TOWARDS THE DEVELOPMENT OF A HYBRID METHODOLOGY FOR MONITORING OF HEAD CHECKS IN RAILWAY INFRASTRUCTURE

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ABSTRACT
In this paper, the first step towards the development of a hybrid methodology for the monitoring of head checks is discussed. The proposed hybrid method combines a data driven approach with physical modelling of the rail in order to obtain an early stage warning for head checks. Rail defect detection at an early stage of the growth can be challenging and the existence of the seed defects can be confused with non-defect objects on the rail. Thus, a physical model is proposed to investigate how head checks, in particular in curved tracks, initiate and evolve. Track characteristics and loading, e.g. track geometry and track tonnage, are considered to analyze crack initiation by using the Whole Life Rail Model (WLRM) for Rolling Contact Fatigue (RCF) relying on meta-models. The results of the physical modelling and the rail defect observations obtained from the data analysis on the eddy current (EC) measurements are then compared. The physics based model only suggests whether a crack will be initiated or not, it does not give information about the size of the crack. Hence, the next step is to develop an evolution model from the EC and Ultrasonic (US) measurements data, from which the crack size can be determined. This combination of physics based and data-driven evolution model is thus regarded as the hybrid method. This hybrid method can be a robust tool for the prediction of rail condition, as it eases the visualization of the rail degradation and keeps infrastructure managers informed of the actual rail condition that can be confirmed with rail inspections. Finally, real-life measurements from a track in the Dutch railway network are used to show the (potential) benefits of the proposed methodology.

INTRODUCTION

For many decades already trains and railway tracks belong to one of the main transportation systems in the world. They are essential for the transportation of people, raw materials and products. Since the beginning of being in operation the railway tracks were suffering from rail wear phenomena. Rail material, rail profiles and running gears have been improved over the years in order to prevent catastrophic wear behavior. However, this introduced another deterioration problem to rails, namely the Rolling Contact Fatigue (RCF) problem (Ekberg, Akesson, & Kabo, 2014). An overview of all problems related to wheel and rail RCF is given by Ekberg and Kabo (2005). The various types of railhead RCF defects are squats, head-checks, gauge corner cracking, shelling, corrugation, wheel burns, tache ovals etc. A complete list of all RCF defects can be found in UIC Code 712R (2002).

Head checks on rails are one of the most occurring RCF defect types globally and can lead to severe accidents when not treated in time (Dollevoet, 2010). Preventive maintenance is being performed to delay the crack initiation and propagation of head checks by means of periodic grinding (Dollevoet, 2010; Jun, Lee, & Kim, 2015). The average material depth to be removed by periodic grinding is between 0.15 – 0.25 mm (Jun et al., 2015). Jun et al. (2015) found that when a certain crack size is exceeded the crack growth rate is higher and the effect of periodic grinding on the delay of the crack growth is negligible. Zacher (2009) also mentioned that grinding is economically feasible only until a crack depth of 2.7 mm is reached. This illustrates the importance of detecting head-checks in their earliest stage as preventive grinding can then take place just in time and rail replacement, which results in higher maintenance costs, can be prevented. Hence, rail inspection is an essential aspect in maintenance planning.

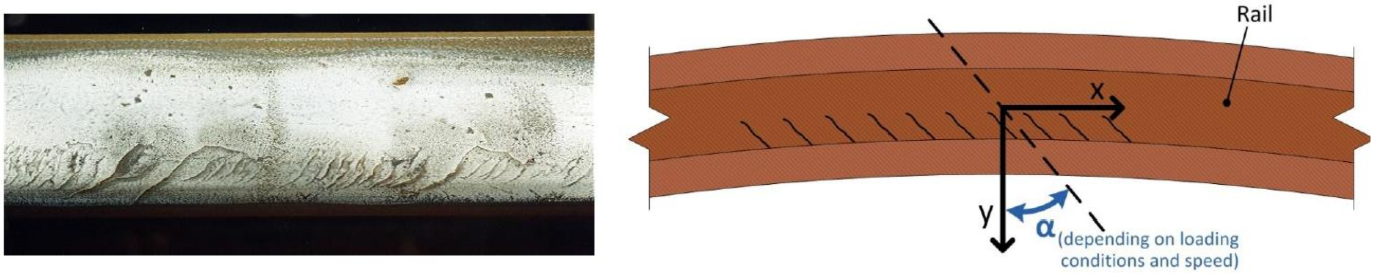
Several rail inspection techniques are available to detect head checks on rail. These techniques are mostly Non Destructive Testing (NDT) techniques like visual inspection, optical camera systems, ultrasonic (US) and eddy current (EC) testing. The US, EC and optical camera systems are usually mounted on measurement trains. However, US testing can also be carried out manually when severe cracks are reported (Popović, Lazarević, Brajović, & Vilotijević, 2015). Currently the train measurements are carried out at most twice a year on the Dutch rail network (Dollevoet, 2010). Dollevoet (2010) concluded from a literature review that head checks initiation depends on track geometry and loading and operating conditions e.g. cant deficiency, traffic load, vehicle types etc. Hence, it is not optimal to schedule rail inspections for different locations in large railway networks at the same frequencies, as the criticality for each track is different. Thus, maintenance planning and strategy should be adjusted to local conditions (condition based maintenance) if one wants to extend rail service life and minimize maintenance costs designated for rail inspections.

