FINITE ELEMENT ANALYSIS OF PHOTONIC CRYSTAL FIBERS

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Abstract. A finite-element-based vectorial optical mode solver, furnished with Bayliss-Gunzburger-Turkel-like transparent boundary conditions, is used to rigorously analyze photonic crystal fibers (PCFs). Both the real and imaginary part of the modal indices can be computed in a relatively small computational domain. The leakage loss, the dispersion properties, the vectorial character, as well as the degeneracy of modes of the fibers can be studied through the finite element results. Results for PCFs with either circular or non-circular microstructured holes, solid- or air-core will be presented, including the air-core air-silica Bragg fiber. Using the mode solver, the single-modeness of a commercial endlessly single-mode PCF was also investigated.

Key-words: finite element analysis, photonic crystal fibers, transparent boundary conditions, leaky modes.

1 Introduction

Since the introduction of the photonic crystal fiber (PCF) [7], various waveguiding structures that utilize the arrangement of microstructured holes [16] or thin layers [4] have been realized. The large variety of possible hole shapes and arrangements demand the use of numerical methods that can handle arbitrary cross-sectional shapes to analyze this kind of structures. Besides, the existence of interfaces with high index-contrast between the solid host material and air holes calls for the use of the vectorial wave equation to accurately model the structure. Finite element method (FEM) is suitable for such analysis as it can handle complicated structure geometries and solve vectorial equations transparently. By incorporating proper boundary conditions, it also can model the leaky behavior of the realistic PCFs.

In this paper, we apply a vectorial optical mode solver based on Galerkin FEM [17], which is furnished with a 1st-order Bayliss-Gunzburger-Turkel-like (BGT-like) transparent boundary conditions (TBC) to rigorously model various kinds of PCFs [18]. Thanks to the boundary conditions, the structure can be analyzed in a relatively small computational domain for its complex-valued modal indices and field profiles. The structures being considered include those with either solid material or air as the core; circular or non-circular microstructured holes arranged around the core. Through the FEM results, we studied the leakage loss, dispersion properties, vectorial character, as well as the degeneracy of modes and single-modeness of particular kinds of PCFs.

2 Formulation of the method

The detail discussions on the formulation of the mode solver has been given elsewhere [17], but for convenience will be briefly reviewed here.
2.1 Finite element formulation

Using the H-field-based vectorial wave-equation, \( \nabla \times \vec{E} \times \vec{H} = k_0^2 \vec{H} \), for longitudinally-invariant structures composed of non-magnetic anisotropic materials with diagonal permittivity tensors and \( \exp(j\omega t) \) time dependence of the field; it is possible to get a vectorial wave-equation expressed only in terms of the transverse components of the magnetic field as follows:

\[
\begin{bmatrix}
\partial_y \left( \frac{1}{\gamma_\omega} \left( \partial_x H_y - \partial_y H_x \right) \right) \\
-\partial_x \left( \frac{1}{\gamma_\omega} \left( \partial_x H_y - \partial_y H_x \right) \right)
\end{bmatrix}
\begin{bmatrix}
H_x \\
H_y
\end{bmatrix}
+ k_0^2 n_{\text{eff}}^2
\begin{bmatrix}
\frac{H_x}{\gamma_\omega} \\
\frac{H_y}{\gamma_\omega}
\end{bmatrix}
= k_0^2
\begin{bmatrix}
H_x \\
H_y
\end{bmatrix}.
\]

Here, the \( x \) and \( y \) denote the transverse Cartesian coordinates associated with the structure cross-section, \( k_0 \) the vacuum wavenumber, \( n_{\text{eff}} \) the complex modal index, \( H_x \) and \( H_y \) the \( x \) and \( y \) components of the magnetic field \( \vec{H} \), while \( n_{11}^x \), \( n_{12}^y \), and \( n_{22}^z \) the non-zero entries located at the diagonal of the relative permittivity tensor \( \varepsilon_r \) associated with the \( x \), \( y \), and \( z \) components of the electric field, respectively. Using the Galerkin procedure and discretizing the computational domain into triangular elements lead to the following discretized weak formulation:

