatrix} Z_I & 0 \\ 0 & Z_{II} \end{pmatrix}.$$

In particular, we have

$$\bar{L}_{Z,1} = Z_I\bar{L}_1 + \bar{L}_1^{\mathrm{T}}Z_I > 0.$$

We define

$$\begin{aligned}p_I &= (\bar{L}_1 \otimes I)\tilde{x}_I + (\bar{L}_2 \otimes I)\tilde{x}_{II} \\ e_I &= p_I - \chi_I.\end{aligned}$$

We get

$$\begin{aligned}\dot{e}_I &= \left[(I \otimes A) + \bar{K}_{I,\varepsilon}(I \otimes C)\right] e + (\bar{L}_1 \otimes B)u_I + (\bar{L}_2 \otimes B)u_{II} \\ \dot{p}_I &= (I \otimes A)p_I + (\bar{L}_1 \otimes B)u_I + (\bar{L}_2 \otimes B)u_{II}.\end{aligned}$$

We define

$$\begin{aligned}V_1 &= e_I^{\mathrm{T}}\left(Z_I\varepsilon_I \otimes Q^{-1}\right)e_I \\ V_2 &= p_I^{\mathrm{T}}(Z_I \otimes I)\bar{P}_{I,\varepsilon}p_I = (e_I + \chi_I)^{\mathrm{T}}(Z_I \otimes I)\bar{P}_{I,\varepsilon}(e_I + \chi_I).\end{aligned}$$

We find

$$\begin{aligned}\dot{V}_1 &= e_I^{\mathrm{T}}\left(Z_I\dot{\varepsilon}_I \otimes Q^{-1}\right)e_I - e_I^{\mathrm{T}}\left(Z_I\varepsilon_I \otimes Q^{-2}\right)e_I - e_I^{\mathrm{T}}\left[(2I - \varepsilon_I)Z_I \otimes C\mathrm{T}C\right]e_I \\ &\quad + 2e_I^{\mathrm{T}}\left(Z_I\varepsilon_I\bar{L}_1 \otimes Q^{-1}B\right)u_I + 2e_I^{\mathrm{T}}\left(Z_I\varepsilon_I\bar{L}_2 \otimes Q^{-1}B\right)u_{II} \\ \dot{V}_2 &= (e_I + \chi_I)^{\mathrm{T}}(Z_I\dot{\varepsilon}_I \otimes I)\dot{\bar{P}}_{I,\varepsilon}(e_I + \chi_I) + (e_I + \chi_I)^{\mathrm{T}}\left[\bar{P}_{I,\varepsilon}(Z_I \otimes A) + (Z_I \otimes A)^{\mathrm{T}}\bar{P}_{I,\varepsilon}\right](e_I + \chi_I) \\ &\quad - u_I^{\mathrm{T}}(\bar{L}_{Z,1} \otimes I)u_I + 2e_I^{\mathrm{T}}\bar{P}_{I,\varepsilon}(Z_I\bar{L}_1 \otimes B)u_I - 2u_I^{\mathrm{T}}(Z_I\bar{L}_2 \otimes I)u_{II} + 2e_I^{\mathrm{T}}\bar{P}_{I,\varepsilon}(Z_I\bar{L}_2 \otimes B)u_{II},\end{aligned}$$

and P_{ε_i} is increasing in ε_i, while ε_i is decreasing over time. Based on Lemma 1, $\dot{\bar{P}}_{I,\varepsilon} > 0$; therefore,

$$(e_I + \chi_I)^{\mathrm{T}}(Z_I\dot{\varepsilon}_I \otimes I)\dot{\bar{P}}_{I,\varepsilon}(e_I + \chi_I) < 0.$$