The rail inspections based on the above mentioned inspection techniques are scheduled at most two times a year. Usually, the planning of the inspection frequency is based on the preventive maintenance strategy. In order to shift from preventive to predictive maintenance and detect early head checks on rails the hybrid method is proposed. The physics based part from the hybrid method predicts head checks initiation and the data driven part predicts the evolution of the initiated crack. This does not only inform about the probability of the occurrence of a crack but also gives information about the size of the crack. Furthermore, the hybrid method also takes into account the criticality of the tracks e.g. if the calculated damage index D (using physics based model) for a certain track is larger than another considered track with same amount of traffic but different geometry the track with highest D value is more critical and needs to be inspected first. Thus, the coupling of the prediction model to a rail inspection system are beneficial in the sense that the results from the damage prediction model are utilized for determining the inspection intervals. In this way, infrastructure managers can adjust their rail inspection planning accordingly.

This paper discusses the first step towards the development of the hybrid methodology which involves the implementation and validation of the physics based model. The implementation is based on the head checks prediction model of M. C. Burstow (2003), namely the Whole Life Rail Model (WLRM), and for the validation, a list of rail defects, as detected with EC measurements in different rail measurements campaigns, associated with their kilometre positions is provided. The structure of the paper is as follows: first the head checks formation mechanisms, prediction models and inspection methods are described, followed by a description of the utilized methodology, the correlation analysis and the conclusions.

HEAD CHECKS FORMATION

Head checks are caused by repetitive wheel-rail contact with increasing speed and higher axle loads (Grohmann & Schoech, 2002). They mainly occur on the surface of the rail head and can usually be detected as a cluster of fine cracks with 1 mm to a few cm in between on the gauge side of the outer rail, see Figure 1. They typically develop on curved tracks with radii between 500 and 3000 meters Dollevoet (2010).



(a) (b)

Figure 1: (a) severe head checks (b) schematic view of head checks on the gauge corner of the outer rail (Dollevoet, 2010).

The formation of head checks is a complex phenomenon. The driving mechanism behind this formation is plastic deformation (or ratchetting). Plastic deformation is caused by the shear stress applied to the rails. Due to these stresses, the material locally deforms and hardens (Dollevoet, 2010). The material response due to cyclic loading which eventually leads to plastic deformation and RCF was first studied by Bower and Johnson (1989) and is shown in Figure 2. This figure shows that a material can behave in 4 different ways depending on the stress level. Process A in Figure 2 corresponds to the elastic behavior of the material as the elastic limit is not exceeded by the maximum applied stress. If the yield stress of the material is exceeded in the first cycles, plastic deformation is triggered. This process (process B) is called elastic shakedown because due to residual stresses and strain hardening the material is still able to support the load and acts elastically. Process C represents the plastic shakedown of the material. Here plastic deformation occurs but without strain accumulation. Finally, during process D (ratchetting) strain accumulates at each cycle and the material keeps on deforming plastically (Coleman, 2014; Babette Dirks, 2015; Dollevoet, 2010). This latter process will eventually lead to head check formation.