\[
\sum_{\text{BoundaryElements}} \int_{\Gamma_e} \left[ \int_{\Omega_e} \left( \partial_y w_y \left( \partial_x H_y - \partial_y H_x \right) + \partial_x w_x \left( \partial_x H_y - \partial_y H_x \right) \right) dx - \int_{\Omega_e} \left( \partial_x w_y \left( \partial_x H_y - \partial_y H_x \right) + \partial_y w_x \left( \partial_x H_y - \partial_y H_x \right) \right) dy \right]
\]

\[
+ \sum_{\text{InterfaceElements}} \int_{\Gamma_{\text{int,e}}} \left[ \int_{\Omega_{\text{int,e}}} \left( \partial_y w_y \left( \partial_x H_y + \partial_y H_x \right) + \partial_x w_x \left( \partial_x H_y + \partial_y H_x \right) \right) dx + \partial_y w_y \left( \partial_x H_y + \partial_y H_x \right) dy \right]
\]

\[
+ \sum_{\text{TriangularElements}} \int_{\Omega_e} \left( \frac{w_x}{\gamma_\omega} \left( \partial_x w_y - \partial_y w_x \right) + \frac{w_y}{\gamma_\omega} \left( \partial_x w_y - \partial_y w_x \right) \right)
\]

\[
\left( \partial_x H_y + \partial_y H_x \right) + k_0^2 n_{\text{eff}}^2 \left( \frac{w_x}{\gamma_\omega} + \frac{w_y}{\gamma_\omega} \right) \left( w_x H_y + w_y H_x \right)
\]

\( dx dy = 0 \) (1)

with \( w_x \) and \( w_y \) denoting the weight functions, \( \Omega_e \) the area in each triangular element, \( \Gamma_{\text{int,e}} \) the line element at the interface between different materials, and \( \Gamma_e \) the line element at the computational boundaries.

Approximating the fields using quadratic nodal-based basis functions will lead to a sparse generalized matrix eigenvalue equation, which can be solved using an eigenvalue solver to obtain the eigenvalues related to the modal indices \( n_{\text{eff}} \) and eigenvectors associated with the transverse components of the magnetic field \( \left[ H_x, H_y \right] \) of the corresponding modes.
2.2 Boundary conditions

The derivatives of the fields occurring in the boundary term in Eq. (1) will be handled through the 1st-order BGT-like [1] TBC to mimic the properties of the fields in the exterior domain properly. We use a vector radiation function

\[
\hat{H}(r, \theta) = \begin{bmatrix} H_x \\ H_y \end{bmatrix} = \sum_{p=0}^{\infty} \frac{1}{r^{p+1/2}} \begin{bmatrix} H_{x,p}(\theta) \exp(-j \kappa_{xx} r) \\ H_{y,p}(\theta) \exp(-j \kappa_{yy} r) \end{bmatrix}
\]

(2)

along the computational boundary \( \Gamma \), which leads to a 1st-order operator on the boundary fields as follows:

\[
B_L \begin{bmatrix} H_x \\ H_y \end{bmatrix} = \left\{ \begin{array}{c} \partial_x + \frac{1}{2r} \left( \begin{array}{c} \kappa_{xx} H_x \\ \kappa_{yy} H_y \end{array} \right) \right\} = O(r^{-5/2}).
\]

(3)

In Eqs. (2) and (3), \( r \) and \( \theta \) are the polar coordinates of the cross-section whereby the center of the core of the waveguide has been taken as the origin, and \( \kappa_{xx} \) and \( \kappa_{yy} \) are the complex transverse wavenumbers associated with the \( x \) and \( y \) components of the field. Solving the wave-equation at the elementwise homogeneous anisotropic exterior domain leads to

\[
k_{xx} \big|_L = k_0 \sqrt{n_{xx}^2 - n_{eff}^2}, \quad \text{and} \quad k_{yy} \big|_L = k_0 \sqrt{n_{yy}^2 - n_{eff}^2},
\]

with \( \text{Re}(\kappa)>0 \) associated with the outward leaking case (the leaky-mode being considered in this paper) and \( \text{Im}(\kappa)<0 \) associated with evanescently decaying case (the guided-mode case). By neglecting the angular dependence of the field at each line element,

\[
\partial_n H_x \big|_L = -\hat{r} \cdot \hat{n} \left( j \kappa_{xx} \frac{1}{2r} \right) H_x \big|_L + O(r^{-5/2})
\]