Then, we have

$$\begin{aligned}\dot{V}_1 &\leq -e_I^{\mathrm{T}}\left(Z_I\varepsilon_I \otimes Q^{-2}\right)e_I - e_I^{\mathrm{T}}\left[(2I - \varepsilon_I)Z_I \otimes C^{\mathrm{T}}C\right]e_I + 2e_I^{\mathrm{T}}\left(Z_I\varepsilon_I\bar{L}_1 \otimes Q^{-1}B\right)u_I + 2e_I^{\mathrm{T}}\left(Z_I\varepsilon_I\bar{L}_2 \otimes Q^{-1}B\right)u_{II} \\ \dot{V}_2 &\leq (e_I + \chi_I)^{\mathrm{T}}\left[\bar{P}_{I,\varepsilon}\left(Z_I\varepsilon_I^{\nu} \otimes BB^{\mathrm{T}}\right)\bar{P}_{I,\varepsilon}\right](e_I + \chi_I) - (e_I + \chi_I)^{\mathrm{T}}(Z_I\varepsilon_I \otimes I)\bar{P}_{I,\varepsilon}(e_I + \chi_I) \\ &\quad - u_I^{\mathrm{T}}(\bar{L}_{Z,1} \otimes I)u_I + 2e_I^{\mathrm{T}}\bar{P}_{I,\varepsilon}(Z_I\bar{L}_1 \otimes B)u_I - 2u_I^{\mathrm{T}}(Z_I\bar{L}_2 \otimes I)u_{II} + 2e_I^{\mathrm{T}}\bar{P}_{I,\varepsilon}(Z_I\bar{L}_2 \otimes B)u_{II},\end{aligned}$$

which yields

$$\begin{aligned}\dot{V}_1 \le & -e_I^{\mathrm{T}}\left(Z_I\varepsilon_I \otimes Q^{-2}\right)e_I - e_I^{\mathrm{T}}\left[(2I-\varepsilon_I)Z_I \otimes C^{\mathrm{T}}C\right]e_I \\ & + 2e_I^{\mathrm{T}}\left(Z_I\varepsilon_I\bar{L}_1 \otimes Q^{-1}B\right)u_I + 2e_I^{\mathrm{T}}\left(Z_I\varepsilon_I\bar{L}_2 \otimes Q^{-1}B\right)u_{II} \\ \dot{V}_2 \le & \, e_I^{\mathrm{T}}\left[\bar{P}_{I,\varepsilon}\left(Z_I\varepsilon_I^{\nu} \otimes BB^{\mathrm{T}}\right)\bar{P}_{I,\varepsilon}\right]e_I + u_I^{\mathrm{T}}\left[\left(Z_I\varepsilon_I^{\nu} - \bar{L}_{Z,1}\right) \otimes I\right]u_I \\ & - 2u_I^{\mathrm{T}}\left[Z_I\left(\varepsilon_I^{\nu} - \bar{L}_1^{\mathrm{T}}\right) \otimes B^{\mathrm{T}}\right]\bar{P}_{I,\varepsilon}e_I - 2u_I^{\mathrm{T}}(Z_I\bar{L}_2 \otimes I)u_{II} + 2e_I^{\mathrm{T}}\bar{P}_{I,\varepsilon}(Z_I\bar{L}_2 \otimes B)u_{II}.\end{aligned}$$

Given (14), we get

$$e_I^{\mathrm{T}}\left[\bar{P}_{I,\varepsilon}\left(Z_I\varepsilon_I^{\nu} \otimes BB^{\mathrm{T}}\right)\bar{P}_{I,\varepsilon}\right]e_I \le \frac{1}{6}e_I^{\mathrm{T}}\left(Z_I\varepsilon_I \otimes Q^{-2}\right)e_I.$$

Moreover,

$$2e_I^{\mathrm{T}}\left(Z_I\varepsilon_I\bar{L}_1 \otimes Q^{-1}B\right)u_I \le 6u_I^{\mathrm{T}}\left(\bar{L}_1^{\mathrm{T}}Z_I\varepsilon_I\bar{L}_1 \otimes B^{\mathrm{T}}B\right)u_I + \frac{1}{6}e_I^{\mathrm{T}}\left(Z_I\varepsilon_I \otimes Q^{-2}\right)e_I$$