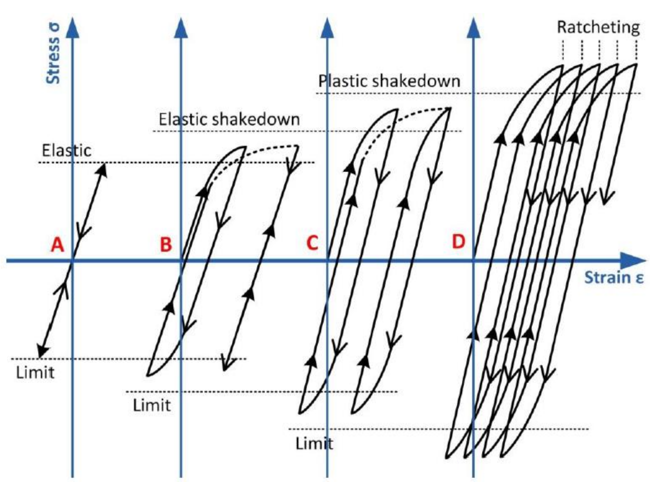


Figure 2: Material response to cyclic loading: (a) purely elastic deformations, (b) elastic shakedown, (c) plastic shakedown and (d) ratchetting (Dollevoet, 2010).

HEAD CHECKS PREDICTION MODELS

This section will give a short description of the most commonly used prediction models for head checks initiation along with their required input parameters. Then, an explanation on the selection of the prediction model for the hybrid methodology is given.

Shakedown diagram

The first damage prediction model of surface initiated cracks like head checks is based on a so called Shakedown diagram. In this diagram the normalized vertical load is plotted against the utilized friction coefficient, see Figure 3. From the shakedown diagram, which is based on the Hertzian contact theory, the effect of shear stress on the rail can be determined (Ekberg et al., 2014). This effect is quantified by the surface fatigue index (FI_{surf}), which functions as a measure of the risk for head check initiation.

$$FI_{surf} \equiv \mu - \frac{2\pi abk}{3\ F_z} \qquad \textit{Equation 1}$$

$FI_{surf} > 0$ corresponds to plastic deformation that leads to head checks initiation. The input for FI_{surf} consists of 3 main parameter groups: the acting loads, contact geometry and material strength. F_z is the normal contact force, k is the yield stress in shear, a and b are the semi-axis of the contact ellipse and μ is the utilized friction coefficient which is determined from the acting tangential (F_x, F_y) and normal (F_z) forces:

$$\mu = \frac{\sqrt{{F_x}^2 + {F_y}^2}}{F_z} \qquad \textit{Equation 2}$$

The input parameters of FI_{surf} related to acting loads and contact geometry are obtained from numerical simulations of vehicle-track interaction which are conducted with multi-body dynamic codes. The shakedown diagram (Figure 3) shows which material response can be expected for certain combinations of normal load and friction coefficient. The surface fatigue index FI_{surf} then quantifies the distance between the working point (WP, the actual situation) and the threshold value (curve BC) for ratchetting. A negative value represents a safe situation. The disadvantage of the shakedown approach focussing on RCF, however, is the deficiency to analyze the interaction between wear and RCF.

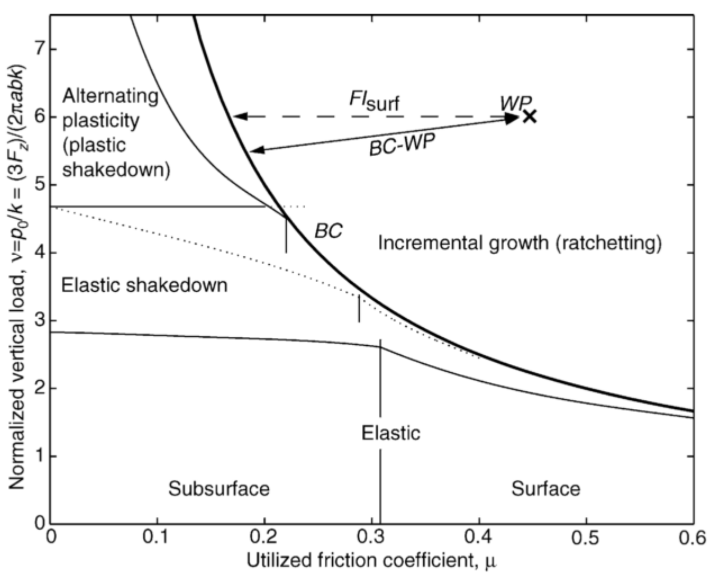


Figure 3: Shakedown diagram (Ekberg et al., 2014).