(4)

\[
\partial_n H_y \big|_L = -\hat{r} \cdot \hat{n} \left( j \kappa_{yy} \frac{1}{2r} \right) H_y \big|_L + O(r^{-5/2})
\]

(5)

Dirichlet to Neumann (DtN) map can be obtained and used for approximating the derivative operators within the boundary terms of Eq. (1), hence allows a proper truncation of the FEM mesh. In Eqs. (4) and (5), the caret (^) notation denotes the unit vector, while \( n \) denotes the normal direction. Note that, these boundary conditions induce non-linearity to the eigenvalue problem due to the appearance of \( n_{eff} \) (the eigenvalue itself) within the DtN. In this work, we have employed linearization by simple iteration technique to enable the use of linear eigenvalue solver and search only for eigenvalues \( n_{eff} \) of interest, i.e those related to low-loss, localized leaky modes within expected range [20].

3 Study of PCF properties through the FEM results

The computed complex-valued mode indices and field profiles associated with the eigenvalues and eigenvectors of the eigenvalue equation discussed in the previous
section can be used to study the properties of the PCFs. The dispersion parameter \( D \) of the PCFs can be obtained from the wavelength dependence of the real part of the mode indices as follows

\[
D = -\frac{\lambda}{c} \frac{\partial^2}{\partial \lambda^2} \text{Re}(n_{eff}) ,
\]

while the attenuation due to leakage loss can be deduced from the imaginary part as follows

\[
\alpha = \text{Loss} / \text{L} = -20k_e \text{Im}(n_{eff}) \log(e) .
\]

As will be shown in the next section, the vectorial character of the modes will show up from the discrepancies of results for modes which are often regarded as the same mode in scalar analysis for weakly guiding structures [5].

Although a rigorous study on the degeneracy and classification of modes requires knowledge on group theory [11], we can also use the FEM results to recognize the degeneracy or non-degeneracy of modes of PCFs. Instead of using mesh refinement [9] or symmetry-preserving mesh generation [13], here we will simply use a visual inspection on the vector plots of the transverse modal field to recognize the degeneracy of a pair of modes. A symmetry operated modal field can be expressed as a linear combination of the orthogonal set of degenerate modes as follows

\[
\sum_{i=1}^{n} a_i \Psi_i = \Psi = \Psi = \sum_{G} G\Psi ,
\]

where \( S \) is such a symmetry operation, \( \{ \Psi_i, \Psi_f \} \) is the orthogonal degenerate modal fields set with \( \Psi_i \in \{ \Psi_i, \Psi_f \} \). A non-degenerate mode requires \( p=1 \) meaning that its symmetry operated modal field will always be linearly dependent with the initial modal field. A failure to fulfill this requirement is already enough to indicate that a mode is degenerate. A pair of two-fold degenerate modes (which is the case for degenerate modes in optical waveguides [11] with rotational structural symmetry \( C_m \) or \( C_{mm} \)) requires \( p=2 \), meaning that we can reconstruct the modal profile of a mode from 2 symmetry-operated modal fields taken from its degenerate pair, provided that these symmetry-operated modal fields are not linearly dependent.

4 Examples

Here, we will demonstrate the application of the FEM leaky mode solver to study the properties of various kinds of PCFs, including those of solid and hollow core, circular and non-circular microstructured holes.