and

$$\begin{aligned}& -2u_I^{\mathrm{T}}\left[Z_I\left(\varepsilon_I^{\nu} - \bar{L}_1^{\mathrm{T}}\right) \otimes B^{\mathrm{T}}\right]\bar{P}_{I,\varepsilon}e_I \\ \le & \, u_I^{\mathrm{T}}\left[\left(\varepsilon_I^{\nu} - \bar{L}_1^{\mathrm{T}}\right)Z_I\varepsilon^{1/2}(\varepsilon_I^{\nu} - \bar{L}_1) \otimes I\right]u_I + e_I^{\mathrm{T}}\left[\bar{P}_{I,\varepsilon}\left(Z_I\varepsilon_I^{-1/2} \otimes BB^{\mathrm{T}}\right)\bar{P}_{I,\varepsilon}\right]e_I \\ \le & \, u_I^{\mathrm{T}}\left[\left(\varepsilon_I^{\nu} - \bar{L}_1^{\mathrm{T}}\right)Z_I\varepsilon^{1/2}\left(\varepsilon_I^{\nu} - \bar{L}_1\right) \otimes I\right]u_I + \frac{1}{6}e_I^{\mathrm{T}}(Z_I\varepsilon_I \otimes Q^{-2})e_I.\end{aligned}$$

For the terms involving u_{II}, we have the following bounds:

$$2e_I^{\mathrm{T}}\left(Z_I\varepsilon_I\bar{L}_2 \otimes Q^{-1}B\right)u_{II} \le 6u_{II}^{\mathrm{T}}\left(\bar{L}_2^{\mathrm{T}}Z_I\varepsilon_I\bar{L}_2 \otimes B^{\mathrm{T}}B\right)u_{II} + \frac{1}{6}e_I^{\mathrm{T}}\left(Z_I\varepsilon_I \otimes Q^{-2}\right)e_I$$

and

$$-2u_I^{\mathrm{T}}(Z_I\bar{L}_2 \otimes I)u_{II} \le \frac{1}{a_1}u_I^{\mathrm{T}}\left(Z_I\bar{L}_2\bar{L}_2^{\mathrm{T}}Z_I \otimes I\right)u_I + a_1u_{II}^{\mathrm{T}}u_{II} \le \frac{1}{2}u_I^{\mathrm{T}}(\bar{L}_{Z,1} \otimes I)u_I + a_1u_{II}^{\mathrm{T}}u_{II},$$

where a_1 is such that $2Z_I\bar{L}_2\bar{L}_2^{\mathrm{T}}Z_I \le a_1\bar{L}_{Z,1}$. Finally,

$$\begin{aligned}2e_I^{\mathrm{T}}\bar{P}_{I,\varepsilon}(Z_I\bar{L}_2 \otimes B)u_{II} \le & \, u_{II}^{\mathrm{T}}\left(\bar{L}_2^{\mathrm{T}}\varepsilon_I^{1/2}Z_I\bar{L}_2 \otimes I\right)u_{II} + e_I^{\mathrm{T}}\left[\bar{P}_{I,\varepsilon}\left(Z_I\varepsilon_I^{-1/2} \otimes BB^{\mathrm{T}}\right)\bar{P}_{I,\varepsilon}\right]e_i \\ \le & \, u_{II}^{\mathrm{T}}\left(\bar{L}_2^{\mathrm{T}}\varepsilon_I^{1/2}Z_I\bar{L}_2 \otimes I\right)u_{II} + \frac{1}{6}e_I^{\mathrm{T}}\left(Z_I\varepsilon_I \otimes Q^{-2}\right)e_I.\end{aligned}$$

Define $V = V_1 + V_2$; then,

$$\begin{aligned}\dot{V} \le & -\frac{1}{6}e_I^{\mathrm{T}}\left(Z_I\varepsilon_I \otimes Q^{-2}\right)e_I - e_I^{\mathrm{T}}(Z_I \otimes C^{\mathrm{T}}C)e_I + 6u_I^{\mathrm{T}}\left[\bar{L}_1^{\mathrm{T}}\varepsilon_IZ_I\bar{L}_1 \otimes B^{\mathrm{T}}B\right]u_I \\ & + 6u_{II}^{\mathrm{T}}\left[\bar{L}_2^{\mathrm{T}}\varepsilon_IZ_I\bar{L}_2 \otimes B^{\mathrm{T}}B\right]u_{II} + u_I^{\mathrm{T}}\left[\left(Z_I\varepsilon_I^{\nu} - \frac{1}{2}\bar{L}_{Z,1}\right) \otimes I\right]u_I \\ & + u_I^{\mathrm{T}}\left[Z_I\left(\varepsilon_I^{\nu} - \bar{L}_1^{\mathrm{T}}\right)\varepsilon_I^{1/2}\left(\varepsilon_I^{\nu} - \bar{L}_1\right) \otimes I\right]u_I + u_{II}^{\mathrm{T}}\left[\left(\bar{L}_2^{\mathrm{T}}Z_I\varepsilon_I^{1/2}\bar{L}_2 + a_1\right) \otimes I\right]u_{II}.\end{aligned}$$