Dang Van criterion

Head checks occur on the surface of the rail, but can be initiated either on the surface or subsurface. Subsurface cracks are rare, but very dangerous as they are more difficult to detect. In the past, subsurface initiated cracks occurred more often then now. Due to improved steel production techniques defects within the material, that can lead to crack initiation, are rarely found. A model that can predict the initiation of head checks due to subsurface fatigue was developed by Dang Van, Cailletaud, Flavenot, Le Douaron, and Lieurade (1989) and is also known as the Dang Van criterion. The subsurface fatigue index (FI_{sub}) derived from the Dang Van criterion (Ekberg, Kabo, & Andersson, 2002), that determines the risk on subsurface head checks initiation, is as follows:

$$FI_{sub} = \frac{F_z}{4\pi ab}(1 + \mu^2) + a_{DV}\,\sigma_{h,res} \qquad \textit{Equation 3}$$

Where a_{DV} is a material parameter and $\sigma_{h,res}$ is the hydrostatic part of the residual stresses, F_z is the normal contact force, a and b are the semi-axis of the contact ellipse and μ is the traction coefficient. Failure, i.e. head checks initiation, occurs when the subsurface fatigue index exceeds the fatigue limit of the material in pure shear (B. Dirks, Enblom, Ekberg, & Berg, 2015). Similar to FI_{surf}, the input parameters of FI_{sub} are also determined from multi-body dynamic simulations and FI_{sub} is also not able to take into account the interaction between wear and RCF.

<u>Ratchetting model</u>

A RCF damage prediction model that takes the interaction of wear and RCF into account is the ratchetting model. This model was first developed to calculate the amount of wear for ductile materials (Kapoor & Franklin, 2000) and is based on plastic strain accumulation. The material is discretized in N rectangular elements and then for each element the plastic strain ($\Delta\gamma^{ij}$) is accumulated depending on the number of cycles and reaches a certain accumulated plastic strain value (γ^{ij}). Ratchetting failure (wear or head check initiation) occurs when the accumulated plastic strain (γ^{ij}) (based on number of cycles and strain per cycle) for one element exceeds a certain critical strain value (γ_c). The considered element is regarded as a weak element and is removed from the material as wear or stays there and is considered as a crack. Franklin, Chung, and Kapoor (2003) explains in a scheme how to distinguish whether a certain weak element is caused by wear or crack initiation.

The plastic strain for each element (in column i and row j) per cycle is calculated as follows (Franklin et al., 2003):

$$\Delta\gamma^{ij} = C\left(\frac{{\tau_{zx(max)}}^{j}}{{k_{eff}}^{ij}} - 1\right) \qquad \textit{Equation 4}$$

And the accumulated plastic strain is the as follows:

$$\gamma^{ij} = \gamma^{ij} + \Delta\gamma^{ij} \qquad \textit{Equation 5}$$

where ${\tau_{zx(\max)}}^{j}$ is the maximum orthogonal shear stress at the depth of row j, C is a material constant and ${k_{eff}}^{ij}$ is the effective shear yield stress that is calculated as follows:

$${k_{eff}}^{ij} = \beta k_0\, max\left\{1, \sqrt{1 - e^{-\alpha\gamma^{ij}}}\right\} \qquad \textit{Equation 6}$$

and α and β are material constants and k_0 is the initial yield stress.

The input parameters required for the plastic strain calculation are obtained from two processes. The first process is the multi-body dynamic simulation to determine the contact conditions like contact pressure and contact geometry. The second process is the calculation of the maximum orthogonal shear stress which is derived from the subsurface stresses of the material. The subsurface stresses can either be calculated by using Finite Element Methods or by using the semi-analytical half space (Santos, Santos Jr., & Bruni, 2004).

Therefore, the ratchetting model is considered as a time-consuming approach if various scenarios need to be evaluated.

Whole Life Rail Model

Another RCF damage prediction model that takes into account the interaction between wear and RCF is the so called Whole Life Rail Model (WLRM), see Figure 4. M. C. Burstow (2003) developed this model from field observations and numerical modelling to determine the probability of RCF on rails of the R220 steel grade. The majority of the tracks installed in the Netherlands are made of R260Mn steel grade and recently Hiensch and Steenbergen (2018) investigated the WLRM for this type of rail material. They concluded that the WLRM of R260Mn coincides with that of R220.

The WLRM is based on the dissipated energy in the wheel-rail contact. The damage index for the WLRM depends on the wear number $T\gamma$ which can be calculated as:

$$T\gamma = T_x\gamma_x + T_y\gamma_y \qquad \textit{Equation 7}$$

Where T_x and T_y represent the tangential or shear forces and γ_x, γ_y the longitudinal and lateral creepages, respectively.