4.1 Solid-core PCF with circular microstructured holes

First, we studied the most widely used kind of PCFs, i.e. those with cladding made up of circular holes arranged in a triangular lattice with core residing in the region at the center formed by missing of hole(s) [18]. We found that adding more rings of holes will be influential only to the leakage loss, while giving practically similar dispersion properties. Hence, for the sake of efficiency, we took structure with only
6 circular holes in the cladding for most of the computations and took more rings of holes to show their effect on the leakage loss, when necessary. Fig. 1 shows the fiber cross sections together with the FEM mesh and size of computational window. Note that, by the help of the boundary conditions, the computation can be carried out in a relatively small computational domain. Also note that by taking advantages of the structure mirror symmetry, we only need a quarter of the structure as the computational domain [20]. The diameter of the holes is \( d = 5 \mu m \) with pitch length of \( \Lambda = 6.75 \mu m \). Pure SiO2 is considered as the host solid material with its refractive index \( n_{bg} \) taken from the Sellmeier’s equation [10], while the refractive index of the air holes \( n_{hole} \) is 1. Fig. 2 shows the real part of \( n_{eff} \) and the dispersion parameter as function of wavelength for the structure of 6 circular holes. Note that, by using the Sellmeier’s equation, the material dispersion effect has been rigorously taken into account in the plots here and in all other wavelength dependent plots in this paper. In this paper we have used similar hybrid mode notation as in ordinary fiber with additional superscript \( a \) and \( b \) to denote the results obtained using perfect magnetic conductor (PMC) and perfect electric conductor (PEC), respectively, as the symmetry boundary conditions at the horizontal symmetry plane.

![Fig. 1. The PCF with (a) 6 circular holes and (b) 60 circular holes forming their cladding, the FEM mesh and their computational window.](image)

The curves in Fig. 2 show three groups of modes, which correspond to the first-three \( LP \)-like modes [5]. By using the FEM mode solver, it is possible to distinguish modes within the same group. The differences between the curves of modes associated with the same group, i.e. the vectorial properties of the modes, are more pronounced for longer wavelength, where the dimension of the structure becomes more comparable to the wavelength. The divergence between curves for \( HE_{11}^{+} \)- and \( HE_{11}^{-} \)-like modes indicates their non-degeneracy, a property which can be intuitively understood by visual inspection on the vector plot of their transverse modal field profile as shown in Fig. 3, whereas the symmetry-operated (e.g. rotated with \( 2\pi m/6 \) rotation angle with \( m \) integer) modal field is always linearly dependent with the initial modal field. Note that these modes are degenerate in ordinary fiber,
a property which can also be understood using the same intuitive procedure. Fig. 2(b) also shows that $HE_{11}$-like mode has zero dispersion wavelength at shorter wavelength than the ordinary fiber. Note that this zero-dispersion wavelength can be engineered by playing with the hole sizes [8], making this kind of fibers attractive for applications like dispersion compensation [3], supercontinuum light generation [14], ultra-flat and ultra-low dispersion [15], etc.

Fig. 2. (a). The real part of $n_{\text{eff}}$ and (b) the dispersion parameter of the structure with 6 circular holes forming its cladding.

Fig. 3. Vector plot of (a) $HE_{31}^{a}$ - and (b) $HE_{31}^{b}$-like modes.

Fig. 4(a) shows the leakage loss as function of wavelength. The vectorial behavior of the modes is even more pronounced through the obvious divergence of curves within the same $LP$-like modes. As wavelength increases, the modes become less quasi-confined; consequently, the leakage loss increases. Fig. 4(b) shows the effect
of adding more rings of holes around the central core. In this case; 2-ring, 3-ring, and 4-ring structures correspond to 18, 36, and 60 holes (see Fig. 1b) in the cladding arranged in triangular lattice setting. The figure shows that adding rings of holes will reduce the leakage loss exponentially.

**Fig. 4.** The leakage loss of the structure with circular holes in the cladding. (a). Leakage loss of the first-ten modes in the 1-ring (6-hole) structure. (b). The effect of adding more rings of holes in the cladding.

**Fig. 5.** The cross-section of the PCF with 3 annular-sector-shaped holes in its cladding, the mesh definition, and the computational window.