Since $\varepsilon_I \to 0$, we know that for t to be sufficiently large, we have

$$\dot{V} \le -a_2u_I^{\mathrm{T}}u_I + a_3u_{II}^{\mathrm{T}}u_{II}$$

for some fixed $a_2, a_3 > 0$ (independent of ε_i), which is clearly contradictory with $u_I \notin \mathcal{L}_2$ and $u_{II} \in \mathcal{L}_2$. Therefore, we find that all ε_i for $i = 1, \ldots, N$ are bounded away from 0, and hence, synchronization is achieved. □

4 | SIMULATION RESULTS

In this section, we will provide an example to verify our adaptive nonlinear dynamic protocol design.

Consider a MAS with five identical agents and an exosystem. The agents are triple integrators, which obviously are weakly unstable and exhibit polynomial growth. The agents ($i = 1, \ldots, 5$) and exosystem models in the form of (1), (3), and (4) are as follows:

$$\begin{cases} \dot{x}_i = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} x_i + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u_i \\ y_i = (1 \ \ 0 \ \ 0) x_i \end{cases}, \qquad \begin{cases} \dot{x}_r = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} x_r \\ y_i = (1 \ \ 0 \ \ 0) x_r \end{cases}$$

The agents are coupled through partial-state coupling networks. In other words, the network provides information for each agent as

$$\zeta_i(t) = \sum_{j=1}^{N} a_{ij}(y_i(t) - y_j(t)),$$

where the adjacency matrix is

$$A = [a_{ij}] = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0. \end{pmatrix}$$

By solving the ARE in (11), the adaptive low-gain protocol would be

$$K_{\varepsilon_i} = -\frac{1}{\varepsilon_i} Q C^{\mathrm{T}} = -\frac{1}{\varepsilon_i} \begin{pmatrix} 2.4142 \\ 2.4142 \\ 1 \end{pmatrix}.$$

For this specific case, we can compute P_ε explicitly as

$$P_\varepsilon = 1/\varepsilon^\nu \begin{pmatrix} \varepsilon^5 & 2\varepsilon^4 & \varepsilon^3 \\ 2\varepsilon^4 & 5\varepsilon^3 & 3\varepsilon^2 \\ \varepsilon^3 & 3\varepsilon^2 & 3\varepsilon. \end{pmatrix}$$