The wear number ($T\gamma$) is obtained from multi-body dynamic simulations and determines which degradation mechanism (RCF or wear) is dominant on a specific piece of track that is subjected to certain loading conditions. The loading conditions between line section A-B in Figure 4 represent the RCF dominant region, C-D the wear dominant region while B-C corresponds with a combination of RCF and wear. Failure (i.e. head check initiation) occurs when the summation of the damage index D_i (i.e. damage contribution per cycle) reaches unity after N load cycles (Palmgren-Miner rule):

$$D = \sum_{i=1}^{N} D_i \qquad \textit{Equation 8}$$

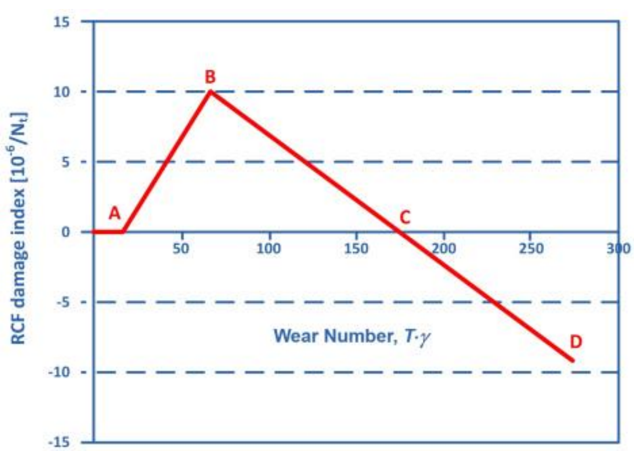


Figure 4: RCF damage index as a function of the wear number (Dollevoet, 2010).

The advantages of the WLRM are: 1) the input parameter required for the damage evaluation for the WLRM is a single variable (e.g. wear number) and 2) the interaction between wear and RCF is incorporated. Therefore, for the present study the WLRM is chosen as the prediction model for head checks initiation.

Zacher (2009) implemented the WLRM for two curves with mixed traffic and found a good correlation between predicted cracks and detected cracks from field observations. However, the disadvantage of this approach is the computation time required to evaluate the different traffic and track geometry scenarios in combination with detailed vehicle models. Furthermore, the practicing engineers do not have complex software like multi-body dynamic codes at their disposal. Therefore, Network Rail developed look-up tables also referred as Vehicle Damage Matrices (VDMs) that present the wear number as function of curve radius and cant deficiency for particular vehicles (M. C. Burstow, Dembosky, Gurule, & Urban, 2008). However, the existing look-up table is limited to a specific type of vehicle.

In this study the relation between wear number and track geometry, vehicle and traffic conditions are obtained from meta-models. By doing so, the limitation of specific vehicle types is eliminated and look-up tables for the wear number as function of several parameters can be developed. The meta-models are derived from physics based models for a relevant set of different scenarios which are created by using the Latin Hypercube Sampling Method and evaluated by the multi-body dynamics software VI-Rail (VI, 2016) (Meghoe, Loendersloot, & Tinga, 2019).

INSPECTION METHODS

Besides the prediction of head checks initiation, the detection of this type of RCF is also important. As mentioned before, there are several rail inspection methods used for damage detection. This section presents a short description of these techniques and mentions which of them will be considered in the present study.

Visual inspection

Visual inspection with the help of photographs and video images is most of the time carried out in parallel with US testing on the same measurement train. The visual inspection data set only contains information about the visual length of the crack and not about the depth of the crack (Boyacioglu, Bevan, & Vickerstaff, 2018). The other limitations of the visual inspection are the number of man-hours required to analyze the data and the poor visibility due to lack of proper lighting and/or contamination (e.g. due to lubrication) (Popović et al., 2015).

Ultrasonic testing

Ultrasonic sound waves are sent in three directions into the rail head, namely 0°, 35° and 70°. Depending on the time that the signal returns to the sensor, the presence of internal cracks is determined. However, the measurements start 4 mm from the rail head, meaning that cracks within the first millimeters (also referred as the dead zone) cannot be detected using US testing. On the other side, head-checks with a depth larger than 4 mm can be spotted (Boyacioglu et al., 2018).

Eddy Current testing

Eddy current testing is based on electromagnetic induction principle where the magnetic field of a coil induces eddy currents when making contact with a conductive material. The presence of defects changes the strength of these currents. The EC testing system uses probes that are only situated at the gauge side of the rail and can only detect defects with a depth up to 3 mm. The depth of defects can only be measured indirectly and the actual crack depth is unknown (Popović et al., 2015). However, video images of the rail head are captured during the measurements and the surface crack length can be computed from this. Hence, using the crack length and a range of crack propagation angles the crack depth is estimated. Furthermore, it can be concluded that between 3 and 4 mm from the railhead no defects can be detected as the EC testing works up to 3 mm of material depth and US testing starts from 4 mm of material depth. EC and US testing results are also combined to find crack depths larger than 5 mm. For the present study the EC measurements are used to validate the prediction model that is part of the hybrid methodology as the confirmation of crack initiation is prioritized over the crack depth.