### 4.2 Solid-core PCF with annular-sector-shaped holes

Next, we consider a PCF with cladding consists of three annular-sector-shaped holes as shown in Fig. 5 together with its mesh definition and the size of the computational window. The annular-sector-shaped holes has an inner radius of
radius $r_1 = 1 \mu m$ and outer radius $r_2 = 2 \mu m$. Again here, we took the pure SiO$_2$ as the host material. Fig. 6 shows the real $n_{\text{eff}}$ and related dispersion parameter of the structure as function of wavelength. Fig. 7 shows the imaginary part of the $n_{\text{eff}}$ and its related leakage loss. Since this structure has much smaller core size and larger (local) air filling fraction than the previous example, the effect of the air core is stronger, leading to a shorter zero-dispersion wavelength (see arguments given in [8]). Also, the vectorial character of the modes is more pronounced as indicated by more divergent curves of $TE_{01}$-, $TM_{01}$-, and $HE_{21}$-like modes (which are all associated to $LP_{11}$-like mode in scalar analysis), both in their real and imaginary part of the $n_{\text{eff}}$. Fig. 6(a) shows that at around 1.483$\mu m$, the real part of $HE_{21}$- and $TM_{01}$-like modes cross over, leading to a rather dissimilar dispersion properties in Fig. 6(b).

**Fig. 6.** (a). The real part of $n_{\text{eff}}$ and (b) the dispersion parameter of the modes of the PCF with 3 annular-sector-shaped holes.

**Fig. 7.** (a). The imaginary part of the effective indices and (b). the leakage loss of the modes of the PCF with 3 annular-sector-shaped holes.
4.3 Hollow-core PCF with rounded-annular-sector-shaped holes

For hollow-core PCF, we take the one with rounded-annular-sector-shaped holes in the cladding, i.e. the air-silica Bragg fiber as proposed by Vienne et al. [21]. The structure is shown in Fig. 8, where $r_{\text{core}}=10\mu m$, $t_{\text{annular}}=2.3\mu m$, $r_{\text{corner}}=t_{\text{annular}}/4$, $t_{\text{ring}}=0.2\mu m$, and $t_{\text{bridge}}=45\text{nm}$. The structure has 24, 34, and 44 holes at the first, second, and third ring of holes. We used computational domain with $r_b=19\mu m$ and wavelength 1.06\mu m. The refractive index of the host silica material is taken from its Sellmeier's equation [10].

![Fig. 8. The model of the air-silica Bragg fiber with 3 rings of annular-sector-shaped holes in the cladding. Also shown are the mesh definition and the computational window size.](image)

Fig. 9 shows the vector plot of the transverse component of the magnetic field of modes of the PCF. In this type of waveguide, the leakage loss of $TE_{01}$-like mode (computed to be 0.015 dB/cm) was found to be lower than that of the $HE_{11}$-like mode (computed to be 0.44 dB/cm). This is a typical property of a Bragg fiber (as the micro-structured cladding can be regarded as alternating air and silica Bragg 'layers'). This lower loss for $TE$-like mode comes from the fact that the Fresnel reflection coefficient for $TE$ is higher than $TM$ polarization. As modes other than $TE$-like modes have some component with $TM$-like polarization, they will exhibit higher leakage loss due to the lower reflection coefficient of the cladding. Fig. 10 shows the longitudinal component of the time averaged Poynting vector of $HE_{11}$- and $TE_{01}$-like modes. Small spots at the solid material in the cladding for $HE_{11}$-like mode indicate the onset of the anti-crossing of this mode with a cladding resonance mode. For hollow-core structure, since the $n_{\text{eff}}$ of interest is below 1, the transverse wavenumber $k_i=k_0\sqrt{n_{\text{air}}^2-n_{\text{eff}}^2}$ is rather large, which enables resonances to take place even in thin solid material of the cladding. This explains why cladding resonance modes are easily observable near to our modes of interest in such hollow-core PCFs.
Fig. 9. The transverse component of the magnetic field of (a). $HE^{a}_{11}$, (b). $HE^{b}_{11}$, (c). $TM^{a}_{01}$, (d). $TE^{a}_{01}$, (e). $HE^{a}_{21}$, and (f). $HE^{b}_{21}$-like modes of the air-silica Bragg fiber.
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Fig. 10. The longitudinal component of the time averaged Poynting vector of (a), $HE_{21}^+$- and (b), $TE_{11}^+$-like modes of the air-silica Bragg fiber.