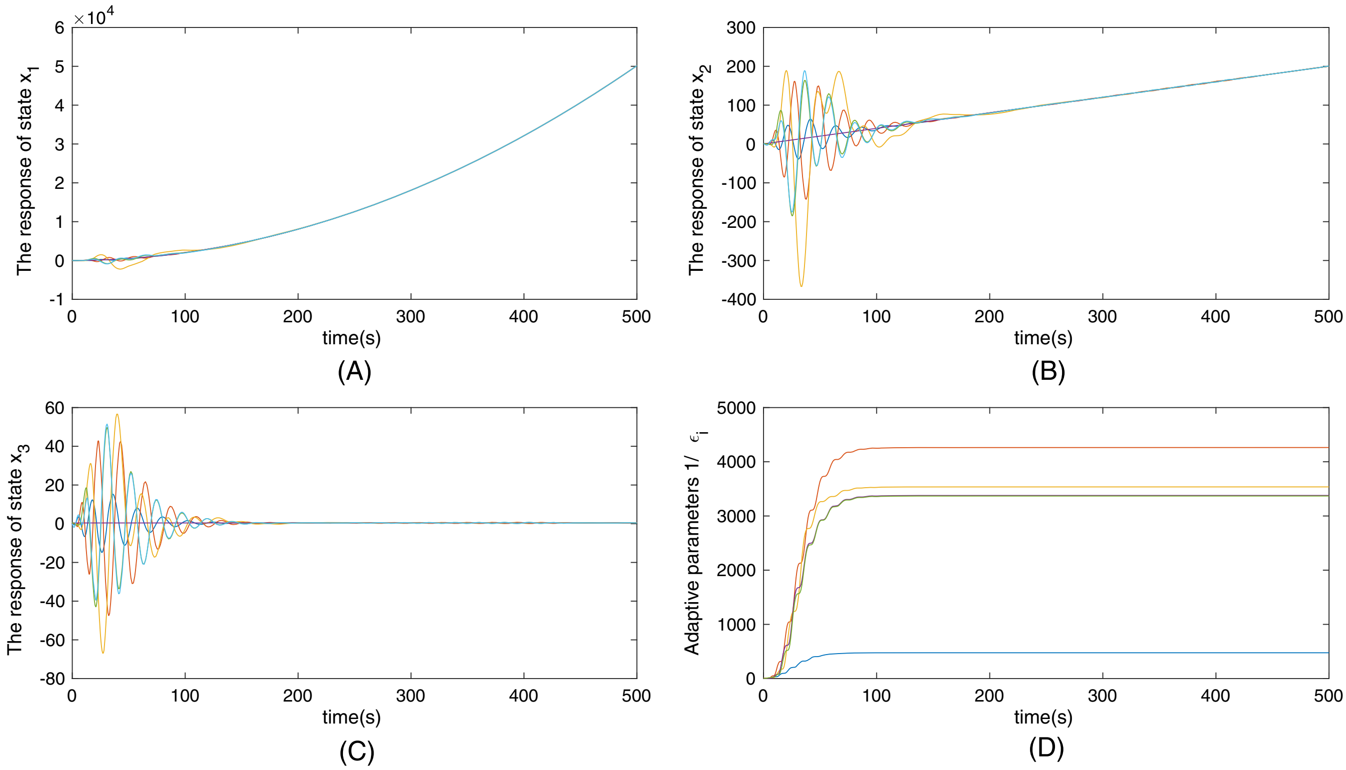

FIGURE 1 Partial-state coupling: (A) to (C) are the agents and exosystem state trajectories and (D) is the trajectories of parameters $\frac{1}{\varepsilon_i}$ [Colour figure can be viewed at wileyonlinelibrary.com]

By solving ARE (12) with $\nu = 0.01$, and $\varepsilon(0) = (1\ 0.\ 2\ 0.1\ 0.2\ 0.1)^{\mathrm{T}}$, we can obtain the following adaptive dynamic protocol:

$$\begin{cases} \dot{\chi}_i &= \left[\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} + \frac{1}{\varepsilon_i}\begin{pmatrix} -2.4142 & 0 & 0 \\ -2.4142 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}\right]\chi_i - \frac{1}{\varepsilon_i}\begin{pmatrix} 2.4142 \\ 2.4142 \\ 1 \end{pmatrix}\bar{\zeta}_i, \\ u_i &= -(0\ \ 0\ \ 1)P_{\varepsilon_i}\chi_i, \\ \dot{\varepsilon}_i &= -\varepsilon_i^2 u_i^{\mathrm{T}} u_i. \end{cases}$$

It can be verified that, for $\nu < 0.4158$, we have P_ε is increasing in ε.

The state trajectories of the MAS are given in Figures 1A to 1C. We see that all the agents achieve regulated state synchronization with the exosystem. In addition, the adaptive parameters converge to constant values, as shown in Figure 1D.

5 | CONCLUSION

In this paper, we have developed regulated state synchronization for homogeneous MASs with weakly unstable agents. The communication network is directed and partial-state coupling. We have designed a low-gain adaptive nonlinear dynamic protocol to achieve regulated state synchronizations for a MAS. Note that the a protocol design does not use any information about the directed communication network.

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