METHODOLOGY

The goal of the hybrid method is early stage detection of head checks by means of an initiation model developed from physics based models (in this case the WLRM) and an evolution model based on a data-driven framework. In this study the first step towards the hybrid method is taken by validating the output of the physics based model with the EC measurements. If damage index $D \geq 1$ is obtained from the physics based model after a certain amount of traffic has passed the considered track, it is expected that the detection of head checks from the EC measurement data is positive. Furthermore, the output of the physics based model can also be used as an input for the EC measurement planning, as this increases the opportunity of detecting early stage head checks. A schematic overview of this method is presented in Figure 5. Early detection of head checks is beneficial for maintenance actions such as grinding when the crack depth is relatively low. As was mentioned in the introduction, grinding activities are no longer economically beneficial after a certain crack size is reached due to the increasing propagation rate (Jun et al., 2015).

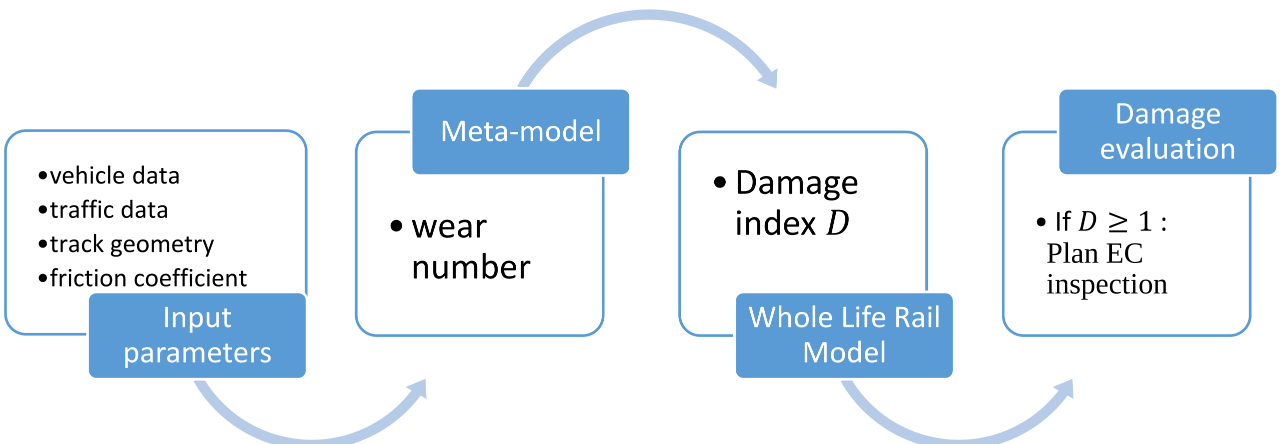


Figure 5: Schematic overview of early stage head checks detection.

From the WLRM subsection it can be concluded that if the wear number for every passing wheel on the track is known, then by accumulating the numbers of wheel passages and using the Palmgren-Miner rule the damage index D can be calculated. The challenge here is then in calculating the wear number for each passing wheel or loading condition as this value is an output of the multi-body dynamics simulations which are computationally expensive. Therefore the multi-body dynamic simulations are replaced by meta-models (Meghoe et al., 2019).

The process to predict head check initiation by using the meta-models is described by the flowchart in Figure 6. The main data sets required for the calculation are:

1. Quo Vadis data: This data set gives information about the number of trains, the number of wheels, the type of train, the velocity of each wheel and the axle load.
2. Rail profile data: This data set gives information about the local condition of the measured rail profiles characterized by the vertical wear depth h.
3. Geographic Information System (GIS) data: This data set gives information about the geometry of the track e.g. rail inclination, location and length of curve radius.