4.4 Modal degeneracy in PCF

Here, we will demonstrate the use of visual inspection of the vector plot of the transverse modal field of the modes of PCF to recognize their degeneracy or non-degeneracy. As an example, we took the $HE_{21}^+$-like modes of the PCF with 3 annular-sector-shaped holes in the cladding as discussed in Section 4.2. Fig. 11 shows the modal profile of the two $HE_{21}^+$-like modes. As shown by Eq. (6) for $p=1$, a non-degenerate mode requires linear dependency of a symmetry-operated modal field with its initial modal field. Visual inspection on Fig. 11(a) or (b) shows that rotating the modal field by $2\pi/3$ (which is a symmetry operation due to the $C_{3v}$ structure symmetry) will not result in a linear dependent modal field with the initial modal field. This fact is already enough as a proof of the degeneracy of such modes.

Fig. 11. Vector plot of the transverse component of magnetic field of (a), $HE_{21}^+$- and (b), $HE_{21}^+$-like modes of the PCF with 3 annular-sector-shaped holes.
Fig. 12. Fields obtained by rotating the transverse modal field of $HE_{21}^0$ by (a). $2\pi/3$ and (b). $4\pi/3$ radiant.

To further show that the $HE_{21}^0$-like mode (Fig. 11(a)) is the degenerate pair of the $HE_{21}^1$-like mode (Fig. 11(b)), we will show that it is possible to construct the modal field of Fig. 11(b) from two symmetry-operated modal fields taken from Fig. 11(a), which is a consequence of Eq. (6) for $p=2$ in the case that these two symmetry-operated modal fields are not linear dependent each other. Fig. 12(a) and (b) show the fields obtained by rotating Fig. 11(a) by $2\pi/3$ and $4\pi/3$ radiant, respectively. Note that these rotation operations are symmetry operations as they leave the structure unchanged. Fig. 13 shows the field obtained by multiplying the field of Fig. 12(a) by -1 and adding the result to the field of Fig. 12(b). Visual inspection on Fig. 13 and Fig. 11(b) shows that they are linear dependent. This fact proves that $HE_{21}^0$- and $HE_{21}^1$-like modes are degenerate pair.

Fig. 13. Fields obtained by multiplying field of Fig. 12(a) by -1 and adding the result to field of Fig. 12(b).
4.5 Single-modeness of an endlessly single-mode PCF

Finally, we will consider the single-modeness of a PCF which is specially designed to have single-mode properties over a wide wavelength range. This type of PCF is known as the endlessly-single-mode (ESM) PCF [2]. An intuitive explanation of this property can be given by the modified-total-internal-reflection model [16] of the PCF. At shorter wavelength, the effective refractive index of the cladding becomes closer to the refractive index of the silica host material. This dispersive property will decrease the effective index contrast between the core and cladding, which in turn compensates the effect of the decrease of the wavelength and keep the structure to be single-moded over a wide wavelength range.

In this section, we will use the FEM leaky mode solver to study such ESM-PCF rigorously using the leakage loss as a measure of its single-modeness [19]. Here, we picked up a commercial ESM-PCF, which is ESM-12-01 fiber made by BlazePhotonics [6]. The structure cross-section of the fiber is shown by Fig. 14, with $d=3.68\,\mu m$ and $\Lambda=8\,\mu m$. We assume pure SiO$_2$ as the host material with its refractive index taken from Sellmeier’s equation [10].

Fig. 14. The cross-section of the ESM-12-01 PCF.

$\bullet$ $\circ$ $\circ$ air holes
$\bullet$ $\circ$ $\circ$ $\circ$ $\circ$ $\circ$
$\bullet$ $\circ$ $\circ$ $\circ$ $\circ$ $\circ$ $\circ$
$\bullet$ $\circ$ $\circ$ $\circ$ $\circ$ $\circ$ $\circ$
$\bullet$ $\circ$ $\circ$ silica $\circ$ $\circ$ $\circ$
$\bullet$ $\circ$ $\circ$ $\circ$ $\circ$ $\circ$ $\circ$ $\circ$
$\bullet$ $\circ$ $\circ$ $\circ$ $\circ$ $\circ$ $\circ$ $\circ$
$\bullet$ $\circ$ $\circ$ $\circ$ $\circ$ $\circ$ $\circ$ $\circ$

Fig. 15 shows the real and imaginary part of the $n_{\text{eff}}$ of the first-few modes of the PCF with attenuation constant of smaller than 10 dB/cm. The figure shows that the curve for $HE_{11}$-like mode is well separated from other modes, indicating the single-mode behavior, which is stronger for longer wavelength. The figure also shows the existence of cladding and core-cladding resonance modes, similar to the one usually found in hollow-core PCF. As the ESM-PCF is designed using small $d/\Lambda$ to obtain low effective index contrast, it has large solid medium between holes, hence small transverse wavenumber is already enough for resonance within this cladding region.

