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The dispersion relation for waves above arbitrary currents

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Abstract: Waves on the surface of a layer of fluid are described in the approximation of small elevations (linear wave theory) by a dispersion relation, which is an algebraic relation between the wave frequency and the wavenumber.

For waves above a uniform (depth-independent) current, the presence of a constant current simply adds a linear contribution to the frequency, reflecting the invariance of the physical phenomenon for translations with a uniform current speed.

When the current profile depends on the depth the situation becomes much more complicated. Nevertheless, for any current profile, a linear wave of (suitably) given frequency can still be found, but its wavenumber depends in a complicated way on the current profile. To find the dispersion relation in that case, the equation for the vertical fluid motion has to be studied (i.e. the so-called Rayleigh equation with impermeable bottom boundary condition). Except in the special case of a (piecewise) linear current, this equation cannot be solved explicitly and approximations have to be found.

We will show that the Rayleigh problem is actually a variational problem (i.e. has optimization properties), and that the relevant functional (or its dual formulation) is directly related to the dispersion relation. Taking trial functions that are in some sense approximations of the solutions and inserting them into the functional lead to approximations of the dispersion relation for depth-dependent currents.