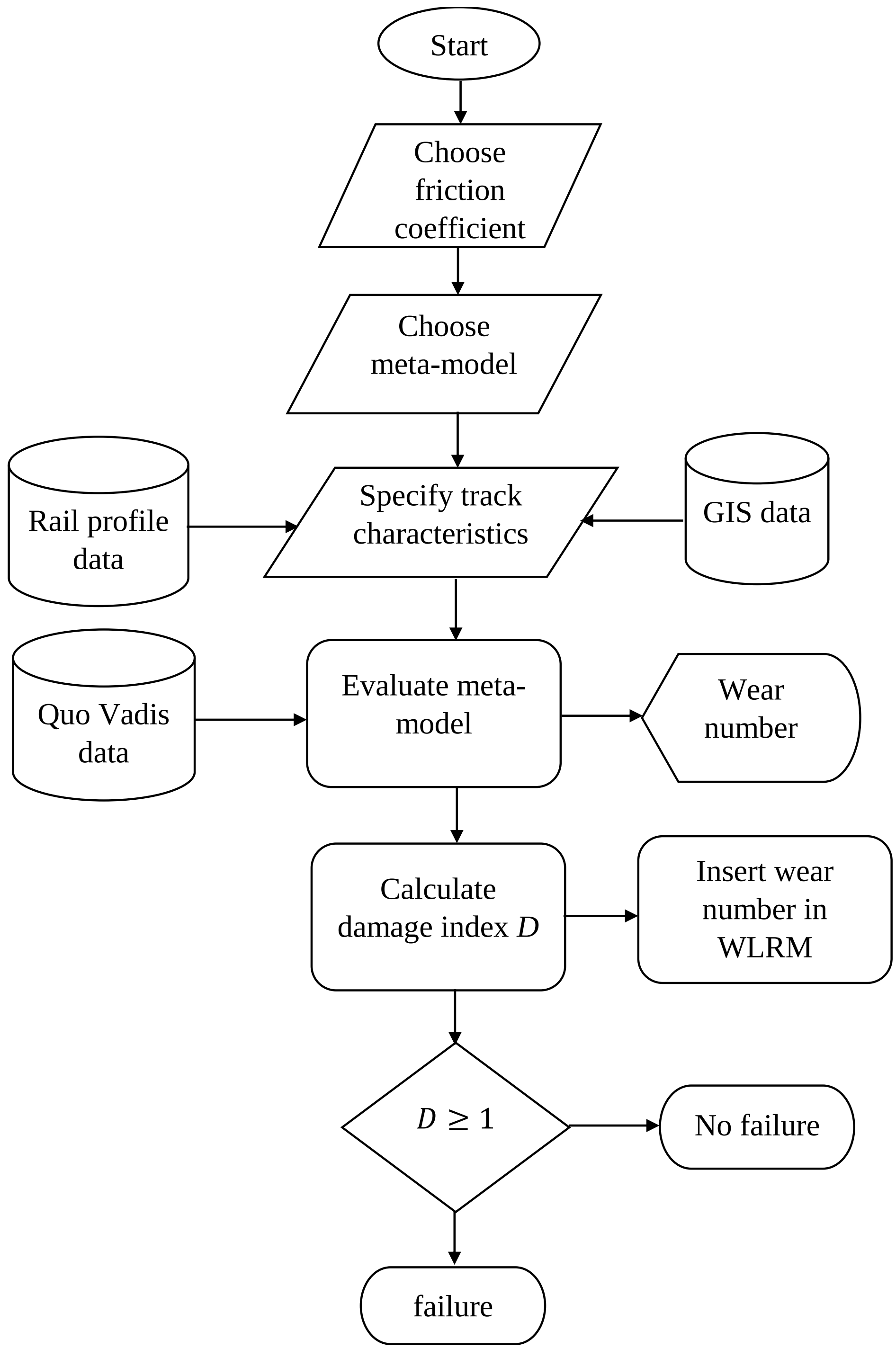


Figure 6: Flowchart describing the head checks prediction process.

CORRELATION ANALYSIS

In order to check to correlation between the prediction model (meta-model) and EC testing a case study from the route between the cities Weesp and Almere has been studied. The track that has been studied is located from km 2.68 until km 3.61 and consists of a curve with radius $R = 1500$ meters, rail cant of 100 mm, rail profile UIC54E1 and steel grade R260Mn. The time period taken into account is from 2nd March 2016 until 3rd March 2017, which corresponds to the time period in between consecutive EC measurements. The crack depth results from these measurement are depicted in Figure 7, where a certain block (i.e. from km 3.6 to 3.8) represents a continuous piece of track with head checks over 200 m. This figure also shows that new head checks occurred within the considered period of one year, and others have extended (both in depth and width). However, between the two measurement periods grinding was conducted on the track and the occurrence of head checks at the same location and that head checks of measurement 2 show greater depths than those of measurement 1 is therefore uncorrelated as a certain crack depth has been removed during the grinding activity.