Fig. 16 shows the dispersion parameter and the attenuation constant of the modes. The curves in Fig 16(a) almost coincide, indicating the dominance of the material dispersion of the bulk silica as the air holes effect is weak due to the small $d/\Lambda$ ratio. Fig. 16(b) shows that due to the low effective index contrast, the vectorial
property of the modes is not so pronounced as indicated by the very similar loss profile of the \(TE_{01}, TM_{01}\), and \(HE_{21}\)-like modes, which are often regarded as \(LP_{11}\)-like scalar mode. The figure also shows that the \(HE_{11}\)-like mode exhibits the lowest loss, hence can be regarded as the most dominant mode.

**Fig. 15.** The (a) real and (b) imaginary part of \(n_{\text{eff}}\) of the first-few modes of the ESM-12-01 PCF. In (b) the curve for \(HE_{11}\)-like mode almost coincides with the horizontal axis.

**Fig. 16.** The (a) dispersion parameter and (b) attenuation constant of the first-few modes of the ESM-12-01 PCF. The curves in (a) almost coincide, indicating the weak waveguide dispersion.

The single-modeness of the ESM-PCF can be rigorously measured through the ability of the PCF to discriminate the dominant mode from the nearest higher-order mode in the sense of leakage loss. For this purpose, we define a quantity called multi-mode rejection ratio (MMRR) as follows

\[
MMRR = 10 \log \left( \frac{P_1}{P} \right) = (\alpha_1 - \alpha_0) L
\]
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with $P_0$ and $P_1$ denoting the power of the dominant fundamental and the nearest higher-order mode (which are assumed to be equally excited), while $\alpha_0$, $\alpha_1$, and $L$ denoting the attenuation constant of the dominant mode, the nearest higher-order mode, and the length of the fiber, respectively. By putting $20 \text{ dB}$ as the minimum $\text{MMRR}$ (meaning that the power of the fundamental mode is at least 100 times larger than the nearest higher order mode) as the single-modeness criterion, we get the minimum length of the fiber for single mode operation are $11.97\text{ m}$, $3.86\text{ m}$, and $0.78\text{ m}$ for wavelength of $0.4\mu\text{m}$, $0.6328\mu\text{m}$, and $1\mu\text{m}$, respectively, as shown in Fig. 17. Allowing the power of the fundamental mode to be only at least $10$ times the nearest higher order mode, the minimum length is just half of those of the previous criterion, a length which is still considerably long for applications like gas/liquid sensing when operated at short wavelength region. Hence, although this fiber geometry does not fulfill the ESM criterion of Mortensen et al. [12], it still can be regarded as an ESM-PCF for long fiber-length applications. While, for short fiber-length applications, especially for short wavelength region, the endlessly single-modeness should be considered with some precaution. Although the attenuation of the fundamental mode is $6$ orders lower (in dB scale) than the nearest higher order modes, the low attenuation of these higher order modes can make them to be quite significant for these particular applications. This fact suggests the requirement of ESM-PCF which is specially designed for short fiber-length applications. As these applications can tolerate higher attenuation, the use of smaller $d/\Lambda$ and less rings of air holes can be incorporated. Otherwise, some manner to strip off higher order modes might be required.

![Fig. 17. Minimum fiber length for single-mode operation by the loss discrimination criterion for minimum $\text{MMRR}$ of 10 and 20 dB](image)

5 Conclusions

We demonstrate the use of FEM leaky mode solver to rigorously analyze various kinds of PCFs, ranging from those of solid core to hollow core, circular to non-circular microstructured holes in the cladding. The dispersion properties, leakage loss, vectorial character, mode degeneracy, and single-modeness of the structures can be well studied through the FEM results.
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