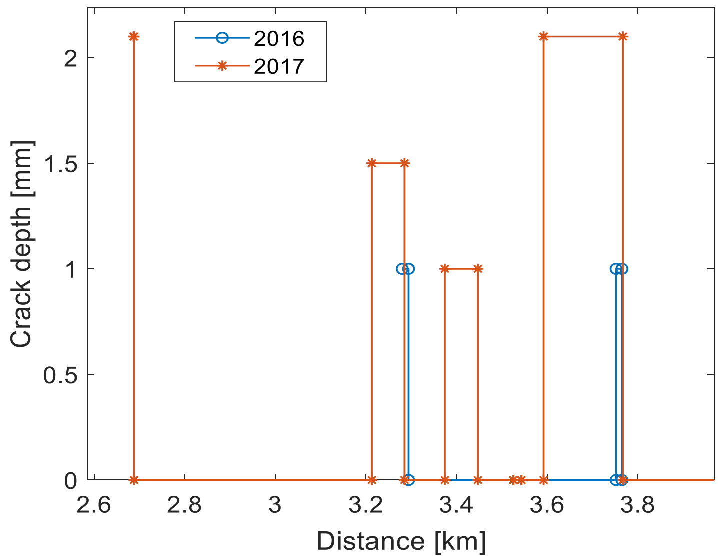


Figure 7: Crack depth measurements from EC testing on 2nd March 2016 and 3rd March 2017.

The initiation of head checks was predicted for rail profiles with a wear depth value h = 1.3 mm (this is the average value for the measured rail profiles from km 2.68 until km 3.61) and friction coefficient of 0.3. The calculated damage indices are presented in Table 1. In between the two measurements the track has been grinded for preventive maintenance purposes on 11th June 2016. From the results presented in Table 1 it can be concluded that worn wheels contribute to the initiation of head checks as expected, because due to the high wear rate of new wheels, the new wheels are worn out faster and can be considered as worn wheels during its remaining lifetime (before the grinding activity of worn wheels).

Furthermore, the calculated damage index $D = 2.41$ for the considered time period of one year suggests that between the two measurements new head checks have occurred on the surface of the rail. This result is confirmed by the EC measurement data, see Figure 7. This figure shows that at several locations new head checks have occurred and for the validation of the prediction model the occurrence of a single new head check is sufficient.

The results in Table 1 also show that the damage index D calculated just before the grinding took place is equal to 0.69. $D < 1$ means that no new head checks have occurred on the track. Hence, in terms of maintenance planning the maintenance activity of grinding took place before it was required as according to the prediction no new head check is initiated. However, grinding is also required to remove the already existing cracks as they have a certain crack growth in a certain period. Therefore, the hybrid method which is the combination of head checks occurrence prediction based on physics based models and crack growth modelling from data-driven framework is beneficial. This hybrid method will not only predict the occurrence of head checks, but will also predict the growth of crack size in time and this information can then be used to schedule the grinding intervals.

Finally, it should be mentioned that the proposed method is not able to predict the head checks locally, but gives a measure for the probability of head check initiation for a given curved track with radius R. Despite this limitation this method can still be considered as a robust methodology for early head check prediction and detection.

Table 1: Calculated damage indices.

	D (new wheels)	*D* (worn wheels)
Period before grinding	0.12	0.69
Period after grinding	0.36	1.71
Entire year	0.51	2.41

CONCLUSION AND FUTURE WORK

The objective of this study was to show how well the prediction of head checks using a physics based model correlate with measurement data, which is the first step of towards the development of the hybrid methodology. From the correlation analysis it can be concluded that the concept of this method is has been proven, because the predicted probability of head check initiation corresponded with the EC testing results. Depending on the results it could also be discussed whether the time in which the grinding activity was performed was required or not. It was then concluded that a hybrid method (crack initiation from physics based model and crack size evolution from data driven framework) will make a more reliable tool for planning the grinding intervals.

Furthermore, in order to prove the reliability of the method, the hit rate defined as the ratio of correct predictions to the total number of observations will be calculated for various case studies. Thereafter, a generic method will be developed which holds true cases when Quo Vadis data is unavailable. For this case Quo Vadis data will be replaced by train schedule information and traffic data expressed in Million Gross Tons (MGT). An analysis upon potential correlation between rail curve, RCF spot location, train speed (acceleration and deaccelerations) and wear number will also be conducted. This can be formulated as an inference system by generating decision rules.

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